UNIVERSITY OF TORONTO
Faculty of Arts and Science
APRIL 2012 EXAMINATIONS
MAT135H1S
Calculus I(A)
Duration - 3 hours

(Please PRINT full name and UNDERLINE surname)

NAME OF STUDENT: _____

STUDENT NO.: _____

SIGNATURE OF STUDENT: _____

(in INK or BALL-POINT PEN)

TUTORIAL SESSION (e.g., T0101A, etc.): _____

TUTORIAL TIME (e.g., M4, R4, etc.): _____

NAME OF YOUR TA: _____

ANSWER BOX	
1.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
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14.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
15.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

NOTE:

1. Before you start, check that this test has 16 pages. There are **NO** blank pages.
2. This test has two parts:

   PART A [40 marks]: 6 written questions

   PART B [60 marks]: 15 multiple choice questions

   Answers to both PART A and PART B are to be given in this booklet.

   Answers to Part B are to be indicated in the ANSWER BOX.

   No computer cards will be used.
3. No aids allowed. No calculators!

DO NOT TEAR OUT ANY PAGES.

FOR MARKERS ONLY								
QUESTION	A1	A2	A3	A4	A5	A6	PART B	TOTAL
MARKS	/6	/8	/7	/6	/6	/7	/60	/100

PART A [40 marks]

Please read carefully:

Present your complete solutions to the following questions in the spaces provided, in a neat and logical fashion, showing all your computations and justifications. **Any answer in PART A without proper justification may receive little or no credit.** Use the back of each page for rough work only. If you must continue your formal solution on the back of a page, you should indicate clearly, in LARGE letters "SOLUTION CONTINUED ON THE BACK OF PAGE". In this case, you may get credit for what you write on the back of that page, but you may also be penalized for mistakes on the back of that page.

MARKS FOR EACH QUESTION ARE INDICATED BY *[ ]*.

DO NOT TEAR OUT ANY PAGES.

1. Find the value of $\lim_{x \to 0} \left(1 + x^2 + x^3\right)^{1/x^2}$.

[6]

2. For this question, simplify your final answers as much as possible.

(a) Find $f'(0)$ if $f(x) = 3^x + 3^{-x}$.

[4]

(b) Find $f'\left(\frac{\pi}{4}\right)$ if $f(x) = \ln\left(\sec x + \tan x\right)$.

[4]

3. Find the tangent line to the curve

$$y + \tan(e^y - x) = x^4 - 1$$

at the point $(1, 0)$.

[7]

4. Each side of a cube is increasing at a rate of 5 cm/s. At what rate is the volume of the cube increasing when the volume of the cube is 8 cm^3?

[6]

5. A bacteria culture initially contains 500 cells. After 1 hour, the bacteria population has grown to 1000. Assuming exponential growth, what will be the population after 3 hours?

[6]

6. Find the dimensions of a rectangle with perimeter 100 cm whose area is as large as possible.

[7]

PART B [60 marks]

Please read carefully:

PART B consists of 15 multiple-choice questions, each of which has exactly one correct answer. Indicate your answer to each question **by completely filling in the appropriate circle in the ANSWER BOX on the front page with a dark pencil.**

MARKING SCHEME: 4 marks for a correct answer, 0 for no answer or a wrong answer. You are not required to justify your answers in PART B. Note that for PART B, **only your final answers (as indicated by the circles you darken) count; your computations and answers indicated elsewhere will NOT count.** If there is any discrepancy between your answers on the front page and the answers you indicate on the inside page, the circles you darken on the front page will be regarded as your final answer.

ADVICE: Once you have done a question, you should indicate your answer on the front page immediately. Do not wait until the end of the examination to transfer your answers to the front page.

DO NOT TEAR OUT ANY PAGES.

1. Find the value of $\lim\limits_{x\to\infty} \dfrac{2x^5+5x^3+1}{x+x^2-6x^5}$.

(A) 2

(B) $-\dfrac{1}{3}$

(C) $-\dfrac{1}{6}$

(D) -6

(E) ∞

2. Find the value of $\lim\limits_{x\to 1} \dfrac{\sqrt{x}-1}{x^2-1}$.

(A) 1

(B) $\dfrac{1}{2}$

(C) $\dfrac{2}{3}$

(D) $\dfrac{1}{4}$

(E) ∞

INDICATE YOUR ANSWERS ON THE FRONT PAGE.
Penalty for not doing so is minus 5 marks!

3. Let $f(x) = \dfrac{x^3 - 1}{2x^3 + 1}$. Then the inverse function $f^{-1}(x) =$

Ⓐ $\sqrt[3]{\dfrac{x+3}{3+2x}}$

Ⓑ $\sqrt[3]{\dfrac{x-1}{1+x}}$

Ⓒ $\sqrt[3]{\dfrac{x-4}{3-x}}$

Ⓓ $\sqrt[3]{\dfrac{x+2}{3-5x}}$

Ⓔ $\sqrt[3]{\dfrac{x+1}{1-2x}}$

4. A point P on the curve $y = x^2 + 2x - 4$ is such that the tangent line at P is parallel to the line $y = 4x + 11$. Find the y-coordinate of the point P.

Ⓐ 1

Ⓑ 3

Ⓒ -1

Ⓓ -2

Ⓔ 2

INDICATE YOUR ANSWERS ON THE FRONT PAGE.
Penalty for not doing so is minus 5 marks!

5. Let $f(x) = \arctan(2x)$. Then $f'(1) =$

(A) $\frac{1}{3}$
(B) $\frac{5}{3}$
(C) $\frac{1}{5}$
(D) $\frac{2}{5}$
(E) $\frac{2}{3}$

6. Let $f(x) = \sqrt{x+1}$ on $[0, 3]$. Find the number c that satisfies the conclusion of the Mean Value Theorem for f on $[0, 3]$.

(A) $\frac{5}{4}$
(B) $\frac{1}{3}$
(C) $\frac{5}{3}$
(D) $\frac{1}{4}$
(E) $\frac{4}{3}$

INDICATE YOUR ANSWERS ON THE FRONT PAGE.
Penalty for not doing so is minus 5 marks!

7. Let $f(x) = x^2 - 2x + 7$ on $[-1, 2]$. If M is the absolute maximum value of f on $[-1, 2]$ and m is the absolute minimum value of f on $[-1, 2]$, then $M + m =$

(A) 17
(B) 7
(C) 16
(D) 10
(E) 13

8. The graph of $y = 4x^3 - 15x^2 - 18x + 6$ has a local minimum at $x =$

(A) -1
(B) 3
(C) $-\frac{1}{2}$
(D) $\frac{3}{2}$
(E) 1

INDICATE YOUR ANSWERS ON THE FRONT PAGE.
Penalty for not doing so is minus 5 marks!

9. The graph of $y = \frac{1}{2}x^4 - x^3 - 36x^2 + 24x + 42$ is concave down on the interval

(A) $(-4, 3)$
(B) $(-\infty, -3)$
(C) $(4, \infty)$
(D) $(-3, 4)$
(E) $(-1, 4)$

10. The graph of $y = \frac{3x^3 + 13x^2 + 19x + 3}{(x+2)^2}$ has a slant asymptote which is the line

(A) $y = 3x - 7$
(B) $y = 3x - 1$
(C) $y = 3x + 7$
(D) $y = 3x$
(E) $y = 3x + 1$

INDICATE YOUR ANSWERS ON THE FRONT PAGE.
Penalty for not doing so is minus 5 marks!

11. The graph of $y = x(3x-1)^{2/3}$ has a point of inflection at $x =$

(A) $\frac{1}{3}$

(B) $\frac{4}{7}$

(C) $\frac{2}{5}$

(D) $\frac{5}{4}$

(E) $\frac{3}{11}$

INDICATE YOUR ANSWERS ON THE FRONT PAGE.
Penalty for not doing so is minus 5 marks!

12. Let $f(x) = \dfrac{(x^2+1)^{5/4}(x^3+e^x)^4}{\sqrt{x^4+2}}$. Then $f'(0) =$

(A) $\sqrt{2}$
(B) $2\sqrt{5}$
(C) $3\sqrt{2}$
(D) $\sqrt{3}$
(E) $2\sqrt{2}$

INDICATE YOUR ANSWERS ON THE FRONT PAGE.
Penalty for not doing so is minus 5 marks!

13. How many vertical asymptotes does the function $f(x) = \dfrac{\sin x}{x(x^2-1)(x^2+2)}$ exactly have?

(A) One
(B) Two
(C) Three
(D) Four
(E) Five

INDICATE YOUR ANSWERS ON THE FRONT PAGE.
Penalty for not doing so is minus 5 marks!

14. Which of the following function(s) is (or are) differentiable at $x = 0$?

I. $f(x) = x|x|$.

II. $g(x) = \begin{cases} \dfrac{x^2}{|x|} & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$

III. $h(x) = \begin{cases} \dfrac{e^x - 1}{x} & \text{if } x \neq 0, \\ 1 & \text{if } x = 0. \end{cases}$

(A) I only.

(B) II only.

(C) I and II only.

(D) I and III only.

(E) I, II and III.

INDICATE YOUR ANSWERS ON THE FRONT PAGE.
Penalty for not doing so is minus 5 marks!

15. Find the value of $\lim_{x \to 0} \dfrac{\cos x - \sqrt{x^2+1}}{x \sin x}$.

(A) 1

(B) 0

(C) -1

(D) $-\dfrac{1}{2}$

(E) ∞