Find parametric representations:

(i) $y^2 = x \quad \rightarrow \quad \vec{r}(t) = (t^2, t)$

or $(t, \sqrt{t})$

(ii) $y = e^{x+y}$

$= e^x e^y$

or $ye^{-y} = e^x \quad \rightarrow \quad \vec{r}(t) = (\ln(te^{-t}), t)$

$\ln(ye^{-y}) = x$

(iii) $x = 2 \quad \rightarrow \quad \vec{r}(t) = (2, t)$

Find $y(x)$:

(i) $\vec{r}(t) = (t, t^2+4)$

$$x(t) = t \rightarrow x^2 = t^2$$
$$y(t) = t^2 + 4$$

$$\boxed{\therefore\ y(x) = x^2 + 4}$$

(ii) $\vec{r}(t) = (e^t, e^{t+1})$

$$x(t) = e^t$$
$$y(t) = e^{t+1} = e \cdot e^t$$

$$\boxed{\therefore\ y(x) = e x}$$

Extra Questions #4

1

9.7 #26

$$z(x,y) = 2000 - 4x^2 - y^2$$

or $\underbrace{4x^2 + y^2}_{\text{ellipse}} = 2000 - z$

re-writing: $\dfrac{x^2}{\frac{(2000-z)}{4}} + \dfrac{y^2}{(2000-z)} = 1$

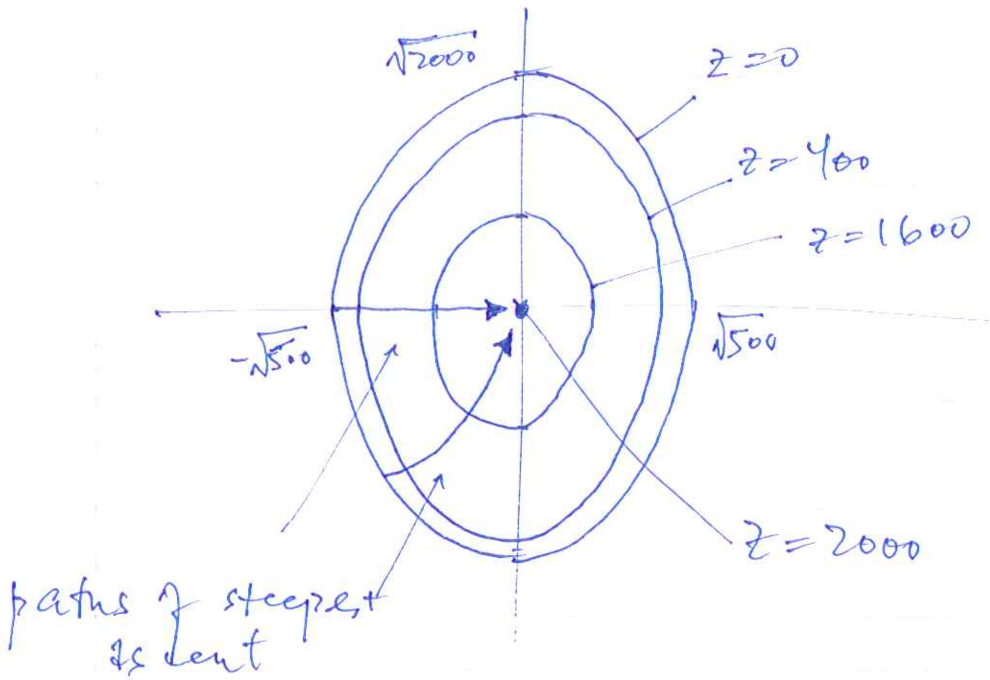
contours

$z = 0$ (sea-level) : $\dfrac{x^2}{500} + \dfrac{y^2}{2000} = 1$

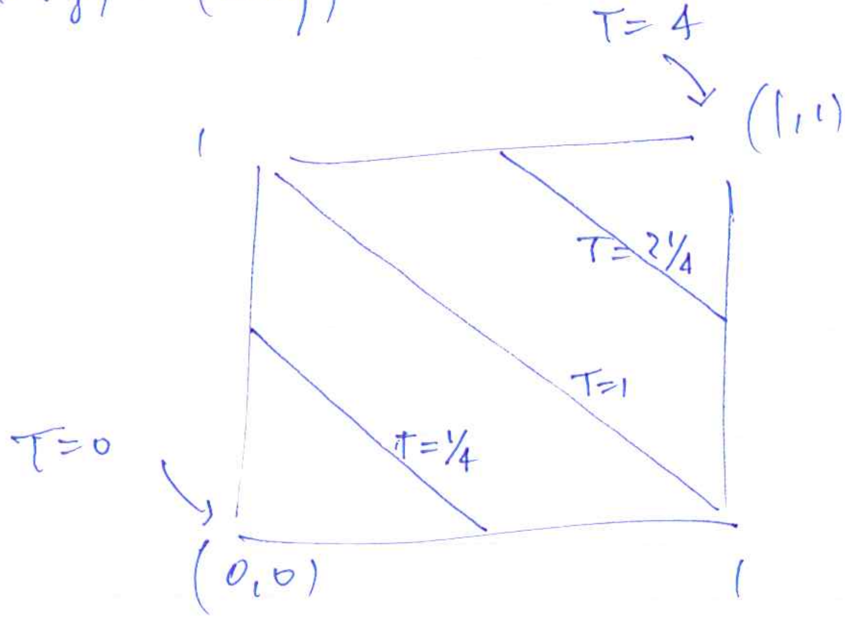
$z = 400$: $\dfrac{x^2}{400} + \dfrac{y^2}{1600} = 1$

$z = 1600$: $\dfrac{x^2}{100} + \dfrac{y^2}{400} = 1$

$z = 2000$: $x^2 + y^2 = 0$ } top of the mountain

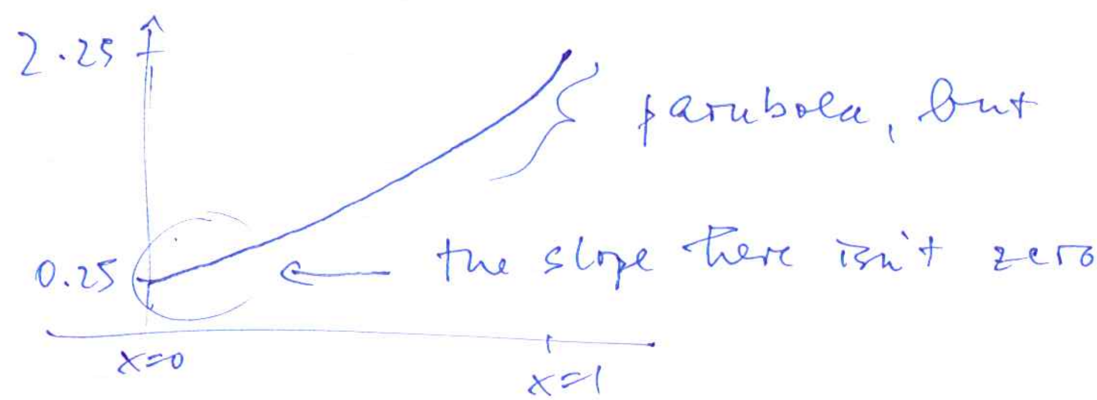

$\sqrt{2000}$
$z=0$
$z=400$
$z=1600$
$-\sqrt{500}$
$\sqrt{500}$
$z=2000$
paths of steepest ascent

$T(x,y) = (x+y)^2$



- plot T horizontally & vertically through $(0.5,\ 0.5)$

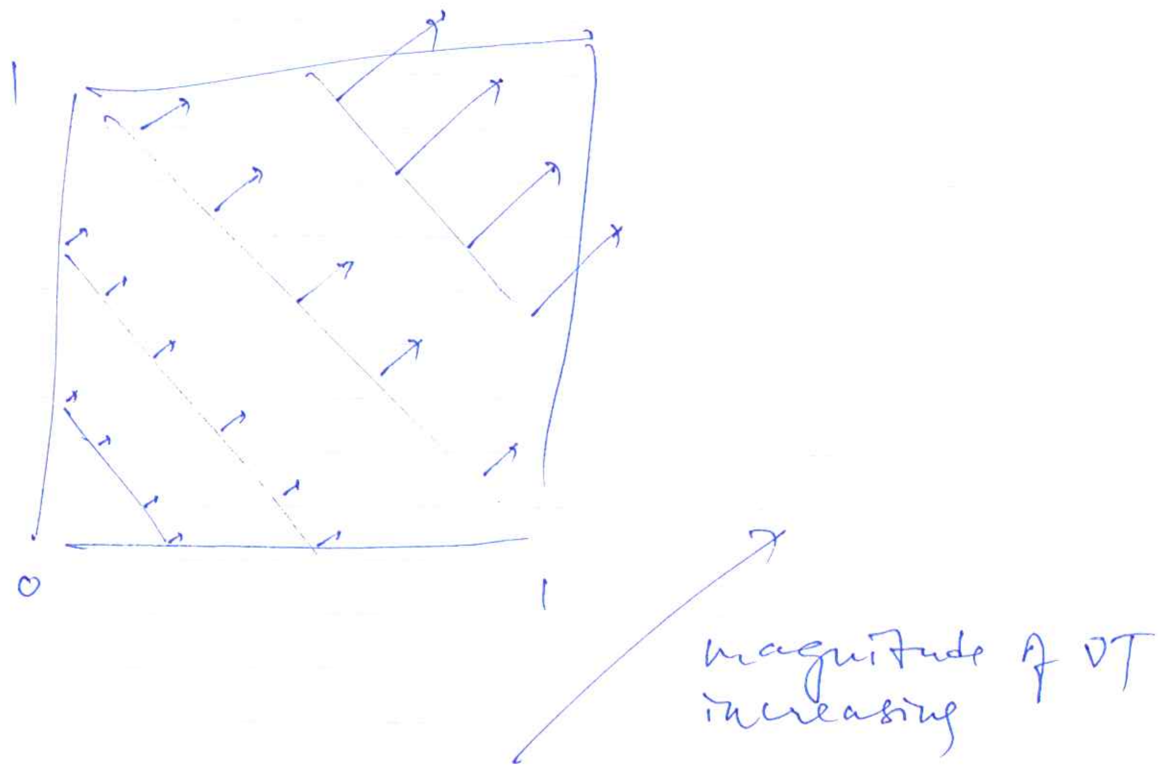
Horizontally: $y = 0.5 \rightarrow T = (x + 0.5)^2$



Vertically: exactly the same as horizontally

$$\nabla T = \text{grad } T = \frac{\partial T}{\partial x}\,\hat{i} + \frac{\partial T}{\partial y}\,\hat{j}$$

$$= (2x+2y)\,\hat{i} + (2x+2y)\,\hat{j}$$



- how does $|\nabla T|$ vary along the diagonal?

along the diagonal: $x = y = t$ (parameter)

$$\therefore\ |\nabla T| = \sqrt{(2x+2y)^2 + (2x+2y)^2}$$

$$= \sqrt{(4t)^2 + (4t)^2} = 4\sqrt{2}\,t$$

at $(0.5, 0.5)$ evaluate Directional Derivatives

(i) $\hat{i}$ $\qquad D_{\hat{i}}T = \underbrace{(1,0)}_{\hat{i}} \cdot \underbrace{(2,2)}_{\substack{\nabla T \text{ at} \\ (0.5,\,0.5)}}$

$$= 2 \; \underbrace{[^\circ/L]}_{\text{degrees per unit length}}$$

---

an alternative?

$$T(x, 0.5) = \underbrace{(x + 0.5)^2}_{\substack{\text{variation of temperature} \\ \text{horizontally through} \\ (0.5,\,0.5)}}$$

$$\frac{dT}{dx} = 2(x + 0.5)$$

at $x = 0.5$, $\frac{dT}{dx} = 2 \quad (= D_{\hat{i}}T)$

(ii) $\hat{j}$: $\qquad D_{\hat{j}} T = (0,1) \cdot (2,2)$

$= 2 \; [°/L]$

(iii) $\underbrace{\hat{i} - \hat{j}}_{\text{along an isotherm,}}$: $\qquad D_{\hat{i}-\hat{j}} T = \dfrac{(1,-1) \cdot (2,-2)}{\sqrt{1^2+1^2}}$

$= 0$

temperature doesn't vary

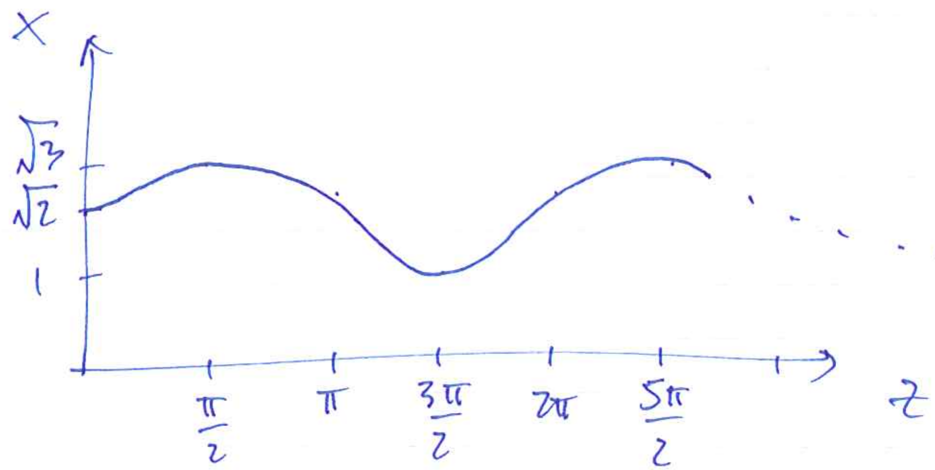
circles of varying radius

$$x^2+y^2 = \sin z + 2$$

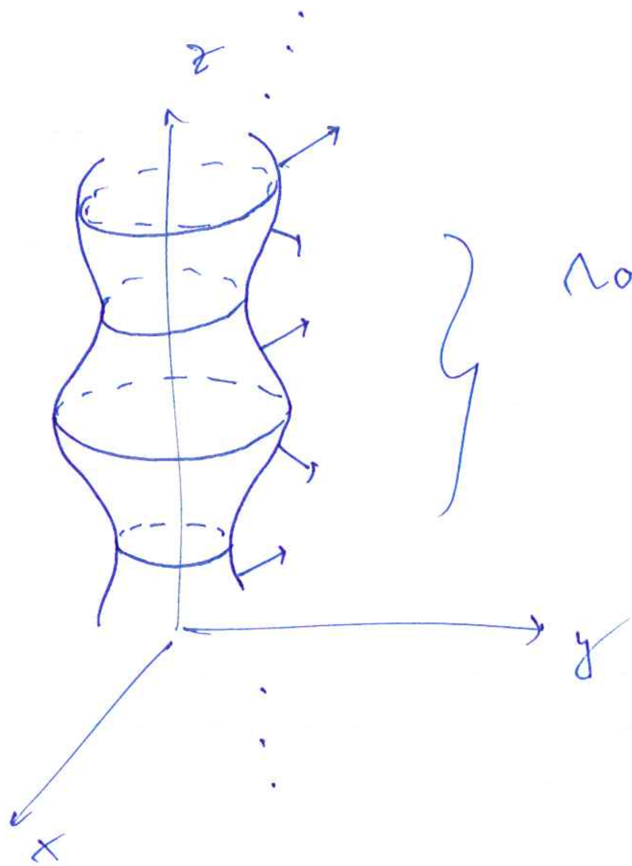
- start by looking at $x-z$ plane (i.e. $y=0$)

$$x^2 = \sin z + 2$$

$$\text{or } x = \sqrt{\sin z + 2}$$



- what we really have is a wavy tube:



rough sketch

scalar function? $f(x,y,z)$

$$= x^2 + y^2 - \sin z$$

$\therefore$ my sketch corresponds to $f = 2$

$$\nabla f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$$

$$= 2x\,\hat{i} + 2y\,\hat{j} - \cos z\,\hat{k}$$

→ normal to isosurfaces of f

do the normals point in or out of the tube?

where's $f=1$? – inside or outside $f=2$?

$$f = x^2+y^2 - \sin z = 1$$

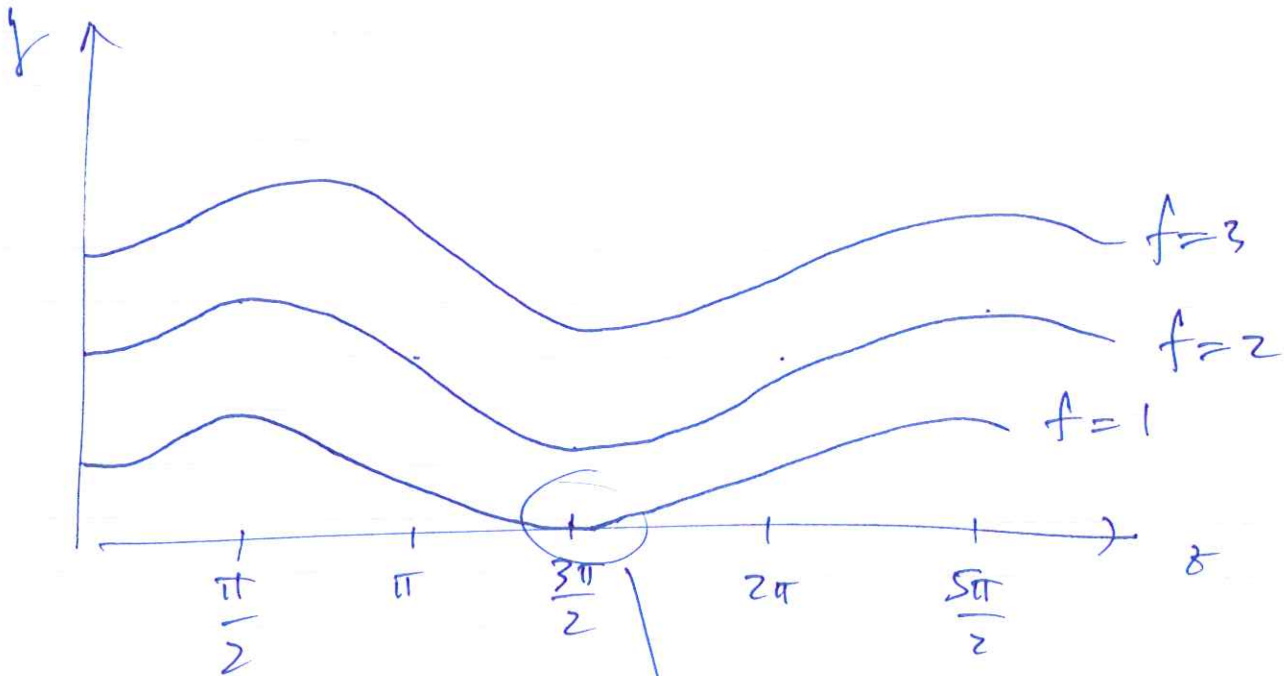
$$x^2+y^2 = \underbrace{1+\sin z}$$

circle radii are smaller

∴ $f=1$ is inside $f=2$

→ since ∇f points towards larger f, ∇f points outwards

(see fig on pg 8)



for $f=1$:

$$x^2+y^2=\underbrace{1+\sin z}_{0 \text{ when } z=\frac{3\pi}{2}}$$