

## Economics of Organization and Management – ECOM40

Fall 2012

Franco

### Instructions for Assignment 2

Due: (on or before): November 22, 2012 before the beginning of the Lecture 2 (IC208).

- **Assignments submitted after 3 p.m. on November 22<sup>nd</sup> WILL NOT BE accepted under ANY circumstance. Assignments may not be faxed or e-mailed, since it is possible that they may not arrive on time. It is the student's responsibility to turn in the assignment to the Professor. If you put it under her office door, there is a possibility that it will not be considered on time and will not be accepted.**
- You may submit this assignment as either part of a group of no more than 5 students or on your own. (One submission per group.) Only students in the same section can be in a group. Assignments may not be faxed or e-mail.
- A title page must be stapled to your assignment. (No paper clips please.) The title page should have all the group members' names, in alphabetical order based on last name, along with their student identification number. Only students listed on the title page will be given credit for the assignment. If you do not include a title page, or if the assignment is not stapled together, your final score on this assignment will be lowered by 10%.
- This assignment is to prepare you for the examinations in this class. So, if you do not show your work, you may not receive credit for a correct answer.
- Make sure that your answers are clear and easy to understand. Your answers should be sufficiently clear that a smart college student without a background in economics or management can understand them.
- The total possible marks for this assignment is 125. There are 13 questions in total, several of which have multiple parts. Answer all of them.

1. (15 points) In a two firm market, let the marginal cost of producing a product be \$20, the market demand be given by the function  $Q=60-P/2$  and the market quantity be equal to  $Q_1+Q_2$ .

- a. Find the best response function for each firm.

**Answer:** First you need to find the marginal revenue for each firm. In order to do that, you need to find the inverse demand (P as a function of Q), substitute market quantity with the sum of output from the two firms, and then multiply that by the firm's quantity.

$$Q=60-P/2$$

$$P = 120 - 2Q$$

$$P = 120 - 2Q_1 - 2Q_2$$

$$TR_1 = (120 - 2Q_1 - 2Q_2)Q_1$$

$$TR_2 = (120 - 2Q_1 - 2Q_2)Q_2$$

Next, we take the derivative of each total revenue function with respect to the firm's output to find the marginal revenue.

$$MR_1 = 120 - 4Q_1 - 2Q_2$$

$$MR_2 = 120 - 2Q_1 - 4Q_2$$

Then we set the marginal revenue equal to the marginal cost and solve for the firm's output, which will be a function of its competitor's output. This is the best response function for the firm.

$$20 = 120 - 4Q_1 - 2Q_2$$

$$20 = 120 - 2Q_1 - 4Q_2$$

$$4Q_1 = 100 - 2Q_2$$

$$4Q_2 = 100 - 2Q_1$$

$$Q_1 = 25 - 1/2Q_2$$

$$Q_2 = 25 - 1/2Q_1$$

- b. Find the Cournot equilibrium quantity for each firm, the total market quantity, the market price and the individual firm profits.

**Answer:** Now we substitute firm 2's quantity into firm 1's best response function and solve for firm 1's output. Note that by firm 2's quantity, I mean firm 2's best response function.

$$Q_1 = 25 - 1/2(25 - 1/2Q_1)$$

$$3/4 Q_1 = 12.5$$

$$Q_1 = 16 \frac{2}{3}$$

When I put this into firm 2's best response function, I get:

$$Q_2 = 25 - 1/2(16 \frac{2}{3}) = 16 \frac{2}{3}$$

Given that the two firms have the same cost structure, this is not surprising. Next, we find total market quantity:

$$Q = 16 \frac{2}{3} + 16 \frac{2}{3} = 33 \frac{1}{3}$$

Then we find the market price:

$$P = 120 - 2(33 \frac{1}{3}) = 53 \frac{1}{3}$$

Finally, we can determine firm profits:

$$\text{Profits per firm} = (53 \frac{1}{3} - 20) * 16 \frac{2}{3} = 555.56$$

- c. Suppose the firms engage in Bertrand competition. What is the prevailing market price and the quantity that each firm sells? What are individual firm profits?

**Answer:** In this case, both firms will announce their marginal cost as the price ( $P = 20$ ) and each firm will sell 25 units ( $Q = 60 - 20/2$ ). Neither firm will make profits since the price is the same as marginal cost.

2. (15 points) An entrepreneur has a venture that will make either \$100 million or \$0. The change that this venture will make \$100 million depends on the effort level expended by the entrepreneur: If she tries hard, the chance of the \$100 million outcome is 0.1. If she does not work hard, the chance of this outcome is 0.02. This entrepreneur is risk averse, with utility function

$\sqrt{x}$  – disutility of effort,  
where  $x$  is the entrepreneur's return and the disutility of effort is 0 if the entrepreneur does not try hard and 500 if she does.

- a. (5 points) Assuming this entrepreneur bears all the risk of this venture, will she try hard or not? What will be her expected utility, net of the disutility of effort (if any)? (Note that expected utility from working hard (not working) can be calculated by multiplying the utility from being successful when working hard with the probability of being successful plus the utility from being unsuccessful when working hard with the probability of being unsuccessful.)

**Answer:** the entrepreneur's expected utility from working hard is:

$$.1 * (\sqrt{100,000,000}) - 500 = 500.$$

The entrepreneur's expected utility from not working hard is:

$$.02 * (\sqrt{100,000,000}) = 200.$$

So, she works hard.

- b. (5 points) A risk-neutral venture capitalist (VC) is prepared to support this venture. Specifically, the VC will pay the entrepreneur a base amount  $B$  up front, in return for which the VC will retain  $X$  out of the \$100 million the venture generates, if the venture succeeds. Assuming this VC is the entrepreneur's only alternative to going it alone (doing whatever you determined the answer was in part a), and assuming the VC can make part of his contract with the entrepreneur a specification of her effort level, what is the optimal contract of this sort for the VC's net expected monetary value with this contract?

**Answer:** Since the VC is risk-neutral and can observe the effort taken on by the entrepreneur, and the entrepreneur is risk averse, the VC will take on all the risk and pay the entrepreneur a flat wage regardless of the outcome. In this case, the marginal cost of effort is zero, since the VC has specified the entrepreneur's effort level. That means that the VC doesn't have to give the entrepreneur any of the return when the project is successful. So,  $X = 100,000,000$ . In order for the entrepreneur to be willing to work for the VC, the entrepreneur must make as much (in utility terms) as she would make from her outside option. Thus, we have

$$\sqrt{B} - 500 = 500$$

$$\sqrt{B} = 1000$$

$$B = 1,000,000$$

- c. (5 points) Unhappily, the VC cannot contractually specify the effort level of the entrepreneur. If the VC wishes to motivate the entrepreneur to try hard, he must do this with the terms  $B$  and  $X$  in the contract he provides. What is the best

contract for the VC to offer the entrepreneur, assuming that if the entrepreneur does not accept this contract, she is stuck going it alone on this venture?

**Answer:** Now, the VC has to make sure that the entrepreneur has an incentive to work hard. So, the VC must offer the entrepreneur enough when the project is successful to insure that the entrepreneur will work hard. This means that the entrepreneur must make more from working hard compared with not working at all:

$$.1 * (\text{sqrt}(100,000,000 - X) + \text{sqrt}(B)) + .9 * (\text{sqrt}(B)) - 500 = .02 * (\text{sqrt}(100,000,000 - X) + \text{sqrt}(B)) + .98 * (\text{sqrt}(B))$$

We can rearrange this and get:

$$\begin{aligned} .1 * (\text{sqrt}(100,000,000 - X)) + \text{sqrt}(B) - 500 &= .02 * (\text{sqrt}(100,000,000 - X)) + \text{sqrt}(B) \\ .08 * (\text{sqrt}(100,000,000 - X)) &= 500 \\ (\text{sqrt}(100,000,000 - X)) &= 6250 \\ 100,000,000 - X &= 39,062,500 \\ 100,000,000 - 39,062,500 &= X \\ 60,937,500 &= X \end{aligned}$$

Next, we set the expected utility from working hard equal the the entrepreneur's outside option, using our solution for X:

$$\begin{aligned} .1 * (\text{sqrt}(39,062,500) + \text{sqrt}(B)) + .9 * (\text{sqrt}(B)) - 500 &= 500 \\ .1 * (\text{sqrt}(39,062,500)) + \text{sqrt}(B) &= 1000 \\ .1 * (6250) + \text{sqrt}(B) &= 1000 \\ 625 + \text{sqrt}(B) &= 375 \\ \text{sqrt}(B) &= 375 \\ B &= 140,625. \end{aligned}$$

Note that the flat fee is significantly less than in the case where the VC could specify the entrepreneur's level of effort.

- (5 points) Suppose Minot Farm Equipment Corp. employs two salespeople. Each covers an exclusive territory; one is assigned to North Dakota and the other to South Dakota. These two neighboring plains states have similar agricultural economies and are affected by the same weather patterns. Durham Tractor Co. also employs two salespeople. One works North Carolina, while the other is assigned to Oregon. Farm products and methods vary considerably across these two states. Each firm uses the dollar value of annual sales as a performance measure for salespeople. Which of the firms do you think would benefit most from basing pay on its salespeople's relative performance? Why?

**Answer:** Relative performance as a basis for compensation is useful for filtering out common shocks. When conditions are good in North Dakota, they are likely to be good in

South Dakota as well. Hence, using relative performance evaluation allows the firm to shield these employees from the risks associated with the quality of the local farm economy. North Carolina and Oregon are likely to be subject to differing shocks, and so using relative pay evaluation will not be appropriate.

4. (5 points) Suppose you were granted a “risky job” of the type studied in the chapter on incentives and performance. The job pays \$40,000 with probability 1/2, and \$160,000 with probability 1/2. Suppose that your utility function is given by  $u(x) = \ln(x)$ . What is your certainty equivalent for this risky payoff? To answer this question, compare this risky job to a safe job paying \$100,000 for sure. Then reduce the value of the safe job in \$1,000 increments until you are indifferent between the safe job and the risky job. What is your certainty equivalent for a job paying \$10,000 or \$190,000, each with equal probability?

**Answer:** The expected utility is  $.5*\ln(40,000) + .5*\ln(160,000) = 11.28978$ . The certainty equivalent would be 80,000, since  $\ln(80,000)=11.28978$ . Note that the expected payoff would be 100,000. This means that the individual is risk averse. The expected utility of the second position is 10.68256. So, the certainty equivalent is 43,589. In this case, the expected payoff would be 100,000, which is the same as the first “risky job” but because of the variance, the certainty equivalent of this position is lower than that of the first job.

5. (5 points) Consider a market in which consumer indifference curves are relatively steep. Firms in this industry are pursuing two positioning strategies: Some firms are producing a basic product that provides satisfactory performance; others are producing an enhanced product that provides performance that is superior to that of the basic product. Consumer surplus parity currently exists in the industry. Are the prices of the basic and the enhanced product likely to be significantly different or about the same? Why? How would the answer change if the consumer indifference curves were relatively flat?

**Answer:** Indifference curves illustrate price—quality combinations that yield the same consumer surplus. The fact that the indifference curve is steep means that consumers are willing to pay significantly more for a good that is of higher quality. Therefore, the prices of the basic and the enhanced product are likely to be significantly different. On the other hand, the prices of the two goods would be about the same if the consumer indifference curves were relatively flat. Flat indifference curves mean that consumers are not willing to pay much more money for a higher benefit.

6. (5 points) “Firms that seek a cost advantage should adopt a learning curve strategy; firms that seek to differentiate their products should not.” Comment on both of these statements.

**Answer:** A learning curve strategy is one in which a firm seeks to reduce costs by learning. This is but one of many ways in which a firm can achieve a cost advantage. Other cost drivers include economies of scale, economies of scope, capacity utilization, economies of density, process efficiency, government policy, and a firm's location.

Learning curves can also confer quality advantages. If there is a first mover advantage in establishing a particular quality position (say the goods are experience goods), then it may pay to push aggressively down the learning curve to gain that quality advantage. Therefore, pursuing a learning curve strategy could be advantageous to both firms that seek a cost advantage and firms that seek to differentiate their products.

7. (15 points) Zellers and Wal-Mart are two of Canada's largest retailers. To reflect the strong position of the Canadian dollar, each firm is considering lowering prices on *some* goods in Canadian stores. The table displays the payoffs for each firm associated with lowering prices (or not), given the other firm's decision.

If Zellers decides to...	And Walmart decides to...	Then, Zeller's profits are...	And Wal-Mart's profits are...
Keep prices the same	Keep prices the same	\$200MM	\$250MM
Keep prices the same	Drop prices	\$150MM	\$280MM
Drop prices	Keep prices the same	\$230MM	\$190MM
Drop prices	Drop prices	\$180MM	\$220MM

- a. Create a game matrix for the case where both firms move simultaneously. Are there any dominant strategies? If so, describe them. What is the Nash equilibrium/a of this game?

**Answer:**

		Zellers	
		Keep prices the same	Drop prices
Wal-Mart	Keep prices the same	\$280M, \$200M	\$190M, \$230M
	Drop prices	\$250M, \$150M	\$220M, \$180M

		Wal-Mart	

		Keep prices the same	Drop prices
Zellers	Keep prices the same	\$200M, \$250M	\$150M, \$280M
	Drop prices	\$230M, \$190M	\$180M, \$220M

Note there are two ways that this matrix can be set up, depending on which player you choose to make the row player. However, regardless, the first number is always the payoff to the row player and the second number is always the payoff to the column player.

Zellers' strategy is to Drop prices when Wal-Mart doesn't, and to drop prices when Wal-Mart drops its prices. So, it has a dominant strategy to always drop its prices.

Wal-Mart's strategy is to drop prices when Zellers doesn't and to drop prices when Zellers drops its prices. So, like Zellers, it has a dominant strategy to always drop its prices.

So, the Nash equilibrium is for both firms to drop their prices.

- b. Consider a game where Zellers can move first before Wal-Mart. What is the Nash equilibrium/a of this game? Explain why your answer to part a differs from your answer to part b.

**Answer:** Now, we need to use a decision tree to determine what Wal-Mart will do in the last period, given what Zellers did in the period before. Suppose that Zellers didn't drop its prices, then Wal-Mart would drop its prices. Suppose that Zellers dropped its prices, then Wal-Mart would drop its prices. So, given that, Zellers will drop its prices. Part of the reason that the equilibrium is the same as before is because Wal-Mart has a dominant strategy to drop its prices.

- c. Given your answer to parts a and b, how much would Zellers be willing to pay for the right to move first?

**Answer:** Since the equilibrium in the a and b is the same, Zellers would not be willing to pay for the right to move first.

8. (5 points) Consider a monopoly producer of a durable good, such as a supercomputer. The good does not depreciate. Once consumers purchase the good from the monopolist,

they are free to sell it in the “second hand” market. Often times in markets for new durable goods, one sees the following price pattern: The seller starts off charging high price but then lowers the price over time. Explain why with a durable good, the monopolist might prefer to commit to keep its selling price constant over time. Can you think of a way that the monopolist might be able to make a credible commitment to do this?

**Answer:** Monopolist of a durable good faces a problem that can be stated in two ways:

- Monopolist of today is in competition with monopolist of tomorrow since the good is durable.
- The good sold today has a substitute in form of good from the past (i.e., secondary market)

Either way, entry of firms or substitutes will erode monopoly profits. At the extreme, monopolist of durable goods has no market power due to the “entry” and “substitution” effect. This causes monopolist to reduce price in the subsequent period. Hence rational consumers will expect lower prices from the monopolists and may decide to wait.

Monopolist making a credible commitment will reduce the above effect. An example would be adding a most favored customer clause (MFCC) in a contract. The monopolist can offer this clause to most of its customers, making a very credible commitment against price reductions. The monopolist can also make commitment in form of increasing  $B$  in every period (i.e., offer more features at the same fixed price) that will reduce the substitution effect.

9. (5 points) An article on price wars by two McKinsey consultants makes the following argument. *“That the (tit-for-tat) strategy is fraught with risk cannot be overemphasized. Your competitor may take an inordinately long time to realize that its actions can do it nothing but harm; rivalry across the entire industry may escalate precipitously; and as the “tit-for-tat” game plays itself out, all of a price war’s detrimental effects on customers will make themselves felt.”* How would you reconcile the views expressed in this quote with the advantages of tit-for-tat claimed in this chapter?

**Answer:** As the argument in the McKinsey article points out, there are risks involved in adopting the tit-for-tat strategy. Misreading pricing moves made by competitors can lead to alternating cooperative and uncooperative responses or all uncooperative responses. When the possibility of misreads exists, it may be beneficial for a firm to adopt a more forgiving strategy to reduce the likelihood of detrimental responses to a competitor’s price deviations. However, adopting the tit-for-tat strategy before a price war ensues can

serve as a powerful deterrent sustaining monopoly pricing at a non-cooperative equilibrium.

10. (20 points) Nella and Lucca are two Italian interior design stores located in the designer section of Toronto. Each is considering whether or not to increase the number of locations or to maintain the one location downtown. The following game matrix provides the payoffs to each store based on its own actions and the action of its competitor.

		Nella	
		Maintain	Expand
Lucca	Maintain	60, 60	15, 90
	Expand	90, 15	45, 45

- a. What is the one-shot Nash Equilibrium of this game?

**Answer:** the Nash equilibrium is for both firms to Expand.

- b. Suppose that the game is repeated 100 times. What is the Nash equilibrium of the last period? What is the Nash equilibrium of the first period? Explain why or why not your answer to part a differs from your answer to part b.

**Answer:** The Nash equilibrium of the last period is the same as the Nash equilibrium of the first period: {Expand, Expand}. Since, both firms can look ahead and reason back, they know that each period is like a one-shot game and the Nash equilibrium of each period is the same as the Nash equilibrium of the one-shot game.

- c. Now suppose that the game is played repeatedly, but the players are uncertain of when the game will end. In order to ensure cooperative behavior between the firms, find the interest rate that is sufficient to ensure the firms engage in cooperative behavior.

**Answer:** Now the firms can either cooperate or cheat. If the firm cheat when the other firm cooperates, it gets 90 in the current period and will get 45 for each subsequent period. So, the payoff from cheating is:

$$90 + 45/i$$

If the firm cooperates, it gets 60 in the current period and in each subsequent period. So, the payoff from cooperating is:

$$60 + 60/i$$

The firms prefer to cooperate as long as the payoff from cooperating is higher than the payoff from cheating:

$$60 + 60/i > 90 + 45/i$$

Now, we solve for the interest rate.

$$15/i > 30$$

$$1/2 > i$$

This means that if the interest rate is less than .5, the firms will prefer to cooperate.

- d. Suppose that each firm could invest in a technology that would lower its profits when it expanded when its competitor maintained its capacity from 90 to 80. The investment in the technology would be visible to its competitor. Would the firm be willing to invest in this technology? How much would it be willing to invest in the technology? Using the language of the course, explain why or why not the firm might choose to do this and how economists might understand these types of investment.

**Answer:** Now we calculate the interest rate given that the firm received 80 in the first period and 45 in all subsequent periods.

$$60 + 60/i > 80 + 45/i$$

Now, we solve for the interest rate.

$$15/i > 20$$

$$3/4 > i$$

This means that if the interest rate is less than .75, the firms will prefer to cooperate. Thus, this increases the range for which the firms will cooperate.

If the interest rate is higher than .75, the firm will not invest in the technology since it won't change the outcome. If the interest rate is between .5 and .75, the firm may

invest in the technology. Let  $T$  be the price of the technology. If the firm buys the technology, its return is:

$$60 + 60/i + T$$

If the firm doesn't buy it, its return is:

$$90 + 45/i$$

This allows us to calculate a range that the firm would be willing to pay for the technology.

$$\begin{aligned} 60 + 60/i + T &> 90 + 45/i \\ 15/i + T &> 30 \\ T &> 30 - 15/i \end{aligned}$$

When the interest rate is  $\frac{1}{2}$ , the firm is willing to pay zero. When the interest rate is  $\frac{3}{4}$ , the firm is willing to pay 10.

This is a strategic commitment, which would be credible, visible and understandable to the other firm.

11. (5 points) An established firm is considering expanding its capacity to take advantage of a recent growth in demand. It can do so in two ways. It can purchase fungible, general-purpose assets that can be resold close to their original value, if their use in the industry proves unprofitable. Or it can invest in highly specialized assets that, once they are put in place, have no alternative uses and virtually no salvage value. Assuming that each choice results in the same production costs once installed, under what choice is the firm likely to encounter a greater likelihood that its competitors will also expand their capacity.

**Answer:** In order to be effective, commitment must be visible, understandable, and irreversible. The key is irreversibility. In general, a significant investment in a highly specialized relationship specific asset has a high commitment value. The value is greater because the asset has no other use. As the question states, once the plant is build the firm has no option but to utilize it within this particular industry. This sends a strong signal to the competition and they behave less aggressively. Hence if the firm invests in a fungible asset there is higher likelihood that its competitors will also expand their capacity.

12. (10 points) Indicate whether the strategic effects of the following competitive moves are likely to be positive (beneficial to the form making them) or negative (harmful to the firm

making them.)

- a. Two horizontally differentiated producers of diesel railroad engines—one located in the U.S. and other located in Europe—compete in European market as Bertrand price competitors. The U.S. manufacturer lobbies the U.S. government to give it an export subsidy, the amount of which is directly proportional to the amount of output the firm sells in the European market.

**Answer:** Given that the two firms are competing as Bertrand price competitors, Stage 2 tactical variables are strategic complements. U.S. government subsidy would reduce U.S. firm's marginal costs in Europe, reducing the price. This makes Firm 1 tough (i.e., no matter what its European counterpart charges Firm 1 would charge a lower price with the subsidy). This shifts Firm 1's reaction curve inwards (see Figure 9.4) moving Bertrand equilibrium southwest. Firm 2 would follow and reduce price (though by not as much as Firm 1), this hurts Firm 1 and strategic effect is negative.

- b. A Cournot duopolist issues new debt to repurchase shares of its stock. The new debt issue will preclude the firm from raising additional debt in the foreseeable future, and is expected to constrain the firm from modernizing existing production facilities.

**Answer:** Since the firms are competing in a Cournot industry their actions are strategic substitutes. Firm 1's decision to buy back its shares of stock is a "soft" move, because it constrains it from making investments in modernizing its production facilities.

Hence this kind of commitment makes Firm 1 "soft". This means that no matter what level of output Firm 2 chooses, Firm 1 will produce less output. This corresponds to inward shift in Firm 1's reaction curve and has a negative strategic effect since this allows Firm 2 to respond more aggressively.

13. (15 points) The dancing machine industry is a duopoly. The two firms, Chuckie B Corp. and Gene Gene Dancing Machines, compete through Cournot quantity-setting competition. The demand curve for the industry is  $P = 100 - Q$ , where  $Q$  is the total quantity produced by Chuckie B and Gene Gene. Currently, each firm has marginal cost of \$40 and no fixed cost.
  - a. Find the equilibrium quantity each firm produces, market price, and the individual firm profits.

**Answer:**

$$(1) \Pi_1 = P_1 Q_1 - TC_1 = (100 - Q_1 - Q_2) Q_1 - 40 Q_1$$

$$(2) \Pi_2 = P_2 Q_2 - TC_2 = (100 - Q_1 - Q_2) Q_2 - 40 Q_2$$

Now take the derivative of each of the above with respect to Q:

$$(1) \text{ Max } \Pi_1 = 100 - 2Q_1 - Q_2 - 40$$

$$Q_1 = 30 - .5Q_2$$

$$(2) \text{ Max } \Pi_2 = 100 - 2Q_2 - Q_1 - 40$$

$$Q_2 = 30 - .5Q_1$$

Now solve the above simultaneously:

$$Q_1 = 30 - .5Q_2$$

$$Q_1 = 30 - .5(30 - .5Q_1)$$

$$Q_1 = 15 + .25Q_1$$

$$.75Q_1 = 15$$

$$Q_1 = 20 \text{ and } Q_2 = 30 - .5(20) = 20$$

To find P we use the market demand curve:

$$P = 100 - Q = 100 - Q_1 - Q_2 = 100 - 20 - 20 = \$60$$

And finally each firm's profits:

$$\Pi_1 = P_1Q_1 - TC_1 = (60*20) - (40*20) = 1200 - 800 = \$400 \text{ and}$$

$$\Pi_2 = P_2Q_2 - TC_2 = (60*20) - (40*20) = 1200 - 800 = \$400$$

- b. Chuckie B Corp. is considering implementing a proprietary technology they have developed. The onetime sunk cost of implementing this process is \$350. Once this investment is made, marginal cost will be reduced to \$25. Gene Gene has no access to this, or any other cost-saving technology, and its marginal cost will remain at \$40. Chuckie B's financial consultant observes that the investment should not be made, because a cost reduction of \$15 on each of the 20 machines results in a savings of only \$300, which is less than the cost of implementing the technology. Is the consultant's analysis accurate? Why or why not? Compute the strategic effect of the investment.

**Answer:**

$$(1) \Pi_1 = P_1Q_1 - TC_1 = (100 - Q_1 - Q_2) Q_1 - 40 Q_1$$

$$(2) \Pi_2 = P_2Q_2 - TC_2 = (100 - Q_1 - Q_2) Q_2 - 25 Q_2$$

Now take the derivative of each of the above with respect to Q:

$$(1) \text{ Max } \Pi_1 = 100 - 2Q_1 - Q_2 - 40$$

$$Q_1 = 30 - .5Q_2$$

$$(2) \text{ Max } \Pi_2 = 100 - 2Q_2 - Q_1 - 25$$

$$Q_2 = 37.5 - .5Q_1$$

Now solve the above simultaneously:

$$Q_1 = 30 - .5Q_2 = Q_1 = 30 - .5(37.5 - .5Q_1)$$

$$Q_1 = 15 \text{ and } Q_2 = 37.5 - .5(15) = 30$$

To find P we use the market demand curve:

$$P = 100 - Q = 100 - Q_1 - Q_2 = 100 - 15 - 30 = \$55$$

And finally each firm's profits:

$$\Pi_1 = P_1Q_1 - TC_1 = (55*15) - (40*15) = 825 - 600 = \$225 \text{ and}$$

$$\Pi_2 = P_2Q_2 - TC_2 = (55*30) - (25*30) = 1650 - 750 = \$900$$

The firm who invested in the cost-saving technology does save \$15 on the original 20 units it produced. However, there are additional effects: The firm is able to use its cost advantage to take shares from its rival.

The firm used to sell 20 units at \$60 (revenue \$1200). Now the firm sells 30 units at \$55 (revenue \$1,650). The firms used to jointly earn \$2,400, now they jointly earn \$2,475. So the consultant failed to count the gain in units (market share) the investing firm would enjoy.

Below we lay out the effects the firm who makes the investment gets:

$$\$20 * 15 = \$300 \text{ cost savings on original 20}$$

$$\$55 * 10 = \$550 \text{ new units sold at new price as a result of having a cost advantage}$$

$$\$-5 * 20 = -\$100 \text{ loss in revenue on original units as a result of selling them at a new price}$$

$$\$-25 * 10 = -\$250 \text{ cost of making 10 more units}$$

$$\text{Total increase in profits: } \$500 (\$900 - \$400)$$

While the consultant acknowledged the gain of \$300, the consultant did not address the other effects.