Code:1671

FACULTY OF ARTS AND SCIENCE
University of Toronto
FINAL EXAMINATIONS, DECEMBER 2011
MAT135H1F
Calculus I(A)
Duration — 3 hours

PLEASE HAND IN

NAME
*(Please PRINT full name, and UNDERLINE surname):*_____

STUDENT NO:_____

SIGNATURE OF STUDENT:_____

This exam has two Parts:

PART A: 7 questions (52 marks).
PART B: 16 multiple choice questions (48 marks).

Indicate your answer to each multiple-choice question in PART B by completely filling in the appropriate circle in the ANSWER BOX on this front page. (Use a dark pencil!)

1. Before you start, check that this test has 17 pages.
2. No aids allowed. **NO CALCULATORS!**
3. **DO NOT TEAR OUT THIS PAGE OR ANY OTHER PAGE.**
4. **COMPUTER CARDS AND ANSWER BOOKS WILL NOT BE USED. NO SCRAP PAPER.**

FOR	MARKERS	ONLY
A1		/7
A2		/7
A3		/7
A4		/7
A5		/8
A6		/8
A7		/8
PART B		/48
TOTAL		/100

ANSWER BOX FOR PART B	
1.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
2.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
3.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
4.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
5.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
6.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
7.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
8.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
9.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
10.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
11.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
12.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
13.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
14.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
15.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
16.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

PART A *[52 marks]*

Answer all questions in PART A in spaces provided. Show all your work for PART A. Any answer in PART A without proper justification may recieve little or no credit. Use the back of each page for rough work. Marks for each question in PART A are indicated by *[ ]*.
DO NOT TEAR OUT ANY PAGES

1. Evaluate $\lim_{x \to 0} \dfrac{e^x - \cos x - x}{1 + 2x - e^{2x}}$

[7]

2. Find the line passing through the point $(0, 1)$ and tangent to the curve $y = \frac{1}{x}$ at some point.

[7]

3. Find $\frac{dy}{dx}$ if $y = x^{\sin x}$

[7]

4. If $y^3 + xy - x^2 = 5$, find the value of $\frac{dy}{dx}$ at the point where $x = 3, y = 2$.

[7]

5. The length of a rectangle is decreasing at 2 cm/sec, while the width is increasing at 5 cm/sec. At the moment when the length is 15 cm and the width is 20 cm, at what rate is the area of the rectangle changing?

[8]

6. A rectangular box has a square base and an open top (i.e., no top). If the total outer surface area of the box is to be 432 sq cm, what will be the largest possible volume of the box? Remember to justify that your answer is indeed a maximum.

[8]

7. **NOTE: This is a hard problem and will be marked extremely strictly. Very little or no credit will be given unless your solution is completely correct.**

Evaluate $\lim_{x\to\infty}(\sqrt[3]{x^3+x^2}-\sqrt[3]{x^3-x^2})$. Remember to fully justify your answer.

[8]

PART B *[48 marks]*

16 multiple-choice questions

PLEASE READ CAREFULLY: Each of the following multiple-choice questions has exactly one correct answer. **Indicate your answer to each question by completely filling in the appropriate circle in the ANSWER BOX on the front page. Use a dark pencil.**

MARKING SCHEME: 3 marks for a correct answer,
0 for no answer, a wrong answer or giving more than one answer.

You are not required to justify your answers in PART B.

NOTE: If there is any discrepancy between the circles you darken on these inside pages and those you darken on the front page, the circles you darken on the front page will be regarded as your final answers. Note that only the circles you darken will count. For PART B, your computations and answers (other than the circles you darken) will NOT count.

WARNING: If you darken the circles on these inside pages but do not darken the circles on the front page, you will still get credit for your correct answers, but there will be a PENALTY of minus 4 marks

YOU MUST NOT TEAR OUT ANY PAGES OF THIS EXAM.

1. Find the value of $\lim_{x\to 0} \dfrac{\sin(2x)\cos(3x)\tan(4x)}{\cos(2x)\sin(3x)\tan(5x)}$

(A) 0
(B) 8/15
(C) 8/5
(D) 4/5
(E) Undefined

2. If $g(0) = 5, g'(0) = 3$, and $f(x) = \frac{g(x)}{e^x}$, then $f'(0) =$

(A) 0
(B) 4
(C) -2
(D) -5
(E) 3

INDICATE YOUR ANSWERS ON THE FRONT PAGE
Penalty for not doing so is MINUS 4 marks!

3. Let

$$f(x) = \begin{cases} kx^2 + 2x & \text{if } x < 2 \\ x^3 - kx & \text{if } x \geq 2 \end{cases}$$

Find the value of the constant k so that f is continuous everywhere.

(A) $1/3$
(B) $-1/4$
(C) $1/2$
(D) $2/3$
(E) $1/5$

4. If $f(x) = \arcsin(\sqrt{x})$, then $f'(\frac{1}{2}) =$

(A) 1
(B) $-1/2$
(C) $\sqrt{2}$
(D) 0
(E) undefined

INDICATE YOUR ANSWERS ON THE FRONT PAGE
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5. The graph of $y = \frac{x^6 - x^3 + 2}{3 - x - 5x^6}$ has a horizontal asymptote. This horizontal asymptote is the line

(A) $y = \frac{1}{3}$
(B) $y = \frac{2}{3}$
(C) $y = -\frac{2}{5}$
(D) $y = \frac{1}{2}$
(E) $y = -\frac{1}{5}$

6. The graph of $f(x) = x^4 + 8x^3$ is concave down on the interval

(A) $(-5, -1)$
(B) $(-4, 0)$
(C) $(0, \infty)$
(D) $(-\infty, -4)$
(E) $(-2, 2)$

INDICATE YOUR ANSWERS ON THE FRONT PAGE
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7. The graph of $f(x) = 4x^5 - 5x^4 - 40x^3 + 7$ has a local max. at $x =$

Ⓐ -2
Ⓑ 0
Ⓒ -3
Ⓓ 4
Ⓔ 3

8. Let $f(x) = \ln(1 + 3x)$ on $[0, 2]$. Find the number c which satisfies the conclusion of the Mean Value Theorem.

Ⓐ $\frac{2}{\ln(7)} - \frac{1}{5}$
Ⓑ $\frac{3}{\ln(7)} - \frac{1}{3}$
Ⓒ $\frac{2}{\ln(7)} - \frac{1}{3}$
Ⓓ $\frac{2}{\ln(7)} - \frac{1}{4}$
Ⓔ $\frac{3}{\ln(7)} - \frac{1}{2}$

INDICATE YOUR ANSWERS ON THE FRONT PAGE
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9. Let $f(x) = 2x^2 - 2x + 3$ on $[-2, 2]$.
Let M be the absolute maximum value of f on $[-2, 2]$ and m be the absolute minimum value of f on $[-2, 2]$. Then $M - m =$

(A) 13
(B) 12
(C) $\frac{23}{2}$
(D) $\frac{25}{2}$
(E) 10

10. At noon, a bacteria culture has 500 bacteria. At 1 p.m., the bacteria population has grown to 600. Assuming exponential growth, what will be the bacteria population at 3 p.m.?

(A) 800
(B) 840
(C) 864
(D) 858
(E) 880

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11. The graph of $y = \frac{3x^2+x+3}{x+1}$ has a vertical asymptote and a slant (i.e., oblique) asymptote. The slant asymptote is the line

(A) $y = 3x$
(B) $y = 3x + 1$
(C) $y = 3x + 2$
(D) $y = 3x + 3$
(E) $y = 3x - 2$

12. Suppose that f is a differentiable function such that for all x, $xf(x) = x + \sin 2x$. Then $f(0) =$

(A) 3
(B) 0
(C) -5
(D) 4
(E) -2

INDICATE YOUR ANSWERS ON THE FRONT PAGE
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13. Suppose that f is a function such that for all x,

$$f''(x) = (x-1)(x+2)^2(x^2-4)(x^2-3x)(x^2-9)^2(x^2-4x+3).$$

How many points of inflection does the graph of f have?

(A) five
(B) two
(C) four
(D) three
(E) more than five

INDICATE YOUR ANSWERS ON THE FRONT PAGE
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14. Let $f(x) = \frac{x^3+5x^2+8x+5}{x^2+4x+4}$.
Find $f^{(75)}(-3)$, i.e., the value of the 75^{th} derivative of f at $x = -3$.

(A) $-(76!)$
(B) $76!$
(C) $77!$
(D) $-(75!)$
(E) $75!$

INDICATE YOUR ANSWERS ON THE FRONT PAGE
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15. The graph of $y = x(x-4)^{2/3}$ has a point of inflection at $x =$

(A) 6
(B) 5
(C) $\frac{11}{2}$
(D) $\frac{16}{3}$
(E) $\frac{24}{5}$

INDICATE YOUR ANSWERS ON THE FRONT PAGE
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16. Find the value of the sum

$$\arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{3}\right).$$

Hint: You can use the trigonometric identity $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$.

(A) $\frac{\pi}{2}$
(B) $\frac{\pi}{3}$
(C) $\frac{\pi}{4}$
(D) $\frac{\pi}{6}$
(E) none of the above