MAT135S – Summer 2012 – FINAL EXAM

Thursday, August 16, 2012
Time: 7:00 pm to 10:00 pm (3 hours)
Instructor: Catalina Anghel

Family Name: ______________________________

Given Name: ______________________________

Student Number: ______________________________

Signature: ______________________________

1. Do not open the exam paper until the instructor asks you to. Then you may check that this test has 16 pages.
2. No aids allowed. NO CALCULATORS.
3. Answer each question in the space provided. You can use the back of each page for rough work. If you want the back of a page marked, clearly indicate this on the front of that page. Do not tear out any pages.
4. This test consists of two parts:
   - PART A: Multiple choice, 11 questions (44 marks)
   - PART B: Long answer, 6 questions (56 marks)

Question	**Mark**
Multiple Choice	/44
Q1	/10
Q2	/10
Q3	/5
Q4	/6
Q5	/10
Q6	/15
Total	/100

PART A: Multiple Choice. *[44 marks]* Circle or shade in the correct answer for each question. Each question is worth 4 marks.

1. Find $\lim_{x\to 0}\left(\frac{\sin 3x}{2x} - \frac{3x}{\sin 4x}\right)$

   (a) 0

   (b) $\frac{3}{4}$

   (c) $-\frac{1}{12}$

   (d) 1

   (e) The limit does not exist.

2. Find $\lim_{x\to\infty} \frac{x^2+1}{e^x}$

   (a) 0

   (b) 1

   (c) 2

   (d) ∞

   (e) e

3. Find $\lim_{x \to 1} \dfrac{1 - \sin\left(\frac{\pi}{2} t\right)}{1 - \cos(1 - t)}$

(a) 0

(b) $\dfrac{\pi^2}{4}$

(c) $-\dfrac{\pi^2}{2}$

(d) π

(e) The limit does not exist.

4. If $g(x) = \dfrac{1 + \ln x}{1 + x}$, then $g'(1) =$

(a) $\dfrac{3}{4}$

(b) $\ln 2$

(c) $\dfrac{1}{4}$

(d) 1

(e) $\dfrac{1}{2}$

5. The equation of the tangent to the curve $4y = x^2$ at the point on the curve where $x = -2$ is

(a) $x + y - 3 = 0$

(b) $x + y - 1 = 0$

(c) $y - 1 = 2x(x + 2)$

(d) $x + y + 1 = 0$

(e) $x - y + 3 = 0$

6. Let $f(x) = x^3 - 3x^2 + 1$. Let M be the absolute maximum of $f(x)$ on the interval $\left[-\frac{1}{2}, 3\right]$ and let m be the absolute minimum of $f(x)$ on the interval $\left[-\frac{1}{2}, 3\right]$. Then $M + m =$

(a) 20

(b) 14

(c) $17\frac{1}{8}$

(d) 18

(e) -2

7. A spherical balloon is being filled with helium at a rate of 100 cm^3/s. At what rate is the radius of the balloon increasing when the radius is 10 cm? The volume of a sphere is $\frac{4}{3}\pi r^3$.

   (a) $\dfrac{1}{4\pi}$

   (b) $\dfrac{10}{3\pi}$

   (c) $\dfrac{1}{100\pi}$

   (d) $\dfrac{100}{\pi}$

   (e) $\dfrac{4}{3\pi}$

8. The product of two positive numbers is 144. What is the minimum value of their sum?

   (a) 26
   (b) 18
   (c) 24
   (d) 20
   (e) 22

9. Let $f(x) = (\ln x)^{\cos x}$. Then $f'(x) =$

(a) $(\cos x)(\ln x)^{\cos x - 1} \cdot \dfrac{1}{x}$

(b) $(\ln x)^{\cos x} \ln(\ln x)$

(c) $-(\ln x)^{\cos x} \left(\ln(\ln x) \sin x + \dfrac{\cos x}{x}\right)$

(d) $(\ln x)^{\cos x} \left(-\sin x \ln(\ln x) + \dfrac{\cos x}{x \ln x}\right)$

(e) The derivative is not defined.

10. If $y = y(x)$ satisfies $y^x = e^y$, then when $(x, y) = (2\sqrt{e}, \sqrt{e})$, the derivative $\dfrac{dy}{dx} =$

(a) $-\dfrac{1}{2}$

(b) $-\dfrac{1}{\sqrt{e}}$

(c) 2

(d) $\sqrt{e}$

(e) $\dfrac{\sqrt{e}}{2}$

11. Let

$$f(x) = \begin{cases} \frac{\ln(2+x)-\ln 2}{x} & \text{when } x > 0 \\ a & \text{when } x = 0 \\ e^{x-b} & \text{when } x < 0 \end{cases}$$

If f is continuous at $x = 0$, then

(a) $a = \ln 2$ and $b = 0$

(b) $a = 1$ and $b = 1$

(c) $a = \dfrac{1}{2}$ and $b = 2$

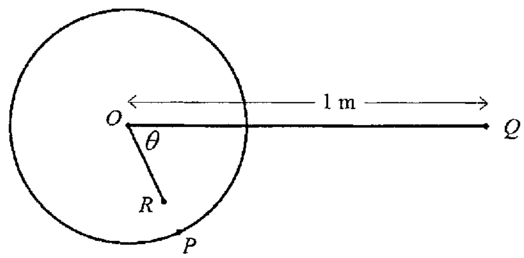
(d) $a = \ln 2$ and $b = \dfrac{1}{2}$

(e) $a = \dfrac{1}{2}$ and $b = \ln 2$

PART B: Long-Answer Questions. Present your complete solutions in the spaces provided, in a neat and logical fashion, showing all your computations and justifications. **Any answer in this part without justification may receive little or no credit.**

1. *[10 marks]* The top and bottom margins of a poster are each 4 cm and the side margins are each 2 cm. If the area of printed material on the poster is fixed at 50 cm^2, find the dimensions of the poster with the smallest area. Check that the value obtained is indeed a minimum.

2. *[10 marks]* The circle below represents a record which is rotating clockwise at 100/3 revolutions per minute. A bug is walking away from the centre of the record directly toward point P on the rim of the record at 1 cm/s. When the bug is at position R, 10 cm from O, the angle θ is $\pi/4$ radians. Find the rate at which the distance from the bug to the fixed point Q is changing when the bug is at R. Simplify as much as possible but your final expression may contain square roots or well-known constants (such as π or e).



3. *[5 marks]* Use logarithmic differentiation to find $\frac{dy}{dx}$ in terms of x given $y = (3x + 5)^x$.

4. *[6 marks]* A bacteria culture initially contains 200 cells. After 1 hour, the bacteria population has grown to 400 cells. Assuming exponential growth, what will be the population after 4 hours? Simplify your answer as much as possible.

5. (a) *[3 marks]* State the Mean Value Theorem.

(b) *[3 marks]* A car is traveling along a straight line. At 2:00 pm the car's speedometer reads 30 km/h. At 2:10 pm it reads 50 km/h. Show that at some time between 2:00 and 2:10 the acceleration was exactly 120 km/h^2.

(c) *[4 marks]* Suppose that $f(0) = -3$ and $f'(x) \leq 5$ for all values of x. What is the largest possible value for $f(2)$?

6. Consider the function

$$f(x) = \frac{x^2 - 1}{x^3}.$$

(a) *[3 marks]* Show that $f'(x) = \dfrac{(3 - x^2)}{x^4}$ and $f''(x) = \dfrac{2(x^2 - 6)}{x^5}$.

(b) *[3 marks]* Find the domain of f. Find the x and y intercepts and horizontal and vertical asymptotes, if any.

(c) *[3 marks]* Find the critical points of f and the intervals of increase and decrease. Find any local maximum and/or minimum points.

(d) *[3 marks]* Find the intervals on which f is concave up, the intervals on which f is concave down, and all inflection points.

(e) *[3 marks]* Sketch the graph of f, labeling intercepts, asymptotes, max/min and inflection points.