Exercise 8

- Repeat the derivation of the multitasking model – using

$$X_1 = \theta_1 e_1 + \varepsilon_1,\ E(\varepsilon_1) = 0 \text{ and } Var(\varepsilon_1) = \sigma_1^2$$
$$X_2 = \theta_2 e_2 + \varepsilon_2,\ E(\varepsilon_2) = 0 \text{ and } Var(\varepsilon_2) = \sigma_2^2$$

Exercise 8

- $W = \alpha + \beta_1 X_1 + \beta_2 X_2$ contract
- $W = \alpha + \beta_1 (X_1 + \gamma X_2)$ $\gamma = [\beta_2/\beta_1]$
- $W = \alpha + \beta_1 \xi$ $\xi = (X_1 + \gamma X_2)$
- $SS = X - C(e_1, e_2) - RP$
- $SS = \theta_1 e_1 + \theta_2 e_2 - (1/2)(e_1^2 + e_2^2) - (1/2)\rho\beta_1^2 \mathrm{var}(\xi)$

Exercise 8

- $var(\xi) = var(X_1 + \gamma X_2)$
- $var(\xi) = Var(X_1) + 2\gamma Cov(X_1, \gamma X_2) + \gamma^2 Var(X_2)$
- $var(\xi) = Var(\theta_1 e_1 + \varepsilon_1) + 2\gamma Cov(\theta_1 e_1 + \varepsilon_1, \gamma(\theta_2 e_2 + \varepsilon_2)) + \gamma^2 Var(\theta_2 e_2 + \varepsilon_2)$
- $var(\xi) = \sigma_1^2 + 2\gamma(r_{12}\sigma_1\sigma_2) + \gamma^2\sigma_2^2$
- So $\gamma^* = - r_{12}\sigma_1/\sigma_2$ **and** $\gamma^* = - \sigma_{12}/\sigma_2^2$

- Same as lecture 8 because $var(\theta_1 e_1) = var(\theta_2 e_2) = 0$

Exercise 8

- **Since** $SS = \theta_1 e_1 + \theta_2 e_2 - (1/2)(e_1^2 + e_2^2) - (1/2)\rho\beta_1^2 var(\xi)$, $e_1 = \beta_1\theta_1$ and $e_2 = \beta_2\theta_2$
- $d[\theta_1(\beta_1\theta_1) + \theta_2(\beta_2\theta_2) - (1/2)((\beta_1\theta_1)^2 + (\beta_2\theta_2)^2) - (1/2)\rho\beta_1^2 var(\xi)\,]/d\beta_1 = 0$
- $\theta_1^2 - \beta_1\theta_1^2 - \rho\beta_1 var(\xi) = 0$
- $\boldsymbol{\beta_1^* = \theta_1^2/(\theta_1^2 + \rho\, var(\xi))}$

- $\boldsymbol{\beta_1^* = \theta_1^2/(\theta_1^2 + \rho\, [1 - r_{12}^2]\sigma_1^2)}$

Exercise 9

Job Design

8.1 How does the presence of multi-tasking explain why piece rates are rare in manufacturing where some tasks are harder to monitor than others'?

8.2 Use of independent contractors involves "high-powered" incentives while employment involves "low-powered" incentives. Explain.

8.3 Employment is more likely to dominate use of an independent contractor as (i) the agent becomes increasingly risk averse, (ii) the variance of his performance measure increases, and (iii) the variance of the change of the asset value increases.

Exercise 9

8.1 Piece Rates – Disadvantages

- Non-uniformity
- Rewards luck – some workers have better areas or machines
- May conflict with worker's participation constraint – uncertainty of income levels
- May conflict with team production – downstream worker is tied to an upstream worker
- May substitute quantity for quality

Milgrom and Roberts - pp. 393 - 395

Exercise 9

8.2 and 8.3 will not be on term test 1

8.2 and 8.3 will be covered next term

Exercise 10

8.4 A risk-neutral principal employs a risk-averse agent to exert effort e_1 on Task 1 and e_2 on Task 2. The worker is paid $w = \alpha + \beta_1 z_1 + \beta_2 z_2$. where $z_1 = e_1 + x_1$, and $z_2 = e_2 + x_2$ and x_1 and x_2 are independent random variables with common means and variances, zero and V, respectively. The agent's coefficient of absolute risk aversion is r, the agent's cost of $\{e_1, e_2\}$ is $.5c(e_1 + e_2)^2$, and the principal's expected revenue is $P(e_1, e_2)$.

a) Describe the principal's and agent's payoffs.

b) Describe the solution to the agent's effort choice problem.

c) Explain why β_1 and β_2 is required to induce the agent to supply positive effort to both tasks.

d) Suppose that $P(e_1, e_2) = +p_1 e_1 + p_2 e_2$. Solve for e_1, e_2, β_1, β_2 when $p_1 > p_2$.

e) Suppose that $P(e_1, e_2) = (e_1)^{1/2} (e_2)^{1/2}$. Solve for e_1, e_2, β_1 and β_2.

f) When the cost function is $.5c(e_1)^2 + .5c(e_2)^2$, $\beta_1 = \beta_2$ is not required to induce the agent to supply positive effort to both tasks. Why does this cost function change the result from part c)?

Exercise 10

- 8.4 (a) – Apply Multi-tasking Model

- $W = \alpha + \beta_1 X_1 + \beta_2 X_2$ contract
- $\pi = X_1 + X_2 - W$ principal's payoff

- NB = W - C - RP
- $NB = \alpha + \beta_1 X_1 + \beta_2 X_2 - (\mathbf{c}/2)(e_1 + e_2)^2 - (1/2)\rho \mathrm{var}(W)$ agent's payoff

Exercise 10

- 8.4 (b) – Apply Multi-tasking Model
- $NB = \alpha + \beta_1 X_1 + \beta_2 X_2 - (\mathbf{c}/2)(e_1 + e_2)^2 - (1/2)\rho var(W)$ agent's payoff
- $d(NB)/de_1 = d[\alpha + \beta_1 e_1 + \beta_2 e_2 - (\mathbf{c}/2)(e_1^2 + 2e_1 e_2 + e_2^2) - (1/2)\rho var(W)]/de_1 = 0$

- $\beta_1 = \mathbf{c}(e_1 + e_2)$
- $\beta_2 = \mathbf{c}(e_2 + e_1)$

Exercise 10

- 8.4 (b) – Apply Multi-tasking Model

$$\beta_1 = \mathbf{c}(e_1 + e_2)$$
$$\beta_2 = \mathbf{c}(e_2 + e_1)$$

$\beta_2 = \beta_1$ is the first order condition

Exercise 10

- 8.4 (c) – Apply Multi-tasking Model

- $SS = e_1 + e_2 – (\mathbf{c}/2)(e_1^2 + 2e_1e_2 + e_2^2) – (1/2)\rho\beta_1^2 var(\xi)$
- $SS = \mathbf{(}\beta_1/\mathbf{c)} – (\mathbf{c}/2)\mathbf{(}\beta_1/\mathbf{c)^2} – (1/2)\rho\beta_1^2 var(\xi)$
- $d(SS)/d\beta_1 = d[\mathbf{(}\beta_1/\mathbf{c)} – (\mathbf{c}/2)\mathbf{(}\beta_1/\mathbf{c)^2} – (1/2)\rho\beta_1^2 var(\xi)]/d\beta_1 = 0$

Exercise 10

- 8.4 (c) – Apply Multi-tasking Model

- $d(SS)/d\beta_1 = 1/\mathbf{c} - \beta_1/\mathbf{c} - \rho\beta_1 var(\xi) = 0$
- $1/\mathbf{c} = \beta_1/\mathbf{c} + \rho\beta_1 var(\xi)$

- $\beta_1 = \beta_2 = 1/[1 + \rho\mathbf{c} var(\xi)]$

Exercise 10

- 8.4 (d)

- $\boldsymbol{\beta}_1* = p_1^2/[p_1^2 + \rho\, \mathbf{c}\mathrm{var}(\xi)]$
- $\boldsymbol{\beta}_2* = p_2^2/[p_2^2 + \rho\, \mathbf{c}\mathrm{var}(\xi)]$

Exercise 10

- 8.4 (e) will not be on term test 1

Exercise 10

- 8.4 (f) – Apply Lecture 8 Multi-tasking Model

- $\beta_1^* = 1/(1 + \rho c[1 - r_{12}^2]\sigma_1^2)$ **– See Exercise 5**
- $\beta_2^* = \gamma^* \beta_1^*$
- $\beta_2^* = [- \sigma_{12} /(\sigma_2^2 + \rho c[1 - r_{12}^2]\sigma_1^2\sigma_2^2)]$

- **The cost function is asymmetric.**

Exercise 11

8.5 Explain why there will tend to be more constraints on an agent's activities (i.e., more rules in the workplace) in situations where performance rewards are weak due to measurement problems.

8.6 (from Lazear (16)) What are the differences between multitasking and multiskilling? Under which circumstances is each likely to be important? When are different tasks likely to be grouped together to define jobs?

Exercise 11

- 8.5 and 8.6 will not be on term test 1
- 8.5 and 8.6 will be covered next term

Exercise 12

With respect to Lincoln Electric , explain why the piece rate system is expected to be complementary with each of (i) the bonus system, (ii) flexible work rules, (iii) work-in-progress inventories and (iv) worker-ownership arrangements.

Exercise 12

- Piece Rates – Advantages
- Simple
- Immediate – can be done on a per shift basis
- Studies show that productivity increases
- Individual Bonus – Performance based on:
  - skills acquired
  - individual output
  - profit or revenue generated

Milgrom and Roberts - p. 392

Exercise 13

- How do the parameters ρ, σ_1^2, k_1, k_2 and κ impact the problem of `moonlighting` in the multitasking model

- Will not be on term test 1

Exercise 14

- What is α in $W_1 = \alpha + \beta_1 X_1$ if there is a ban on moonlighting
- What is α in $W_1 = \alpha + \beta_1 X_1$ if there is no ban on moonlighting

- Will not be on term test 1

Exercise 15

- Will not be on term test 1
- Prove:

- $\alpha_1^* = (2\rho_1\kappa_1\sigma_{w1}^2 - 1) / (4\kappa_1 (1 + 2\rho_1\kappa_1\sigma_{w1}^2)$
- $\alpha_2^* = (2\rho_2\kappa_2\sigma_{w2}^2 - 1) / (4\kappa_2 (1 + 2\rho_2\kappa_2\sigma_{w2}^2)$

Exercise 16

- Will not be on term test 1
- Derive
- $\alpha_1^* = \beta_1\beta_2^2\sigma_{12} / \sigma_2^2 - (1/2)\beta_1^2(1 - \rho(\sigma_1^2 - 2\sigma_{12}^2/\sigma_2^2 + \sigma_{12}^2\sigma_2^2))$
- $\alpha_2^* = \beta_2\beta_1^2\sigma_{12} / \sigma_1^2 - (1/2)\beta_2^2(1 - \rho(\sigma_2^2 - 2\sigma_{12}^2/\sigma_1^2 + \sigma_{12}^2\sigma_1^2))$

Exercise 17

- Reflect on the differences between the 2 team production models
- In what sense could the agents collude and what impact would this have on the model

Exercise 17

- Equilibrium when $\mathbf{e_1 = e_2 = 1}$:
- **SS = 1 – (1⁄2) – 0 = 0.5**
- Equilibrium when $\mathbf{e_1 = e_2 = 0.5}$:
- **SS = 0.5 – (1⁄2)(0.25) – 0 = 0.5 – 0.375 = 0.125**
- Nash Equilibrium is $\mathbf{e_1 = e_2 = 1}$:

Exercise 17

Agent 2

Agent 1	$e_2 = 1$	$e_2 = 0.5$
$e_1 = 1$	9.5, 9.5	12.83, 6.54
$e_1 = 0.5$	6.54, 12.83	9.875, 9.875

Nash Equilibrium

Pareto Optimal Equilibrium

$NB_1 = (1/(0.5+1)/2)*10 - e_1^2/2$, $NB_2 = (1/(0.5+1)/2)*10 - e_2^2/2$

Exercise 17

		Agent 2	
		$e_2 = 1$	$e_2 = 0.5$
Agent 1	$e_1 = 1$	9.5, 9.5	12.83, 6.54
	$e_1 = 0.5$	6.54, 12.83	9.875, 9.875

Nash Equilibrium

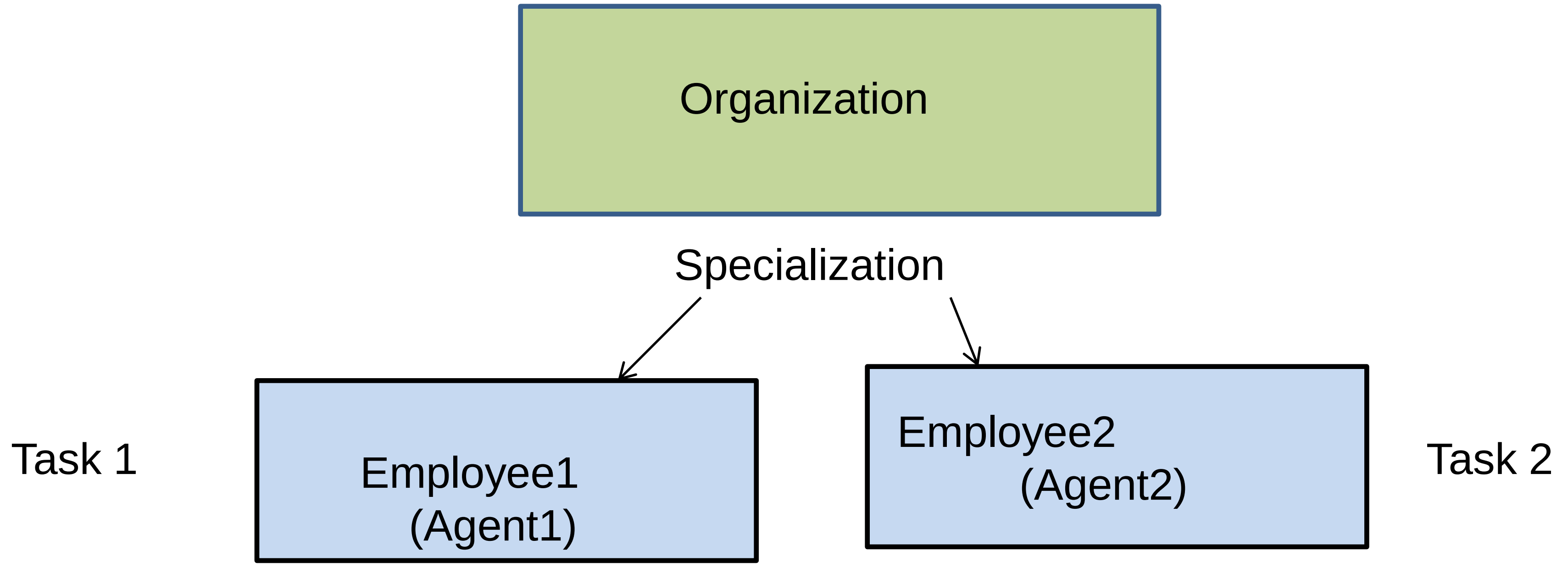
Pareto Optimal Equilibrium

$NB_1 = \mathbf{(1/(1+1)/2)}*10 - e_1^2\mathbf{/2}$, $NB_2 = \mathbf{(1/(1+1)/2)}*10 - e_2^2\mathbf{/2}$

Exercise 18

Look at two employees in organizations. Their outputs are given by $X_1 = 2e_1 + \varepsilon_1$ and $X_2 = 2e_2 + \varepsilon_2$. The exogenous shocks are identically normally distributed with zero means, variance $\sigma^2 = 0.5$ and covariance $\sigma_{12} = 0.25$. Workers are identical in terms of their preferences: CARA preferences and the risk-aversion coefficient $\rho = 4$, cost function $C_i = 0.5e^2$, and zero reservation utility.

Exercise 18


Organization
Specialization
Task 1
Employee1
(Agent1)
Employee2
(Agent2)
Task 2

Exercise 18

- Relative Performance Evaluation (Linear) Contract:

Components of the Model:

$C(e_1,e_2) = (1/2)(e_1^2 + e_2^2)$ effort of two (2) agents

$W = \alpha + \beta_1 X_1 + \beta_2 X_2$ contract

Risk Premium = $(\rho/2)(\beta_1^2 + 2\tau\beta_1\beta_2 + \beta_2^2)\sigma^2$

* $X_1 = \theta_1 e_1 + \varepsilon_1$, $\theta_2 X_2 = e_2 + \varepsilon_2$

Exercise 18

- Relative Performance Evaluation (Linear) Contract:

$e_1 = \theta_1\beta_1$, $e_2 = \theta_1\beta_2$, incentive compatibility constraints

$$\beta_1 = \theta_1^2/[\theta_1^2 + \rho\sigma_1^2(1 - \tau^2)]$$

$$\text{Risk Premium} = (1/2)\rho\sigma^2(1 - \sigma_{12})$$

Exercise 18

- $\beta_1 = \theta^2/[\theta^2 + \rho\sigma^2(1 - \sigma_{12})]$
- Contrast with $\beta_1 = \theta^2/[\theta^2 + \rho\sigma^2(1 - \tau^2)]$ for multi-tasking

- $-\sigma_{12} = \tau^2$ $\quad\quad - 0.25 = -(0.5)^2$

Team

Exercise 18

- $\beta_1 = \theta^2/[\theta^2 + \rho\sigma^2(1 - \sigma_{12})]$
- $\beta_1 = (2)^2/[(2)^2 + (4)(0.5)(1 – (0.25))]$
- $\beta_1 = 4/[4 + 2(0.75)]$
- $\beta_1 = 4/[4 + 1.5]$
- $\beta_1 = 4/5.5 = 0.7273$
- $\beta*_2 = \beta*_1 = 0.7273$

Team

Exercise 18

- $NB = W – C – R = 0$ (Participation constraint)
- $C = (1/2)e^2 = (1/2)(2\beta)^2 = 2\beta^2$
- $R = (1/2)\rho\beta^2\sigma^2(1 - \tau^2)$

  $R = (1/2)(4)\beta^2(1/2)(1 - \tau^2) = \beta^2(1 - \tau^2)$
- $W = \alpha+\beta X = \alpha + \beta(2)(2\beta)(1 – \tau)= \alpha+4\beta^2(1 – \tau)$
- $W – C – R = 0$
- $\alpha + 4\beta^2(1 – \tau) – 2\beta^2 – \beta^2(1 - \tau^2) = 0$

Team

Exercise 18

- $NB = W – C – R = 0$ (Participation constraint)
- $\alpha + 4\beta^2(1 – \tau) – 2\beta^2 – \beta^2(1 - \tau^2) = 0$
- $\alpha = 2\beta^2 + \beta^2(1 - \tau^2) - 4\beta^2(1 – \tau)$
- $\alpha = \beta^2(1 + 2 - 4 - \tau^2 + 4\tau)$
- $\alpha = \beta^2(4\tau - \tau^2 – 1)$
- $\alpha = (0.7273)^2(4(0.5) – (0.25) – 1)$
- $\alpha^* = (0.7273)^2(0.75) = 0.5455$

Team

Exercise 18

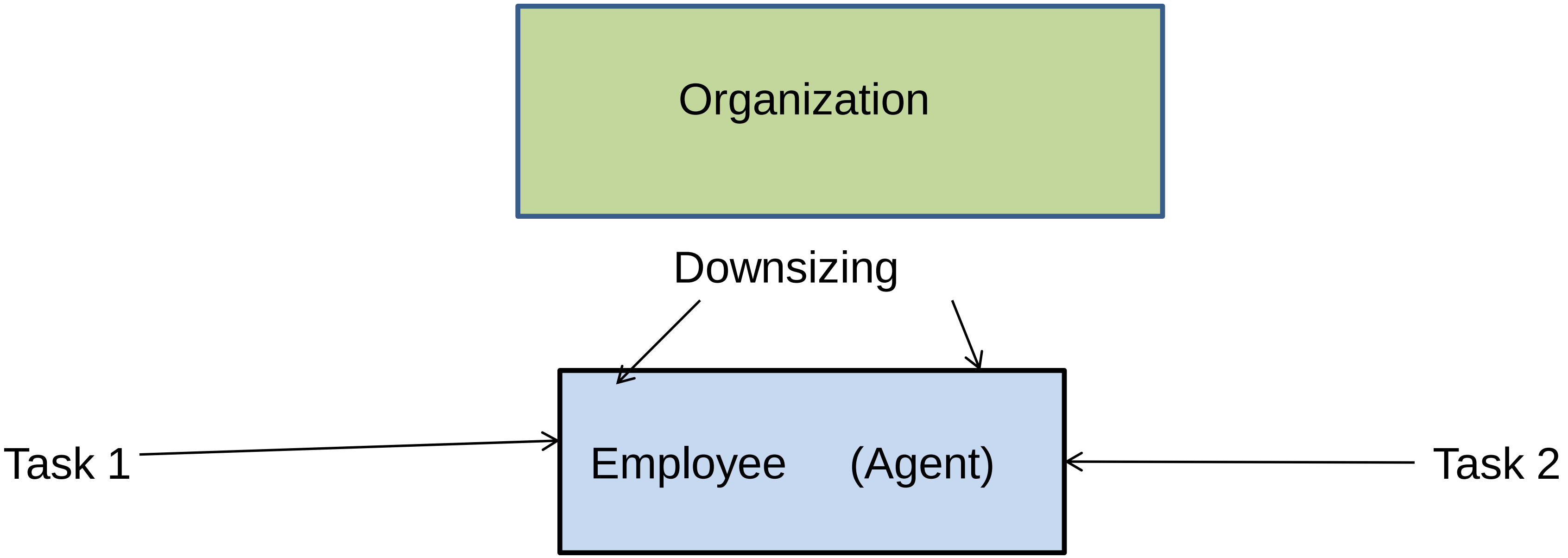
- Organization`s expected surplus:
- $S = E(\pi_1) + E(NB_1) + E(\pi_2) + E(NB_2)$
- $S = (2)(X - W + NB)$
- $S = 2(X – C – R)$
- $S = 2(4\beta – 2\beta^2 – \beta^2(1 - \tau^2))$
- $S = 2(4\beta – \beta^2(3 - \tau^2))$
- $S = 2(0.7273)(4 – (0.7273)(3 – 0.25))$
- $S^* = (1.4546)(2) = 2.9092$

Team

Exercise 19

Now, suppose that the firm can combine the two positions above into one and offer the resulting multi-tasked job an individual with the cost preferences $C = 0.5e_1^2 + 0.5e_2^2$. The firm can still observe individual performance measures.

Exercise 19



Exercise 19

- $\beta_2 = -\tau\beta_1$
- $\beta_1 = \theta^2/[\theta^2 + \rho\sigma^2(1 - \tau)] = \theta^2/[\theta^2 + \rho\sigma_1{}^2 - \tau\sigma_2{}^2]$
- $\tau = -0.5$ (substitute skill)
- $\beta_1 = \theta^2/[\theta^2 + \rho\sigma^2(1 - \tau)]$
- $\beta_1 = (2)^2/[(2)^2 +$

  $(4)(1/2)(1 - (-0.5))]$
- $\beta_1 = 4/[4 + 2(1.5)] = 0.5714$

Multitasking

Exercise 19

- $NB = W - C - R = 0$ (Participation constraint)
- $C = (1/2)(e_1^2 + e_2^2) = (1/2)[(2\beta)^2 + (2\beta)^2] = 4\beta^2$
- $R = \rho\beta^2\sigma^2(1 - \tau) = 2[(1/2)(4)\beta^2(0.5)(1.5)] = 3\beta^2$
- $W = \alpha + \beta_1 X_1 + \beta_2 X_2 = \alpha + \beta(\theta_1 e_1 + \theta_2 e_2)$

  $= \alpha + \beta(2(2\beta) + 2(2\beta))$

  $= \alpha + 8\beta^2$

- $W - C - R = 0$
- $\alpha + 8\beta^2 - 4\beta^2 - 3\beta^2 = 0$

Multitasking

Exercise 19

- $NB = W - C - R = 0$ (Participation constraint)
- $\alpha = 4\beta^2 + 3\beta^2 - 8\beta^2$
- $\alpha = -\beta^2$
- $\alpha = -(0.5714)^2 = -0.3265$

Multitasking

Exercise 19

- Organization`s expected surplus:
- $S = E(\pi) + E(NB)$
- $S = (X - W + NB)$
- $S = (X – C – R)$
- $S = 8\beta – 4\beta^2 – \beta^2(1 + t)$
- $S = 8\beta – 4\beta^2 – \beta^2(1.5)$
- $S = 8(0.5714) – 5.5(0.5714)^2$

Team

Exercise 19

- Organization`s expected surplus:

- $S = 8(0.5714) - 5.5(0.5714)^2$
- $S = 4.5712 - 5.5(0.3265)$
- $S = 4.5712 - 1.7957$
- $S = 4.5712 - 1.7957 = 2.7755$

Team

Exercise 19

- Compare total surplus under specialization and multi-tasking. Which alternative is preferred by the firm?

- Surplus under specialization = 2.9092
- Surplus under downsizing = 2.7755

Exercise 19

Multitasking - Downsizing

- $W_{Multitasking} = \alpha + 8\beta^2$
- $W_{Multitasking} = (-0.3265) + 8(0.5714)^2$
- $W_{Multitasking} = 7(0.3265) = 2.2855$

Team - Specializing

- $W_{Specialization} = \alpha + 4\beta^2(1 - \tau)$
- $W_{Specialization} = (0.5455) + 4(0.7273)^2(0.5)$
- $W_{Specialization} = (0.5455) + 2(0.5290)$
- $W_{Specialization} = 1.6035$