Chapter 1

Introduction
and
Vectors

About Physics

- Provides a quantitative understanding of certain phenomena that occur in our universe
- Based on experimental observations and mathematical analysis
- Used to develop theories that explain the phenomena being studied and that relate to other established theories

Theory and Experiments

- Should complement each other
- When a discrepancy occurs, theory may be modified
  - Theory may apply to limited conditions
    - Example: Newtonian Mechanics is confined to objects traveling slowly with respect to the speed of light
  - Used to try to develop a more general theory

Standards of Quantities

- SI – Systéme International
  - Main system used in this text
  - Consists of a system of definitions and standards to describe fundamental physical quantities

Length

- Units
  - SI – meter, m
- Historically length has had many definitions
- Length is now defined in terms of a meter – the distance traveled by light in a vacuum during a given time
- See table 1.1 for some examples of lengths

Mass

- Units
  - SI – kilogram, kg
- Defined in terms of kilogram, based on a specific cylinder kept at the International Bureau of Weights and Measures
- See table 1.2 for masses of various objects

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Time

- Units
  - Seconds, s
- Historically defined in terms of a solar day, as well as others
- Now defined in terms of the oscillation of radiation from a cesium atom
- See table 1.3 for some approximate time intervals

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Number Notation

- When writing out numbers with many digits, spacing in groups of three will be used
  - No commas
- Examples:
  - 25 100
  - 5.123 456 789 12

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Reasonableness of Results

- When solving problem, you need to check your answer to see if it seems reasonable
- Reviewing the tables of approximate values for length, mass, and time will help you test for reasonableness

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Systems of Measurements, SI Summary

- SI System
  - Most often used in the text
    - Almost universally used in science and industry
  - Length is measured in meters (m)
  - Time is measured in seconds (s)
  - Mass is measured in kilograms (kg)

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Systems of Measurements, US Customary

- US Customary
  - Everyday units
  - Length is measured in feet
  - Time is measured in seconds
  - Mass is measured in slugs

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Prefixes

- Prefixes correspond to powers of 10
- Each prefix has a specific name
- Each prefix has a specific abbreviation

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Prefixes, cont.

- The prefixes can be used with any base units
- They are multipliers of the base unit
- Examples:
  - 1 mm = 10^{-3} m
  - 1 mg = 10^{-3} g

TABLE 1.4
Some Prefixes for Powers of Ten

Power	Prefix	Abbreviation
10^{-24}	yocto	y
10^{-21}	zepto	z
10^{-18}	atto	a
10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P
10^{18}	exa	E
10^{21}	zetta	Z
10^{24}	yotta	Y

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Fundamental and Derived Quantities

- In mechanics, three *fundamental quantities* are used
  - Length
  - Mass
  - Time
- Will also use *derived quantities*
  - These are other quantities that can be expressed as a mathematical combination of fundamental quantities

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Density

- Density is an example of a ***derived*** quantity
- It is defined as mass per unit volume

$$\rho = \frac{m}{V}$$

- Units are kg/m^3

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Dimensional Analysis

- Technique to check the correctness of an equation or to assist in deriving an equation
- Dimensions (length, mass, time, combinations) can be treated as algebraic quantities
  - Add, subtract, multiply, divide
- Both sides of equation must have the same dimensions

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Basic Quantities and Their Dimension

- Dimension has a specific meaning – it denotes the physical nature of a quantity
- Dimensions are denoted with square brackets
  - Length – L
  - Mass – M
  - Time – T

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Dimensional Analysis, cont.

- Cannot give numerical factors: this is its limitation
- Dimensions of some common quantities are given below

TABLE 1.5 Units of Area, Volume, Velocity, and Acceleration

System	Area (L^2)	Volume (L^3)	Velocity (L/T)	Acceleration (L/T^2)
SI	m^2	m^3	m/s	m/s^2
U.S. customary	ft^2	ft^3	ft/s	ft/s^2

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Dimensional Analysis, example

- Given the equation: $x = 1/2\ a\ t^2$
- Check dimensions on each side:

$$L = \frac{L}{T^2} \cdot T^2 = L$$

- The T^2's cancel, leaving L for the dimensions of each side
  - The equation is dimensionally correct
  - There are no dimensions for the constant

Conversion of Units

- When units are not consistent, you may need to convert to appropriate ones
- Units can be treated like algebraic quantities that can cancel each other out
- See Appendix A for an extensive list of conversion factors

Conversion

- Always include units for every quantity, you can carry the units through the entire calculation
- Multiply original value by a ratio equal to one
  - The ratio is called a **conversion factor**
- Example $15.0\,\text{in} = ?\,\text{cm}$

$$15.0\,\text{in}\left(\frac{2.54\,\text{cm}}{1\,\text{in}}\right) = 38.1\,\text{cm}$$

Order of Magnitude

- Approximation based on a number of assumptions
  - May need to modify assumptions if more precise results are needed
- Order of magnitude is the power of 10 that applies
- In order of magnitude calculations, the results are reliable to within about a factor of 10

Uncertainty in Measurements

- There is uncertainty in every measurement, this uncertainty carries over through the calculations
  - Need a technique to account for this uncertainty
- We will use rules for significant figures to approximate the uncertainty in results of calculations

Significant Figures

- A significant figure is one that is reliably known
- Zeros may or may not be significant
  - Those used to position the decimal point are not significant
  - To remove ambiguity, use scientific notation
- In a measurement, the significant figures include the first estimated digit

Significant Figures, examples

- 0.0075 m has 2 significant figures
  - The leading zeroes are placeholders only
  - Can write in scientific notation to show more clearly: 7.5×10^{-3} m for 2 significant figures
- 10.0 m has 3 significant figures
  - The decimal point gives information about the reliability of the measurement
- 1500 m is ambiguous
  - Use 1.5×10^3 m for 2 significant figures
  - Use 1.50×10^3 m for 3 significant figures
  - Use 1.500×10^3 m for 4 significant figures

Operations with Significant Figures – Multiplying or Dividing

- When multiplying or dividing, the number of significant figures in the final answer is the same as the number of significant figures in the quantity having the lowest number of significant figures.
- Example: 25.57 m x 2.45 m = 62.6 m^2
  - The 2.45 m limits your result to 3 significant figures

Operations with Significant Figures – Adding or Subtracting

- When adding or subtracting, the number of decimal places in the result should equal the smallest number of decimal places in any term in the sum.
- Example: 135 cm + 3.25 cm = 138 cm
  - The 135 cm limits your answer to the units decimal value

Operations With Significant Figures – Summary

- The rule for addition and subtraction are different than the rule for multiplication and division
- For adding and subtracting, the ***number of decimal places*** is the important consideration
- For multiplying and dividing, the ***number of significant figures*** is the important consideration

Rounding

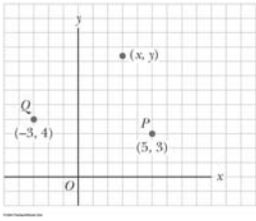
- Last retained digit is increased by 1 if the last digit dropped is 5 or above
- Last retained digit is remains as it is if the last digit dropped is less than 5
- If the last digit dropped is equal to 5, the retained should be rounded to the nearest even number
- Saving rounding until the final result will help eliminate accumulation of errors

Coordinate Systems

- Used to describe the position of a point in space
- Coordinate system consists of
  - A fixed reference point called the origin
  - Specific axes with scales and labels
  - Instructions on how to label a point relative to the origin and the axes

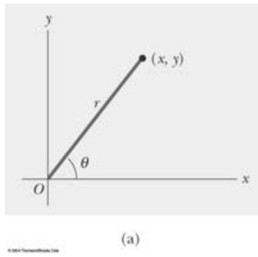
Cartesian Coordinate System

- Also called rectangular coordinate system
- x- and y- axes intersect at the origin
- Points are labeled (x,y)



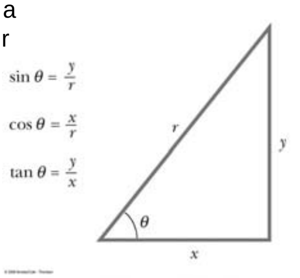
Polar Coordinate System

- Origin and reference line are noted
- Point is distance r from the origin in the direction of angle θ, ccw from reference line
- Points are labeled (r,θ)



Polar to Cartesian Coordinates

- Based on forming a right triangle from r and θ
- x = r cos θ
- y = r sin θ



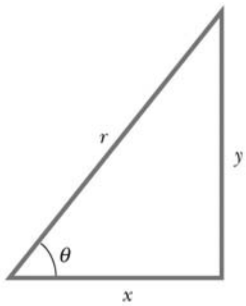
Cartesian to Polar Coordinates

- r is the hypotenuse and θ an angle

$$\tan\theta = \frac{y}{x}$$

$$r = \sqrt{x^2 + y^2}$$

- θ must be ccw from positive x axis for these equations to be valid



Vectors and Scalars

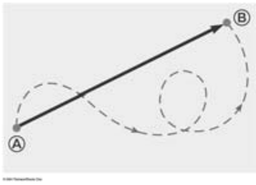
- A **scalar** is a quantity that is completely specified by a positive or negative number with an appropriate unit and has no direction.
- A ***vector*** is a physical quantity that must be described by a magnitude (number) and appropriate units plus a direction.

Some Notes About Scalars

- Some examples
  - Temperature
  - Volume
  - Mass
  - Time intervals
- Rules of ordinary arithmetic are used to manipulate scalar quantities

Vector Example

- A particle travels from A to B along the path shown by the dotted red line
  - This is the ***distance*** traveled and is a scalar
- The ***displacement*** is the solid line from A to B
  - The displacement is independent of the path taken between the two points
  - Displacement is a vector



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Other Examples of Vectors

- Many other quantities are also vectors
- Some of these include
  - Velocity
  - Acceleration
  - Force
  - Momentum

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Vector Notation

- When handwritten, use an arrow: $\vec{A}$
- When printed, will be in bold print with an arrow: $\vec{\mathbf{A}}$
- When dealing with just the magnitude of a vector in print, an italic letter will be used: A or $|\vec{\mathbf{A}}|$
- The magnitude of the vector has physical units
- The magnitude of a vector is always a positive number

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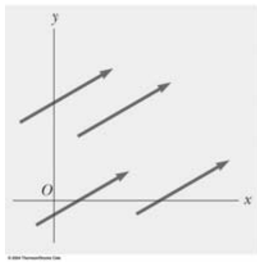
Equality of Two Vectors

- Two vectors are ***equal*** if they have the same magnitude and the same direction
- $\vec{\mathbf{A}} = \vec{\mathbf{B}}$ if $A = B$ and they point along parallel lines
- All of the vectors shown are equal



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Adding Vectors

- When adding vectors, their directions must be taken into account
- Units must be the same
- Graphical Methods
  - Use scale drawings
- Algebraic Methods
  - More convenient

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Adding Vectors Graphically

- Choose a scale
- Draw the first vector with the appropriate length and in the direction specified, with respect to a coordinate system
- Draw the next vector with the appropriate length and in the direction specified, with respect to a coordinate system whose origin is the end of vector $\vec{\mathbf{A}}$ and parallel to the coordinate system used for $\vec{\mathbf{A}}$

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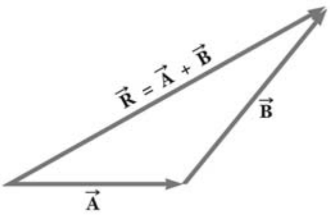
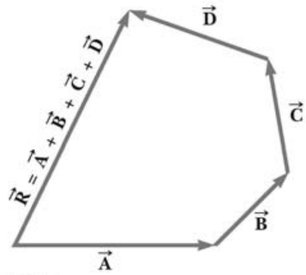
Adding Vectors Graphically, cont.

- Continue drawing the vectors "tip-to-tail"
- The resultant is drawn from the origin of $\vec{\mathbf{A}}$ to the end of the last vector
- Measure the length of $\vec{\mathbf{R}}$ and its angle
  - Use the scale factor to convert length to actual magnitude



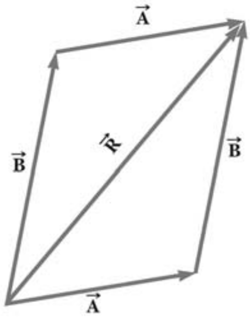
Adding Vectors Graphically, final

- When you have many vectors, just keep repeating the process until all are included
- The resultant is still drawn from the origin of the first vector to the end of the last vector



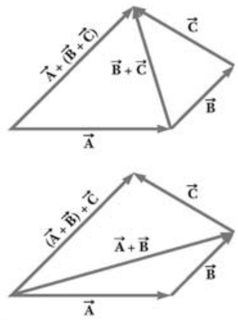
Adding Vectors, Rules

- When two vectors are added, the sum is independent of the order of the addition.
  - This is the ***commutative law of addition***
  - $\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{B}} + \vec{\mathbf{A}}$



Adding Vectors, Rules cont.

- When adding three or more vectors, their sum is independent of the way in which the individual vectors are grouped
  - This is called the ***Associative Property of Addition***
  - $\vec{\mathbf{A}} + (\vec{\mathbf{B}} + \vec{\mathbf{C}}) = (\vec{\mathbf{A}} + \vec{\mathbf{B}}) + \vec{\mathbf{C}}$



Adding Vectors, Rules final

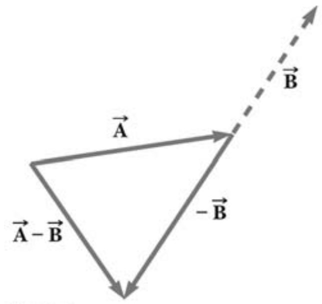
- When adding vectors, all of the vectors must have the same units
- All of the vectors must be of the same type of quantity
  - For example, you cannot add a displacement to a velocity

Negative of a Vector

- The negative of a vector is defined as the vector that, when added to the original vector, gives a resultant of zero
  - Represented as $-\vec{\mathbf{A}}$
  - $\vec{\mathbf{A}} + (-\vec{\mathbf{A}}) = 0$
- The negative of the vector will have the same magnitude, but point in the opposite direction

Subtracting Vectors

- Special case of vector addition
- $\vec{\mathbf{A}} - \vec{\mathbf{B}} = \vec{\mathbf{A}} + \left(-\vec{\mathbf{B}}\right)$
- Continue with standard vector addition procedure



Multiplying or Dividing a Vector by a Scalar

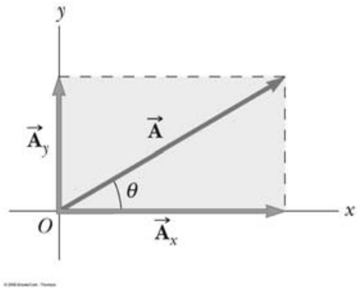
- The result of the multiplication or division is a vector
- The magnitude of the vector is multiplied or divided by the scalar
- If the scalar is positive, the direction of the result is the same as of the original vector
- If the scalar is negative, the direction of the result is opposite that of the original vector

Multiplying Vectors

- Two vectors can be multiplied in two different ways
  - One is the **scalar product** $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = AB\cos\theta$
    - Also called the dot product
  - The other is the **vector product**
    - Also called the cross product $\vec{\mathbf{A}} \times \vec{\mathbf{B}} = AB\sin\theta$
- These products will be discussed as they arise in the text

Components of a Vector

- A **component** is a part
- It is useful to use **rectangular components**
  - These are the projections of the vector along the x- and y-axes



Vector Component Terminology

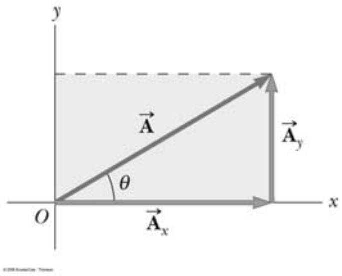
- $\vec{\mathbf{A}}_x$ and $\vec{\mathbf{A}}_y$ are the ***component vectors*** of $\vec{\mathbf{A}}$
  - They are vectors and follow all the rules for vectors
- A_x and A_y are scalars, and will be referred to as the ***components*** of **A**
- The combination of the component vectors is a valid substitution for the actual vector

Components of a Vector, 2

- The x-component of a vector is the projection along the x-axis
  $A_x = A\cos\theta$
- The y-component of a vector is the projection along the y-axis
  $A_y = A\sin\theta$
- When using this form of the equations, θ must be measured ccw from the positive x-axis

Components of a Vector, 3

- The y-component is moved to the end of the x-component
- This is due to the fact that any vector can be moved parallel to itself without being affected
  - This completes the triangle



Components of a Vector, 4

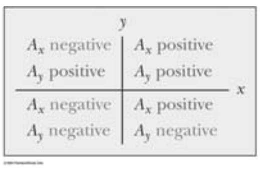
- The components are the legs of the right triangle whose hypotenuse is $\vec{A}$

$$A = \sqrt{A_x^2 + A_y^2} \quad \text{and} \quad \theta = \tan^{-1}\frac{A_y}{A_x}$$

  - May still have to find θ with respect to the positive x-axis
  - Use the signs of A_x and A_y

Components of a Vector, final

- The components can be positive or negative and will have the same units as the original vector
- The signs of the components will depend on the angle

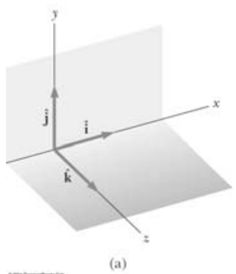


Unit Vectors

- A ***unit vector*** is a dimensionless vector with a magnitude of exactly 1.
- Unit vectors are used to specify a direction and have no other physical significance

Unit Vectors, cont.

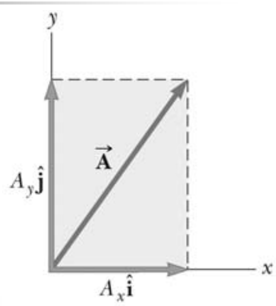
- The symbols $\hat{i}, \hat{j}, \text{and } \hat{k}$ represent unit vectors in the x, y and z directions
- They form a set of mutually perpendicular vectors



Unit Vectors in Vector Notation

- $\vec{A}_x$ is the same as $A_x\hat{i}$ and $\vec{A}_y$ is the same as $A_y\hat{j}$ etc.
- The complete vector can be expressed as

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$



Adding Vectors Using Unit Vectors

- Using $\vec{R} = \vec{A} + \vec{B}$
- Then

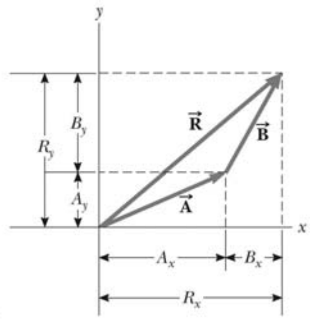
$$\vec{R} = (A_x\hat{i} + A_y\hat{j}) + (B_x\hat{i} + B_y\hat{j})$$

$$\vec{R} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$

- Then $R_x = A_x + B_x$ and $R_y = A_y + B_y$

$$R = \sqrt{R_x^2 + R_y^2} \qquad \theta = \tan^{-1}\frac{R_y}{R_x}$$

Adding Vectors with Unit Vectors – Diagram



Adding Vectors Using Unit Vectors – Three Directions

- Using $\vec{R} = \vec{A} + \vec{B}$

$$\vec{R} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) + (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$\vec{R} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$$

$$\vec{R} = R_x + R_y + R_z$$

- $R_x = A_x + B_x$, $R_y = A_y + B_y$ and $R_z = A_z + B_z$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} \qquad \theta_x = \tan^{-1}\frac{R_x}{R} \text{ etc.}$$

Trig Function Warning

- The component equations ($A_x = A \cos \theta$ and $A_y = A \sin \theta$) apply only when the angle is measured with respect to the x-axis (preferably ccw from the positive x-axis).
- The resultant angle ($\tan \theta = A_y / A_x$) gives the angle with respect to the x-axis.
  - You can always think about the actual triangle being formed and what angle you know and apply the appropriate trig functions

Model Building

- A ***model*** is a simplified substitution for the real problem that allows us to solve the problem in a relatively simple way
  - Make predictions about the behavior of the system
    - The predictions will be based on interactions among the components and/or
    - Based on the interactions between the components and the environment
    - As long as the predictions of the model agree with the actual behavior of the real system, the model is valid

Particle Model

- The particle model allows the replacement of an extended object with a particle which has mass, but zero size
- Two conditions for using the particle model are
  - The size of the actual object is of no consequence in the analysis of its motion
  - Any internal processes occurring in the object are of no consequence in the analysis of its motion

Categories of Models

- There are four categories of models that will be used in this book
  - Geometric model
  - Simplification model
  - Analysis model
  - Structural model

Geometric Models

- Form a geometric construction that represents the real situation
- Perform the analysis of the geometric construction

Simplification Model

- The details that are not significant in determining the outcome of the problem are ignored
- The particle model is an example
- Another example is to assume friction is negligible in many cases

Analysis Model

- Based on general types of problems that have been solved before
- You cast a new problem into the form of one seen (and solved) before

Structural Model

- Used to understand the behavior of a system that is different in scale from our macroscopic world
- Can be used for actual systems much larger or much smaller
- Used for systems you cannot interact with directly

Representation

- Related to modeling is that of forming **alternative representations** of the problem
- A representation is a method of viewing or presenting the information relating to the problem
- Considering alternative representations can help you think about the information in the problem in several different ways to help you understand and solve it.

Types of Representations

- Mental representation
- Pictorial representation
- Simplified pictorial representation
- Graphical representation
- Tabular representation
- Mathematical representation

Mental Representation

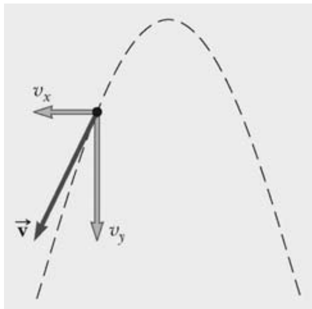
- Imagine the scene described
- Let time progress so you understand the problem and can make predictions about changes that will occur over time
- A critical step in solving every problem

Pictorial Representation

- Drawing a picture of the situation
- The representation describes what you would see if you were observing the situation described in the problem
- Any coordinate system included would be the x- and y-axes

Simplified Pictorial Representation

- Redraw the pictorial representation without complicating details
- Using the particle model could be part of the simplification
- This representation will be used extensively throughout the text



Graphical Representation

- A graph can be very useful in describing a situation
- The axes of the graph may be any two related variables
- A graphical representation is generally not something you would see when actually observing the situation

Tabular Representation

- Organizing information in a table form may make it clearer
- Periodic table is an example
- A table of known and unknown quantities may be helpful in solving a problem

Mathematical Representation

- The ultimate goal in solving problems is often the mathematical representation
- Through various representations that help in understanding the problem, you can arrive at the one or more equations that represent the situation
- The result can be solved for mathematically

General Problem Solving Strategy

- Conceptualize
- Categorize
- Analyze
- Finalize

Problem Solving – Conceptualize

- Read the problem
  - At least twice
  - Be sure you understand the situation and the nature of the problem
- Make a quick drawing of the situation
  - This is the pictorial representation
  - Generate other representations that would be helpful
- Focus on the expected result
  - Think about units
  - Think about what a reasonable answer should be

Problem Solving – Categorize

- Simplify the problem
  - Draw a simplified representation
    - Can you ignore air resistance?
    - Model objects as particles
- Classify the type of problem
  - Plug-in problem – just substitute numbers
- Try to identify similar problems you have already solved
  - The analysis representation

Problem Solving – Analyze

- Select the relevant equation(s) to apply
- Solve for the unknown variable
- Substitute appropriate numbers
- Calculate the results
  - Include units
- Round the result to the appropriate number of significant figures

Problem Solving – Finalize

- Check your result
  - Does it have the correct units?
  - Does it agree with your conceptualized ideas?
- Look at limiting situations to be sure the results are reasonable
- Compare the result with those of similar problems

Problem Solving – Some Final Ideas

- When solving complex problems, you may need to identify sub-problems and apply the problem-solving strategy to each sub-part.
- These steps can be a guide for solving problems in this course