York University

Math 1019

Class test #1

February 2, 2006

Last Name:____
First Name:_______________________________________
Student #:____

Question	Points	Marks
1	10	
2	14	
3	13	
4	10	
5	14	
6	12	
7	12	
8	15	

Notes. 1. The duration of the exam is 75 minutes sharp. When the exam is over, stop writing **instantly**, close your testbook and pass it to the person in your row sitting next to the aisle.

2. Since you will have to stop writing once the test is over, write down your name and student number **in ink (pencil is not acceptable)** before you start working.

3. Answer all questions.

4. Present your solutions in a clear and well-organized manner.

5. Have a photo ID and a sessional student card on your desk ready for checkup.

1. (10pts) Are $\neg p \leftrightarrow q$ and $p \leftrightarrow \neg q$ logically equivalent (circle the correct answer)? YES NO
   Justify your answer (e.g., by a truthtable).

2. (14pts) Show that $(\forall x)(P(x) \to Q(x))$ and $(\forall x)P(x) \to (\forall x)Q(x)$ are not logically equivalent. This requires specifying domain of discourse and interpreting predicates P and Q.

3. (13pts) Prove that if m and n are integers and mn is even, then at least one of m and n is even. (You can use the rules of arithmetic as well as the fact that every integer is either odd or even.)

4. (10pts)

   (a) Express each of the following statements using quantifiers. Enter your answer in the space provided.

      i. For every $x \in A$ and for every $y \in A$ distinct from x we have $f(x) \neq f(y)$.

      ____

      ii. For every $y \in B$ there is $x \in A$ such that $f(x) = y$. ______________________________

   (b) Now write negations of the above statements and simplify then using DeMorgan laws.

      i. ______________________________

      ii. ______________________________

5. (14pts) You encounter two people, A and B. You know that each one of them is either a knave (i.e., he always lies) or a knight (i.e., he always tells the truth). They address you in the following way:

   A: We are both knights.

   B: A is a knave.

   Based on this, can you determine whether A is a knight? Can you determine whether B is a knight? Check the appropriate box and then justify your answers.

	knave	knight	don't know
A	☐	☐	☐
B	☐	☐	☐

6. (12pts) Find the value of $\sum_{i=10}^{20}(2i+1)$. You can use the fact that $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$. Show all work.

7. (12pts) Let A, B, and C be arbitrary sets. Show that $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$. (Venn diagrams are not acceptable as a proof. You need to verify that for an arbitrary x, x belongs to $A \setminus (B \cup C)$ iff x belongs to $(A \setminus B) \cap (A \setminus C)$.)

8. (15pts) In each of the cases below, you need to clearly state what is the function and to justify why it has the required properties. (No formal proof is necessary; for example, to show a function is not one-to-one, you only need to exhibit distinct x and y in the domain such that $f(x) = f(y)$.)

   (a) Give an example of a function $f_1 \colon \mathbb{N} \to \mathbb{N}$ that is one-to-one, but not onto.

   (b) Give an example of a function $f_2 \colon \mathbb{N} \to \mathbb{N}$ that is onto, but not one-to-one.

   (c) Give an example of a function $f_3 \colon \mathbb{N} \to \mathbb{N}$ that is neither one-to-one nor onto.