MAT 1330, Fall 2012 Assignment 2
Due Friday October 5, 3:00pm.
Late assignments will not be accepted; nor will unstapled assignments.

Instructor (circle one): Frithjof Lutscher Olga Vasilyeva Catalin Rada Cameron Browne

DGD (circle one): 1 2 3 4

Student Name ______________________ Student Number ______________________

By signing below, you declare that this work was your own and that you have not copied from any other individual or other source.

Signature ______________________

QUESTION 1. Construct a table of values (six terms are enough) to find $\lim_{x\to 0}\left(\frac{x^4+\sin 4x}{40}\right)$.

$x_1 = 0.05$ $f(x_1) = 0.0049668851$
$x_2 = 0.0005$ $f(x_2) \simeq 0.0000499996$
$x_3 = 0.00005$ $f(x_3) \simeq 0.0000049999$
$x \to 0^+$

$x_1' = -0.05$ $f(x_1') \simeq -0.0049668851$
$x_2' = -0.0005$ $f(x_2') \simeq -0.0000499996$
$x_3' = -0.00005$ $f(x_3') \simeq -0.0000049999$
$x \to 0^-$

Answer: $\boxed{0}$

QUESTION 2.

Consider the function $f(x) = \begin{cases} a\sin(\pi x) + b & \text{if } x \le 0 \\ x^2 - a & \text{if } 0 < x \le 1 \\ b\cos(\frac{\pi x}{2}) + a & \text{if } x > 1 \end{cases}$.

Find the values of a and b for which the function is continuous everywhere.

$\lim_{x \to 0^-} [a\sin(\pi x) + b] = b$

$\lim_{x \to 0^+} [x^2 - a] = -a$

$f(x)$ is continuous at $x = 0$ if $\boxed{a = -b}$

$\lim_{x \to 1^-} [x^2 - a] = 1 - a$

$\lim_{x \to 1^+} [b\cos(\frac{\pi x}{2}) + a] = a$

$f(x)$ is continuous at $x = 1$ if $1 - a = a$

$a = \frac{1}{2} \Rightarrow b = -\frac{1}{2}$

QUESTION 3.

Evaluate $\lim_{x \to \infty} \frac{\pi x^5 + 2x^3 + \pi}{x^5 + x^2}$ without using sequences of numerical values for x.

$$\lim_{x \to \infty} \frac{\pi + \frac{2}{x^2} + \frac{\pi}{x^5}}{1 + \frac{1}{x^3}} = \frac{\lim_{x \to \infty} \pi + \lim_{x \to \infty} \frac{2}{x^2} + \lim_{x \to \infty} \frac{\pi}{x^5}}{\lim_{x \to \infty} 1 + \lim_{x \to \infty} \frac{1}{x^3}} = \pi$$

QUESTION 4.

Evaluate $\lim_{x \to 0} \frac{\sqrt{2 - x} - \sqrt{2}}{2x}$ without using sequences of numerical values for x.

$$\lim_{x \to 0} \frac{(\sqrt{2-x} - \sqrt{2})(\sqrt{2-x} + \sqrt{2})}{2x\,(\sqrt{2-x} + \sqrt{2})} =$$

$(a-b)(a+b) = a^2 - b^2$

$$= \lim_{x \to 0} \frac{2-x-2}{2x(\sqrt{2-x}+\sqrt{2})} =$$

$$= \lim_{x \to 0} \frac{-x}{2x(\sqrt{2-x}+\sqrt{2})} = \lim_{x \to 0} -\frac{1}{2(\sqrt{2-x}+\sqrt{2})} =$$

$$= -\frac{1}{4\sqrt{2}}$$

QUESTION 5.

Let $F(x) = \dfrac{1-x^2}{|x-1|} + |x+1|$.

a) Without using sequences of numerical values for x, find $\lim\limits_{x \to 1^+} F(x)$.

$$|x+1| = \begin{cases} x+1 & \text{if } x+1 \geq 0 \text{ or } x \geq -1 \\ -(x+1) & \text{if } x < -1 \end{cases}$$

$$\lim_{x \to 1^+} \frac{1-x^2}{|x-1|} + |x+1| = \lim_{x \to 1^+} \frac{1-x^2}{x-1} + (x+1) = \frac{(1-x)(1+x)}{x-1} + (x+1) =$$

$$= \lim_{x \to 1^+} \left[-(1+x) + (x+1) \right] = 0$$

b) Without using sequences of numerical values for x, find $\lim\limits_{x \to 1^-} F(x)$.

$$\lim_{x \to 1^-} \left[\frac{1-x^2}{|x-1|} + |x+1| \right] = \lim_{x \to 1^-} \left[\frac{1-x^2}{-(x-1)} + (x+1) \right] =$$

$$= \lim_{x \to 1^-} \left[\frac{(1-x)(1+x)}{(1-x)} + (x+1) \right] = \lim_{x \to 1^-} 2(x+1) = 4$$

c) Does $\lim_{x \to 1} F(x)$ exist? Answer: $\boxed{\text{No}}$

Justify your answer:

$$\lim_{x \to 1^+} F(x) \neq \lim_{x \to 1^-} F(x)$$