## APM466 Midterm 2 - March 17, 2010. 9:30 am to 11:00 am

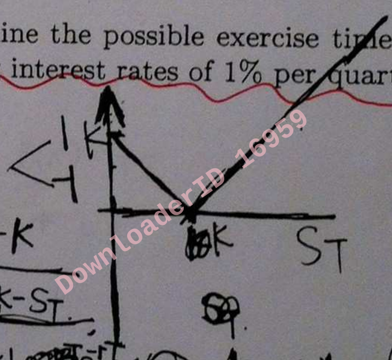
**Two questions of equal value**

**Question 1.** Determine which of the two quantities mentioned below is **larger** (justify your answer):

- A call option price on a stock with a strike price of 1, or a call option price on the same stock with strike price 2.
- A call option price on a stock with volatility 1, or the price of a similar call option on a stock with volatility 2.
- A call option price on a stock *plus* a put option on the same stock with the same strike price, or the price of the stock today (interest rates are strictly positive).

**Question 2.** A stock $S$ is valued at \$1 today. At the end of every quarter, for the next year, it can go up or down by 20%. An 2-down (put) swing option gives the option holder $(\$1 - S)_+$ at the end of every quarter over the next year period (4 quarters altogether), and the option can be exercised twice although not both times at the end of the same quarter.

Find the price of the option today, and determine the possible exercise times when each of the two option exercises will occur, assuming interest rates of 1% per quarter.




$S_T > K$. $S_T$ $S_T - K$

S. $S_T < K$ $S_T$ $K - S_T$.

C+P.

$|S_T - K| e^{-rT} < |S_T - K|$ $|S_T$.

$Ke^{-rT} = p + S - c$.



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① The call with $K=1$ has larger price than the call with $K=2$.
Because the payoff of a call is alwayse $(S-K)^+$ ; when K is lower, the payoff should be higher then the call price should also be higher.

② The call with $\sigma=2$ has larger price than the call with $\sigma=1$.
Because when $\sigma$ is larger, the ~~price of a~~ chance that the stock will do very well or very poorly increases. The owner of a call benefits from price increases but has limited downside risk in the event of price decreases because the most the owner can lose is the price of the option.

③. ~~If $S_T > K$, the payoff will be $S_T - K$.~~
~~If $S_T < K$, the payoff will be $K - S_T$.~~

$$c(t,K,\sigma,r) = S_0 N(d_1) - Ke^{-rT} N(d_2)$$

$$p(t,K,\sigma,r) = Ke^{-rT} N(-d_2) - S_0 N(-d_1) \quad \leftarrow \text{put option price}$$

$$= Ke^{-rT}(1-N(d_2)) - S_0(1-N(d_1)).$$

So the price of call plus put is

$$\pi = S_0 N(d_1) - Ke^{-rT} N(d_2) + Ke^{-rT}(1-N(d_2)) - S_0(1-N(d_1))$$

$$= 2S_0 N(d_1) - S_0 + Ke^{-rT}(1-2N(d_2)).$$

Now the stock price is $S_0$.

$$\text{So } \pi - S_0 = 2S_0 N(d_1) - 2S_0 + Ke^{-rT}(1-2N(d_2))$$

$$= 2S_0\underbrace{(N(d_1)-1)}_{\le 0} + Ke^{-rT}\underbrace{(1-2N(d_2))}_{<0}.$$

So $\pi < S$.
Therefore the price of the stock is larger.

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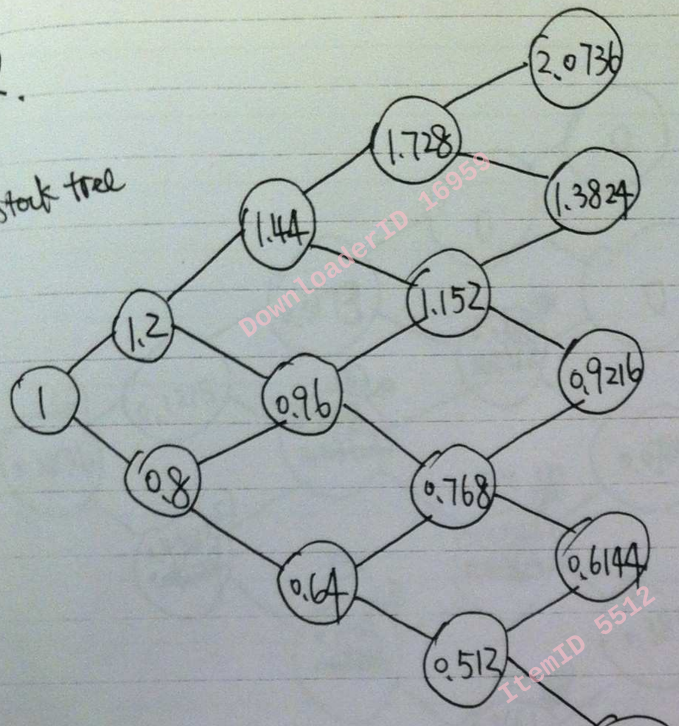
2.

stock tree




current price of underlying asset

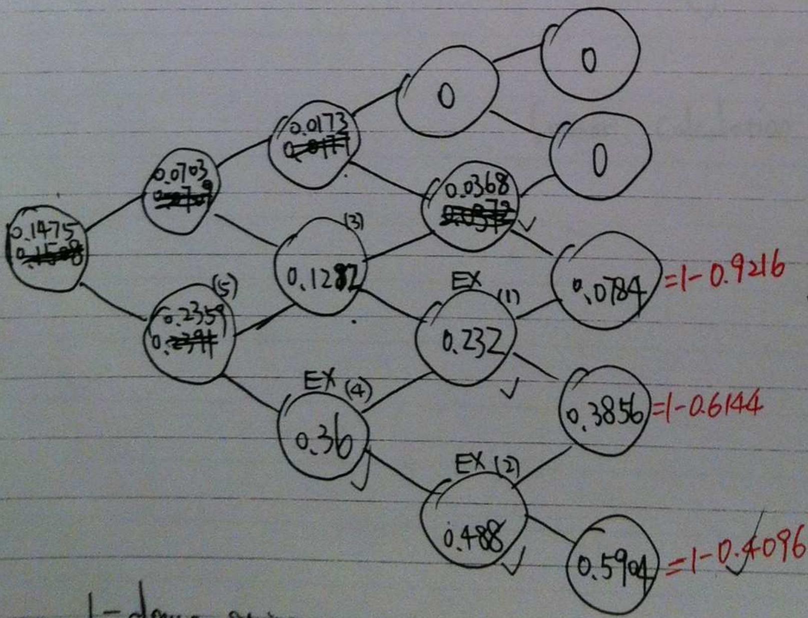
Determine risk-neutral prob. p. $S = (pS_{up} + (1-p)S_{down})e^{-rt}$

$S = (puS + (1-p)dS)e^{-rt}$ $(S=1)$

Now $\boxed{e^{-rt} = e^{-0.01}} = 0.99$

$e^{rt} = pu + d - pd.$

$$p = \frac{e^{rt} - d}{u-d} = \frac{e^{0.01} - 0.8}{1.2 - 0.8} = 0.5251 \quad \therefore q = 0.4749.$$




(1) EX : $1 - 0.768 = 0.232$ ✓
notex : $(0.0784 \times 0.5251 + 0.3856 \times 0.4749) \times 0.99 = 0.22$

(2). EX : $1 - 0.512 = 0.488$ ✓
notex : $(0.3856 \times 0.5251 + 0.5904 \times 0.4$ $= \cancel{0.4829}$ $0.47$

(3) EX : $1 - 0.96 = 0.04$
notex : ~~0.037~~ ~~0.1297~~ 0.1282 ✓

(4) EX : 0.36 ✓
notex : ~~0.3536~~ 0.3501

(5) EX : 0.2
notex : ~~0.2394~~ 0.2359 ✓

1-down swing

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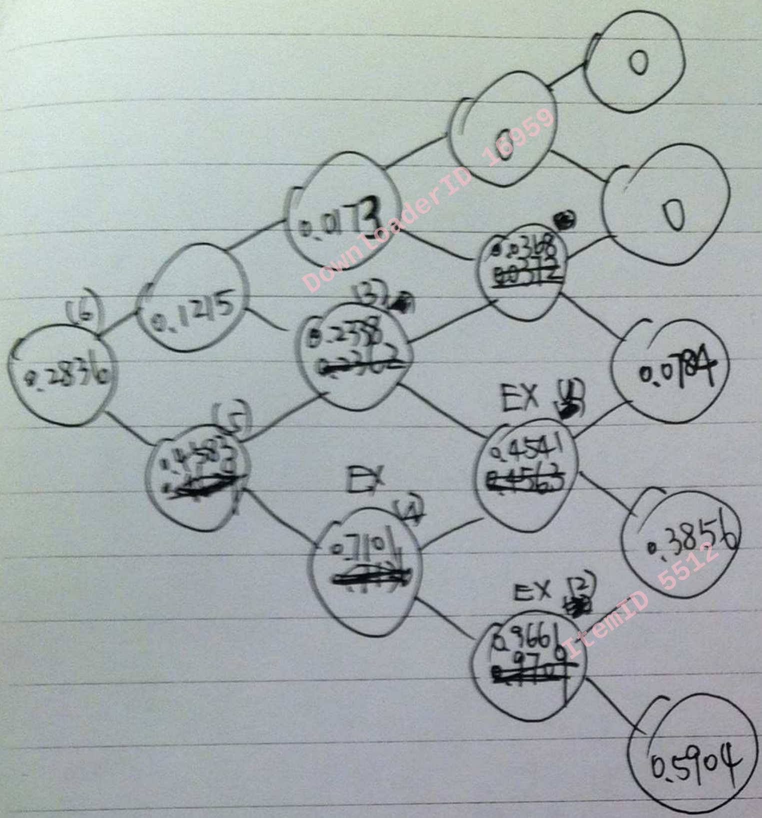


NOT $(1-0.768)+0.232$ since 0.232 is the payoff if we exercise the last one!

~~(1) notex: 0.0372~~

☆(1) EX: $(1-0.768)+(0.0784\times0.5251+0.3856\times0.4749)\,\cancel{0.4563}\times0.99=0.4541$

notex: $\cancel{0.2243}\ 0.2221$

(2) EX: $0.488+\cancel{0.4829}\ 0.4781=\cancel{0.9709}\ 0.9661$

notex: $0.4829$

(3) EX: $0.04+(0.0372\times0.5251+0.232\times0.4749)\times0.99=0.1697$

notex: $(0.0372\times0.5251+0.4563\times0.4749)\times0.99=0.2362$

(4) EX: $0.36+(0.232\times0.5251+0.488\times0.4749)\times0.99=\cancel{0.7136}\ 0.7101$

notex: $(0.4563\times0.5251+0.9709\times0.4749)\times0.99=0.7007$

(5) EX: $0.2+(0.1297\times0.5251+0.36\times0.4749)\times0.99=\cancel{0.4391}\ 0.4359$

notex: $(0.2362\times0.5251+0.7136\times0.4\ldots)\times0.99=\cancel{0.4629}\ 0.4583$

(6) EX: $0+(0.0709\times0.5251+0.2391\times0.4\ldots)\times0.99=0.1508$

notex: $(0.1215\times0.5251+0.4629\times0.474\ldots)\times0.99=0.2836$

(minor Calculation errors)

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