Exercise 1

- Compute the equilibrium values of e*, X*, W*, π* and SS* when

$$\beta^* = \theta^2/(\theta^2 + \rho\sigma^2)$$

$$\alpha^* = (\theta^4/2)[\rho\sigma^2 - \theta^2]/[\rho\sigma^2 + \theta^2]^2$$

$$e^* = \theta\beta$$

$$e^* = \theta^3/(\theta^2 + \rho\sigma^2)$$

$$X^* = \theta e^* = \theta^4/(\theta^2 + \rho\sigma^2)$$

$$W^* = \alpha^* + \beta^* X^*$$

$$W^* = \alpha^* + \beta^* X^*$$

$$W^* = \alpha^* + [\theta^2/(\theta^2 + \rho\sigma^2)][\theta^4/(\theta^2 + \rho\sigma^2)]$$

$$W^* = \alpha^* + \theta^6/(\theta^2 + \rho\sigma^2)^2$$

$$W^* = (\theta^4/2)[\rho\sigma^2 - \theta^2]/[\rho\sigma^2 + \theta^2]^2 + \theta^6/(\theta^2 + \rho\sigma^2)^2$$

$$W^* = (\theta^4/2)[\rho\sigma^2 - \theta^2 + 2\theta^2]/[\rho\sigma^2 + \theta^2]^2$$

$$W^* = [(\theta^4/2)(\rho\sigma^2 + \theta^2)]/(\theta^2 + \rho\sigma^2)^2$$

$$W^* = [\theta^4/2][1/(\theta^2 + \rho\sigma^2)]$$

$$W^* = \theta^4/[2(\theta^2 + \rho\sigma^2)]$$

$$\pi^* = X^* - W^*$$

$$X^* = \theta^4/(\theta^2 + \rho\sigma^2)$$

$$W^* = \theta^4/[2(\theta^2 + \rho\sigma^2)]$$

$$\pi^* = \theta^4/(\theta^2 + \rho\sigma^2) - \theta^4/[2(\theta^2 + \rho\sigma^2)]$$

$$\pi^* = \theta^4/[2(\theta^2 + \rho\sigma^2)]$$

$$SS^* = (X^* - W^*) + (W^* - (1/2)e^{*2} - RP^*)$$

$$SS^* = X^* - (1/2)e^{*2} - RP^*$$

$$SS^* = \theta^4/(\theta^2 + \rho\sigma^2) - (1/2)[\theta^3/(\theta^2 + \rho\sigma^2)]^2 - (1/2)\rho[\theta^2/(\theta^2 + \rho\sigma^2)]^2\sigma^2$$

$$SS^* = \theta^4(\theta^2 + \rho\sigma^2)/(\theta^2 + \rho\sigma^2)^2 - (1/2)[\theta^3/(\theta^2 + \rho\sigma^2)]^2 - (1/2)\rho[\theta^2/(\theta^2 + \rho\sigma^2)]^2\sigma^2$$

$$SS^* = (1/2)[2\theta^4(\theta^2 + \rho\sigma^2) - \theta^6 - \rho\theta^4\sigma^2]/(\theta^2 + \rho\sigma^2)^2$$

- SS* = $(1/2)[2\theta^6 + 2\rho\theta^4\sigma^2 - \theta^6 - \rho\theta^4\sigma^2]/(\theta^2 + \rho\sigma^2)^2$
- SS* = $(1/2)[\theta^6 + \rho\theta^4\sigma^2]/(\theta^2 + \rho\sigma^2)^2$
- SS* = $(\theta^4/2)[\theta^2 + \rho\sigma^2]/(\theta^2 + \rho\sigma^2)^2$

- SS* = $\theta^4/2(\theta^2 + \rho\sigma^2)$

Exercise 2

- Compute the equilibrium values of β^*, α^*, e^*, X^*, W^*, π^* and SS^* when perfect information between principal and agent applies (for example – $RP^* = 0$)

In the first best solution, effort e is observable. The principal only needs to pay a fixed salary α to the agent to guarantee that the agent is willing to participate in accordance with the individual agent's rationality constraint.

- Under perfect information, the net benefit (NB) for the agent is determined to be:

$$\alpha + \beta\theta e - (1/2)(e)^2$$

- The first order condition is:

$$d[\alpha + \beta\theta e - (1/2)(e)^2]/de = 0$$

$$0 + \beta\theta + 0 - (e)(1) - 0 = 0$$

$$e = \beta\theta$$

this is the incentive compatibility constraint and is binding on the principal

- $S = (X - W) + NB$
- $S = X - \alpha - \beta\theta e + \alpha + \beta\theta e - (1/2)(e)^2$
- $S = X - (1/2)(e)^2$
- $S = \theta e - (1/2)(e)^2$
- Setting $e = \beta\theta$
- $S = \beta\theta^2 - (1/2)(\beta\theta)^2$

- The first order condition of the second stage:

$$d[\beta\theta^2 - (1/2)(\beta\theta)^2]/d\beta = 0$$

$$\theta^2 - \beta\theta^2 = 0$$

$$1 - \beta = 0$$

$$\beta^* = 1$$

- Applying the agent's reservation utility

$\alpha + \beta\theta e - (1/2)(e)^2 \geq 0$

- Take the smallest cost

$\alpha + \beta\theta e - (1/2)(e)^2 = 0$

$\alpha = (1/2)(\theta\beta)^2 - \beta\theta(\beta\theta)$

$\alpha = - (1/2)(\theta\beta)^2$

$\alpha* = - (1/2)(\theta)^2$

- Now compute the equilibrium values of e*, X*, W*, π* and SS* when

$$\beta^* = 1$$

$$\alpha^* = -(\theta^2/2)$$

$$e^* = \theta\beta$$

$$e^* = \theta\mathbf{(1) =} \theta$$

$$X^* = \theta e^* = \theta^2$$

$$W^* = \alpha^* + \beta^* X^*$$

$$W^* = -(\theta^2/2) + (1)(\theta^2)$$

$$W^* = (\theta^2/2) = C(e)$$

Note: **Under perfect information**

$W^* = C(e)^*$ **and** $NB^* = 0$

Does this make sense?

Why?

$$\pi^* = X^* - W^*$$

$$\pi^* = \theta^2 - (\theta^2/2) = (\theta^2/2)$$

$$SS^* = X^* - C(e)^* - RP^*$$

$$SS^* = \theta^2 - (\theta^2/2) - \mathbf{0}$$

$$SS^* = (\theta^2/2)$$

Note: **Under perfect information** $SS^* = \pi^*$

Does this make sense?

Why?

Exercise 3

- What happens if instead of maximizing social surplus, the principal maximizes profit?

  (The principal would optimize $\pi = X - W$ where $W = \alpha + \beta X$)

- Under imperfect information, the net benefit (NB) for the agent is still:

$$\alpha + \beta\theta e + \beta\varepsilon - (1/2)(e)^2 - (1/2)\rho\beta^2\sigma^2$$

- So the incentive compatibility constraint is still $e = \beta\theta$ and is binding on the principal

- However, the organization maximizes π not SS:

- $\pi = X - W$
- $\pi = X - \alpha - \beta X - \mathbf{(1/2)}\beta^{\mathbf{2}}\rho\sigma^2$
- $\pi = \theta e - \alpha - \beta\theta e - \mathbf{(1/2)}\beta^{\mathbf{2}}\rho\sigma^2$
- $\pi = \theta(\beta\theta) - \alpha - \beta\theta(\beta\theta) - \mathbf{(1/2)}\beta^{\mathbf{2}}\rho\sigma^2$
- $\pi = \beta\theta^2 - \alpha - \beta^2\theta^2 - \mathbf{(1/2)}\beta^{\mathbf{2}}\rho\sigma^2$

- The organization maximizes π by taking the first order condition:
- $d\pi/d\beta = d(\beta\theta^2 - \alpha - \beta^2\theta^2 - \mathbf{(1/2)}\beta^{\mathbf{2}}\rho\boldsymbol{\sigma}^2)/d\beta$
  $= 0$
- $\theta^2 - 2\beta\theta^2 - \beta\rho\boldsymbol{\sigma}^2 = 0$
- $2\beta\theta^2 + \beta\rho\boldsymbol{\sigma}^2 = \theta^2$
- $\beta^\pi = \theta^2/(2\theta^2 + \rho\boldsymbol{\sigma}^2) < \theta^2/(\theta^2 + \rho\boldsymbol{\sigma}^2) = \beta^{SS}$

Note: If the agent's rationality constraint is binding on the profit-maximizing principal, then $\beta^\pi = \beta^{\mathbf{SS}}$

- Applying the agent's reservation utility and taking the smallest cost:

- $\alpha^{\pi} = (1/2)(\beta^{\pi})^2 [\rho\sigma^2 - \theta^2]$
- $\alpha^{\pi} = (1/2)[\theta^2/(2\theta^2 + \rho\sigma^2)]^2[\rho\sigma^2 - \theta^2]$
- $\alpha^{\pi} = (1/2)[\theta^4/(2\theta^2 + \rho\sigma^2)^2[\rho\sigma^2 - \theta^2]$
- $\alpha^{\pi} = (\theta^4/2)[\rho\sigma^2 - \theta^2]/[\rho\sigma^2 + 2\theta^2]^2$

$$e^{\pi} = \theta\beta^{\pi}$$

$$e^{\pi} = \theta^3/(2\theta^2 + \rho\sigma^2) < e^{SS}$$

$$X^{\pi} = \theta e^* = \theta^4/(2\theta^2 + \rho\sigma^2) < X^{SS}$$

$$SS^{\pi} = X^{\pi} - (1/2)(e^{\pi})^2 - RP^{\pi}$$

$$SS^{\pi} = \theta^4/(2\theta^2 + \rho\sigma^2) - (1/2)[\theta^3/(2\theta^2 + \rho\sigma^2)]^2 - (1/2)\rho[\theta^2/(2\theta^2 + \rho\sigma^2)]^2\sigma^2$$

- $SS^{\pi} = (1/2)[2\theta^4(2\theta^2 + \rho\sigma^2) - \theta^6 - \rho\theta^4\sigma^2]/(2\theta^2 + \rho\sigma^2)^2$
- $SS^{\pi} = (1/2)\ [3\theta^6 + \rho\theta^4\sigma^2]/(2\theta^2 + \rho\sigma^2)^2$
- $SS^{\pi} = (\theta^4/2)\ [3\theta^2 + \rho\sigma^2]/(2\theta^2 + \rho\sigma^2)^2 < SS^{SS}$

Note: If the agent's rationality constraint is binding on the profit-maximizing principal, then $SS^{\pi} = SS^{SS}$

Note: If the agent's rationality constraint is binding on the profit-maximizing principal, then

$$
\begin{aligned}
E(\pi) &= E(X - W) \\
&= e - \underline{\alpha} - \beta e \\
&= (1-\beta)e - \underline{\alpha} \\
&= (1-\beta)e - \underline{W} - (1/2)e^2 - (1/2)\rho\beta^2\sigma^2 \\
&= SS
\end{aligned}
$$

Exercise 4

3. The principal hires the agent to complete the project. The risk-averse agent supplies effort e to produce output $X = e + \epsilon$, where $\epsilon \sim N(0, \sigma^2)$ is a normally distributed exogenous noise component. The agent has CARA utility preferences, with the coefficient of risk aversion ρ. His cost function is given by $C = (e - T)^2$, where T is a constant. The agent's reservation utility is $U_0 = 0$. Find the optimal linear incentive contract $\{a, b\}$ which is offered by the principal.

$$\alpha = ([4T + 1]\rho\sigma^2 + 2T - 1) / (1 + 2\rho\sigma^2)^2$$

$$\beta = 1/(1 + 2\rho\sigma^2)$$

Exercise 4

- Under imperfect information, the net benefit (NB) for the agent is determined to be:

$$NB = E(W) – C - (1/2)\rho \text{ var}(W)$$

$$NB = \alpha + \beta e - (e - T)^2 – (1/2)\rho\beta^2\sigma^2$$

Exercise 4

$$d[NB]/de = 0$$

$$d[\alpha + \beta e - (e - T)^2 - (1/2)\rho\beta^2\sigma^2]/de = 0$$

$$0 + \beta - 2(e - T) - 0 = 0$$

$$2(e - T) = \beta$$

$$2e = \beta + 2T$$

$$e = \beta/2 + T$$

Exercise 4

- $SS = (X - W) + NB$
- $SS = (X - W) + (W - C - RP)$
- $SS = X - C - RP$
- $SS = X - (e - T)^2 - (1/2)\rho\beta^2\sigma^2$
- $SS = e - (e - T)^2 - (1/2)\rho\beta^2\sigma^2$
- $SS = \beta/2 + T - (\beta/2)^2 - (1/2)\rho\beta^2\sigma^2$

Exercise 4

- $d(SS)/d\beta = d(\beta/2 + T - \beta^2/4 - (1/2)\rho\beta^2\sigma^2)/d\beta = 0$
- $1/2 - \beta/2 - \rho\beta\sigma^2 = 0$
- $1 - \beta - 2\rho\beta\sigma^2 = 0$
- $\beta(1 + 2\rho\sigma^2) = 1$
-
- $\beta = 1/(1 + 2\rho\sigma^2)$

Exercise 4

Let $NB* = 0$

Setting NB = 0 makes profit maximization equivalent to the (surplus) value maximization principle

$NB = W – C – RP = 0$

$NB = \alpha + \beta e - (e - T)^2 – (1/2)\rho\beta^2\sigma^2 = 0$

$\alpha = (e - T)^2 + (1/2)\rho\beta^2\sigma^2 - \beta e$

$\alpha = \beta^2/4 + (1/2)\rho\beta^2\sigma^2 – \beta^2/2 + \beta T$

Exercise 4

$\alpha = (1/2)\rho\beta^2\sigma^2 - \beta^2/4 + \beta T$

$\alpha = (1/2)\beta^2[\rho\sigma^2 - 1/2] + \beta T$

$\alpha = (\rho\sigma^2 - 1)/2(1 + 2\rho\sigma^2)^2 + 2T(1 + 2\rho\sigma^2)$
$/(1 + 2\rho\sigma^2)^2$

$\alpha = ([4T + 1]\rho\sigma^2 + 2T - 1) /(1 + 2\rho\sigma^2)^2$

Exercise 5

- Compute the equilibrium values of β^*, α^*, e^*, X^*, W^*, π^* and SS^* when imperfect information between principal and agent applies, $X = \theta e + \varepsilon$ and $C(e) = (\kappa/2)(e)^2$ where κ measures the agents aversion to work

Exercise 5

- $\beta* = \theta^2/\theta^2 + \rho\kappa\sigma^2$
- $\alpha* = (\theta^4)(\rho\kappa\sigma^2 - \theta^2)/2\kappa(\theta^2 + \rho\kappa\sigma^2)^2$
- $e* = (\theta/\kappa)(\theta^2/\theta^2 + \rho\kappa\sigma^2)$
- $X* = (\theta^4/\kappa)(\theta^2 + \rho\kappa\sigma^2)$
- $W* = \theta^4/2\kappa(\theta^2 + \rho\kappa\sigma^2)$
- $SS* = \pi* = \theta^4/2\kappa(\theta^2 + \rho\kappa\sigma^2)$

Exercise 5

- Under imperfect information, the net benefit (NB) for the agent is determined to be:

  $NB = E(W) – C - (1/2)\rho \ var(W)$

  $NB = E(\alpha + \beta\theta e + \beta\varepsilon) - (\kappa/2)(e)^2 – (1/2)\rho\beta^2\sigma^2$

Exercise 5

$d[NB]/de = 0$

$d[\alpha + \beta\theta e - (\kappa/2)(e)^2 - (1/2)\rho\beta^2\sigma^2]/de = 0$

$0 + \beta\theta + 0 - \kappa e - 0 = 0$

$e = \beta\theta/\kappa$

Exercise 5

- $SS = (X - W) + NB$
- $SS = (X - W) + (W - C - RP)$
- $SS = X - C - RP$
- $SS = X - (\kappa/2)(e)^2 - (1/2)\rho\beta^2\sigma^2$
- $SS = \theta e - (\kappa/2)(e)^2 - (1/2)\rho\beta^2\sigma^2$
- $SS = \theta\beta\theta/\kappa - (\kappa/2)(\beta\theta/\kappa)^2 - (1/2)\rho\beta^2\sigma^2$

Exercise 5

- $d(SS)/d\beta = d[\theta^2\beta/\kappa - \beta^2\theta^2/2\kappa - (1/2)\rho\beta^2\sigma^2]/d\beta = 0$
- $\theta^2/\kappa - \beta\theta^2/\kappa - \rho\beta\sigma^2 = 0$
- $\theta^2/\kappa = \beta[\theta^2/\kappa + \rho\sigma^2]$
- $\beta = [\theta^2/\kappa\ ]/[\theta^2/\kappa + \rho\sigma^2]$

- $\beta* = \ \theta^2/\theta^2 + \rho\kappa\sigma^2$

Exercise 5

Let NB$*$ = 0

Setting NB = 0 makes profit maximization equivalent to the (surplus) value maximization principle

$\alpha* + \beta\theta e - (\kappa/2)(e)^2 - (1/2)\rho\beta^2\sigma^2 = 0$

Since $e = \beta\theta/\kappa$

$\alpha* + \beta\theta(\beta\theta/\kappa) - (\kappa/2)(\beta\theta/\kappa)^2 - (1/2)\rho\beta^2\sigma^2 = 0$

$\alpha* + \beta^2\theta^2/\kappa - \beta^2\theta^2/2\kappa - (1/2)\rho\beta^2\sigma^2 = 0$

$\alpha* + \beta^2\theta^2/2\kappa - (1/2)\rho\beta^2\sigma^2 = 0$

$\alpha* = (1/2)\rho\beta^2\sigma^2 - \beta^2\theta^2/2\kappa$

Exercise 5

- $\alpha* = (1/2)\rho\beta^2\sigma^2 - \beta^2\theta^2/2\kappa$
- $\alpha* = (\beta^2/2)(\rho\sigma^2 - \theta^2/\kappa)$
- $\alpha* = (\beta^2/2)(\rho\sigma^2 - \theta^2/\kappa)$
- Since $\beta* = \theta^2/\theta^2 + \rho\kappa\sigma^2$
- $\alpha* = (1/2\kappa)(\theta^2/\theta^2 + \rho\kappa\sigma^2)^2(\rho\kappa\sigma^2 - \theta^2)$
- $\alpha* = (\theta^4)(\rho\kappa\sigma^2 - \theta^2)/2\kappa(\theta^2 + \rho\kappa\sigma^2)^2$

Exercise 5

- d[NB]/de = 0
- $e^* = \beta * \theta/\kappa$
- $e^* = (\theta/\kappa)(\theta^2/\theta^2 + \rho\kappa\sigma^2)$

- $X^* = \theta e^*$
- $X^* = (\theta^2/\kappa)(\theta^2/\theta^2 + \rho\kappa\sigma^2)$
- $X^* = (\theta^4/\kappa)(\theta^2 + \rho\kappa\sigma^2)$

Exercise 5

- $W^* = \alpha^* + \beta^* X^*$

- $W^* = (\theta^4)(\rho\kappa\sigma^2 - \theta^2)/2\kappa(\theta^2 + \rho\kappa\sigma^2)^2 + (\theta^2/\theta^2 + \rho\kappa\sigma^2)(1/\kappa)(\theta^4/\theta^2 + \rho\kappa\sigma^2)$
- $W^* = (\theta^4)(\rho\kappa\sigma^2 - \theta^2 + 2\theta^2)/2\kappa(\theta^2 + \rho\kappa\sigma^2)^2$
- $W^* = (\theta^4)(\rho\kappa\sigma^2 + \theta^2)/2\kappa(\theta^2 + \rho\kappa\sigma^2)^2$
- $W^* = \theta^4/2\kappa(\theta^2 + \rho\kappa\sigma^2)$

Exercise 5

- $\pi^* = X - W$
- $\pi^* = \theta^4/\kappa(\theta^2 + \rho\kappa\sigma^2) - \theta^4/2\kappa(\theta^2 + \rho\kappa\sigma^2)$
- $\pi^* = \theta^4/2\kappa(\theta^2 + \rho\kappa\sigma^2)$
- $SS^* = \pi^* + NB$
- $SS^* = \theta^4/2\kappa(\theta^2 + \rho\kappa\sigma^2) + 0$
- $SS^* = \theta^4/2\kappa(\theta^2 + \rho\kappa\sigma^2)$
- Setting NB = 0 makes profit maximization equivalent to the (surplus) value maximization principle

Exercise 5

- $SS^* = \theta^2\beta/\kappa - \beta^2\theta^2/2\kappa - (1/2)\rho\beta^2\sigma^2$
- $SS^* = (1/2\kappa)2\theta^2(\beta) - (1/2\kappa)(\beta)^2(\theta^2 - \rho\kappa\sigma^2)$
- $SS^* = (1/2\kappa)2\theta^2(\theta^2/\theta^2 + \rho\kappa\sigma^2) - (1/2\kappa)(\theta^2/\theta^2 + \rho\kappa\sigma^2)^2(\theta^2 - \rho\kappa\sigma^2)$
- $SS^* = (\theta^4/2\kappa)2(1/\theta^2 + \rho\kappa\sigma^2) - (\theta^4/2\kappa)(1/\theta^2 + \rho\kappa\sigma^2)^2(\theta^2 - \rho\kappa\sigma^2)$
- $SS^* = 2\theta^4(\theta^2 + \rho\kappa\sigma^2)/(2\kappa)(\theta^2 + \rho\kappa\sigma^2)^2 - \theta^4(\theta^2 - \rho\kappa\sigma^2)/(2\kappa\ )(\theta^2 + \rho\kappa\sigma^2)^2$

Exercise 5

- $SS^* = 2\theta^4(\theta^2 + \rho\kappa\sigma^2)/(2\kappa)(\theta^2 + \rho\kappa\sigma^2)^2 - \theta^4(\theta^2 - \rho\kappa\sigma^2)/(2\kappa)(\theta^2 + \rho\kappa\sigma^2)^2$
- $SS^* = \theta^4(2\theta^2 - \theta^2 + 2\rho\kappa\sigma^2 - \rho\kappa\sigma^2)/(2\kappa)(\theta^2 + \rho\kappa\sigma^2)^2$
- $SS^* = \theta^4(\theta^2 + \rho\kappa\sigma^2)/(2\kappa)(\theta^2 + \rho\kappa\sigma^2)^2$
- $SS^* = \theta^4/(2\kappa)(\theta^2 + \rho\kappa\sigma^2)$
- $SS^* = \theta^4/2\kappa(\theta^2 + \rho\kappa\sigma^2)$

Exercise 6

- Compute the equilibrium values of β*, α*, e*, X*, W*, π* and SS* when utility is expressed as U(W – C(e)) and X = F(e) + ε

See Zhou, Xianming "A Graphical Approach to the Standard Principal-Agent", The Journal of Economic Education, Vol. 33, No. 3 (Summer, 2002), pp. 265-276

Exercise 6

- Under imperfect information, the net benefit (NB) for the agent is determined to be:

$$NB = U(W - C(e)) - (1/2)\rho \text{ var}(W) = 0$$

$$NB = U(\alpha^* + \beta^*(F(e) + \varepsilon) - C(e)) - (1/2)\rho \text{ var}(\alpha^* + \beta^*(F(e) + \varepsilon)) = 0$$

$$NB = U(\alpha^* + \beta^*(F(e) + \varepsilon) - C(e)) - (1/2)\rho\beta^2\sigma^2 = 0$$

Exercise 6

$$d[NB]/de = 0$$

$$d[U(\alpha + \beta(F(e) + \varepsilon) - C(e)) - (1/2)\rho\beta^2\sigma^2]/de = 0$$

$$U'(\alpha + \beta(F(e) + \varepsilon) - C(e))(\beta F'(e) - C'(e)) = 0$$

$$\beta F'(e) = C'(e) \qquad C'(e) = dC/de$$

$$F'(e) = dF/de$$

$\beta* = C'(e)/F'(e)$

So $\beta* = C'(e)/F'(e)$ is the incentive compatibility constraint and is binding on the principal

Exercise 6

- $SS = (X - W) + NB$
- $SS = (X - W) + (W - C - RP)$
- $SS = F(e) - \alpha - \beta F(e) + U(W - C(e)) - (1/2)\rho\beta^2\sigma^2$
- $SS = (1-\beta)F(e) - \alpha + U(W - C(e)) - (1/2)\rho[C'(e)/F'(e)]^2\sigma^2$
- $SS = (1- C'(e)/F'(e))F(e) - \alpha + U(W - C(e)) - (1/2)\rho[C'(e)/F'(e)]^2\sigma^2$

Exercise 6

- The first order condition of the second stage:

- $d[(1- C'(e)/F'(e))F(e) - \alpha + U(\alpha + (C'(e)/F'(e))F(e) - C(e)) - (1/2)\rho[C'(e)/F'(e)]^2\sigma^2]/de = 0$
- $1- C'(e) + [d(1- C'(e)/F'(e))/de]F(e) - 0 + U'(\alpha + (C'(e)/F'(e))F(e) - C(e))d[\alpha + (C'(e)/F'(e))F(e) - C(e)]/de - [C'(e)/F'(e)][d(1- C'(e)/F'(e))/de]\rho\sigma^2 = 0$

Exercise 6

- $[d(C'(e)/F'(e))/de][U'(W - C(e)) - F(e) + \rho\sigma^2] = C'(e) - 1$
- Using $d(C'(e)/F'(e))=[F'(e)C''(e) - C'(e)F''(e)/(F'(e))^2]$
- $[F'(e)C''(e) - C'(e)F''(e)][U'(W - C(e)) - F(e) + \rho\sigma^2] = [C'(e) - 1](F'(e))^2$
- Using $\beta* = C'(e)/F'(e)$ or $\beta*F'(e) = C'(e)$

Exercise 6

- $[F'(e)C''(e) - \beta*F'(e)F''(e)][U'(W - C(e)) - F(e) + \rho\sigma^2] = [\beta*F'(e) - 1](F'(e))^2$
- $C''(e)[U'(W - C(e)) - F(e) + \rho\sigma^2] - \beta*F''(e)[U'(W - C(e)) - F(e) + \rho\sigma^2] = \beta*(F'(e))^2 - F'(e)$
- $C''(e)[U'(W - C(e)) - F(e) + \rho\sigma^2] + F'(e) = \beta*[F''(e)(U'(W - C(e)) - F(e) + \rho\sigma^2) + (F'(e))^2]$
- $\beta* = [F''(e)(U'(W - C(e)) - F(e) + \rho\sigma^2) + (F'(e))^2]/ [C''(e)(U'(W - C(e)) - F(e) + \rho\sigma^2) + F'(e)]$

Exercise 6

- Using $\alpha* = (1 – \beta*)F(e)$
- $\alpha* = (1 – [F''(e)(U'(W – C(e)) - F(e) + \rho\sigma^2) + (F'(e))^2]/ [C''(e)(U'(W – C(e)) - F(e) + \rho\sigma^2) + F'(e)]])F(e)$
- $e* = F^{-1}(\alpha*/(1 – \beta*)$
- $X* = F(e*)$
- $W* = \alpha* + \beta* X*$
- $SS* = (1- C'(e*)/F'(e*))F(e*) – \alpha + U(W – C(e*)) – (1/2)\rho[C'(e*)/F'(e*)]^2\sigma^2$

Exercise 7

4. Discuss the properties of the optimal linear individual incentive contract offered to a risk-neutral agent whose performance signal is given by $X = \theta e + \varepsilon$, where θ is a productivity parameter, e is the agent's effort, $\varepsilon \sim N(0, \sigma^2)$ is the error measurement term. The agent's cost function is $C = ke^2$, where k is the marginal cost parameter. The agent's utility preferences are given by $U = Y = W(e) - C(e)$. Find the agent's optimal effort.

Exercise 7

- Under risk neutral agency, the net benefit (NB) for the agent is determined to be:

$$NB = W - C - RP \text{ where } RP = 0$$

$$NB = E(W) - C$$

$$NB = \alpha + \beta\theta e - \kappa e^2$$

Exercise 7

$$d[NB]/de = 0$$

$$d[\alpha + \beta\theta e - \kappa e^2]/de = 0$$

$$0 + \beta\theta - 2\kappa e - 0 = 0$$

$$2\kappa e = \beta\theta$$

$$e = \beta\theta/2\kappa$$

Exercise 7

- SS = (X – W) + NB
- SS = (X – W) + (W – C – RP) but RP = 0
- SS = X – C
- SS = $\theta e - \kappa e^2$
- SS = $\theta^2\beta/2\kappa - \kappa(\theta\beta/2\kappa)^2$

Exercise 7

- $d(SS)/d\beta = d(\theta^2\beta/2\kappa - \kappa(\theta\beta/2\kappa)^2)/d\beta = 0$
- $d(SS)/d\beta = d(\beta\theta^2/2\kappa - \kappa(\theta^2\beta^2/4\kappa^2)/d\beta = 0$
- $\theta^2/2\kappa - \beta\theta^2/2\kappa = 0$
- $1 - \beta = 0$
- $\beta* = 1$
- $e^* = \beta*\theta/2\kappa = \theta/2\kappa$
- Since the agent is risk-neutral, then it is possible for the principal to design the contract so that the agent accepts all the risk. This is precisely what $\beta* = 1$ gives.

Exercise 7

Let $NB^* = 0$

Setting NB = 0 makes profit maximization equivalent to the (surplus) value maximization principle

$NB = W – C = 0$

$NB = \alpha + \beta\theta e – \kappa e^2 \quad = 0$

$\alpha = \kappa e^2 - \beta\theta e$

Exercise 7

$\alpha = \kappa e^2 - \beta\theta e$

$\alpha = \kappa[\beta\theta/2\kappa]^2 - \beta\theta[\beta\theta/2\kappa]$

$\alpha = \beta^2\theta^2/4\kappa - \beta^2\theta^2/2\kappa$

$\alpha = -\beta^2\theta^2/4\kappa$

The agent buys the project from
the principal and becomes the residual claimant of the project.