Department of Civil Engineering
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Structures, Materials, and Design

CIV101F

Complementary Notes

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1 AXIAL STRESS AND STRAIN, HOOKE'S LAW AND BUCKLING

1.1 INTRODUCTION

In 1676, Robert Hooke, an English scientist, discovered that, if an axial force was applied to a spring, the elongation of the spring was proportional to the force. Thus for the first time, a relationship was established between the loads acting on a mechanical system and the behaviour of the system. Later other investigators discovered that the same relationship existed for rods such as shown in Figure 1.1.

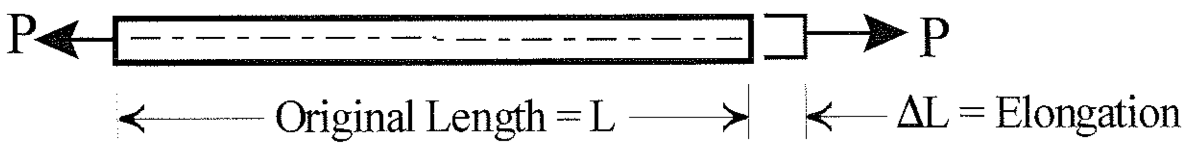


Figure 1.1 Elongation of Axially-Loaded Spring and Bar

Although interesting, this proportional relationship between force and elongation is by itself of relatively little use in analysing or designing structural systems. What is required for analysis and design is an equation that defines the behaviour response (displacement) of the system in terms of material and shape to the action of an applied force. The two quantities that are used to define behaviour are stress and strain.

Axial stress, σ, is defined as force per unit of cross-sectional area:

$$\sigma = P/A \text{ N/mm}^2 \text{ (MPa)} \qquad \text{............Eq. 1.1}$$

where σ is the axial stress in MPa (Newtons per square millimeter)

P = axial force in N (Newtons)

A = cross-sectional area of the member in mm^2 (perpendicular to P)

Axial strain or intensity of deformation at a point, ϵ, is often defined simply as "engineering" strain, or the elongation ("change in length") divided by the original length of the bar:

$$\epsilon = \Delta L/L \text{ (mm/mm)}$$

where ΔL = elongation of the bar (change in length)

L = original length of the prismatic bar

If the axial force on a prismatic bar is proportional to elongation, then as shown below, axial stress must be proportional to axial strain because the cross-sectional area, A, and the length, L, are constant.

$$P \propto \Delta L \rightarrow \frac{P}{A} \propto \frac{\Delta L}{L} \rightarrow \sigma \propto \epsilon$$

This linear relationship between axial stress and axial strain is called Hooke's Law.

The next step in the evolution of the relationship between stress and strain was to establish the constant of proportionality. Credit for that is given to another British scientist, Thomas Young, who in his "Lectures on Natural Philosophy" published in 1807 defined the modulus of elasticity, E, such that:

$$\sigma = E \epsilon \qquad \text{.......................... Eq. 1.2}$$

Strain, ϵ, is dimensionless, so the modulus of elasiticity has the same units as stress, that is, MPa or N/mm^2. Experimentally it has been demonstrated that the modulus of elasticity is a unique constant or property of a particular material that is used to describe its elastic performance. The modulus of elasticity is often referred to as Young's modulus of elasticity, and sometimes simply as Young's modulus.

A common laboratory experiment in the study of engineering materials is to establish the stress-strain diagram for different materials. A typical plot of tension stress vs. strain for low-carbon steel that is used in buildings and bridges is shown in Figure 1.2.

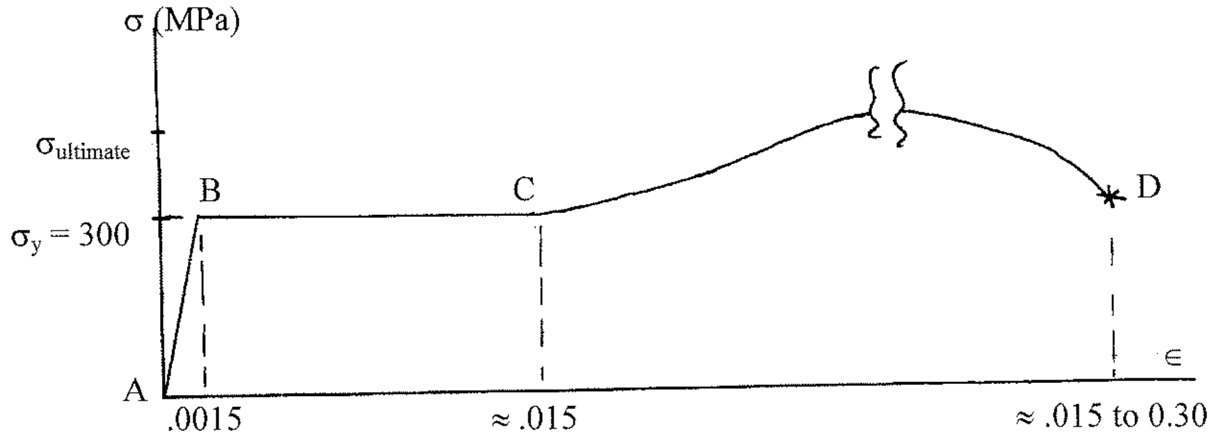


Figure 1.2 Typical Tensile Stress-Strain Diagram for Low-Carbon Steel

As can be seen, the linear relationship between stress and strain only exists in the initial part of the plot, that is, from A to B. We refer to such behaviour as being "elastic" because within this region the bar returns to its original shape after the load is removed. The stress at point B is called the "yield stress", σ_y. Region BC is called the "plastic" region or the yield plateau and here strain occurs without any apparent corresponding increase in stress. Finally in the region CD, stress increases at a rate lower than E, the modulus of elasticity, in a process called strain-hardening. The ultimate stress holds nearly constant for significantly increasing strain and this calculated stress finally decreases as rupture approaches. This simple test provides the basic engineering properties that are required to qualify a material for structural designs and applications.

Other materials produce different stress-strain diagrams but for most material there is at least an initial region in which the stress and strain are proportional, that is, there is linearly elastic behaviour. The slope of the stress-strain diagram in the elastic region is the modulus of elasticity, E.

Material such as the low-carbon steel shown in Figure 1.2 is said to be "ductile" because it can undergo considerable deformation before it fails. Cast iron for which a typical stress-strain diagram is shown in Figure 1.3 is a considered a "brittle" material because it fails suddenly without significant observable deformation. Concrete and rock are also brittle materials unless suitably reinforced with a ductile material such as steel.

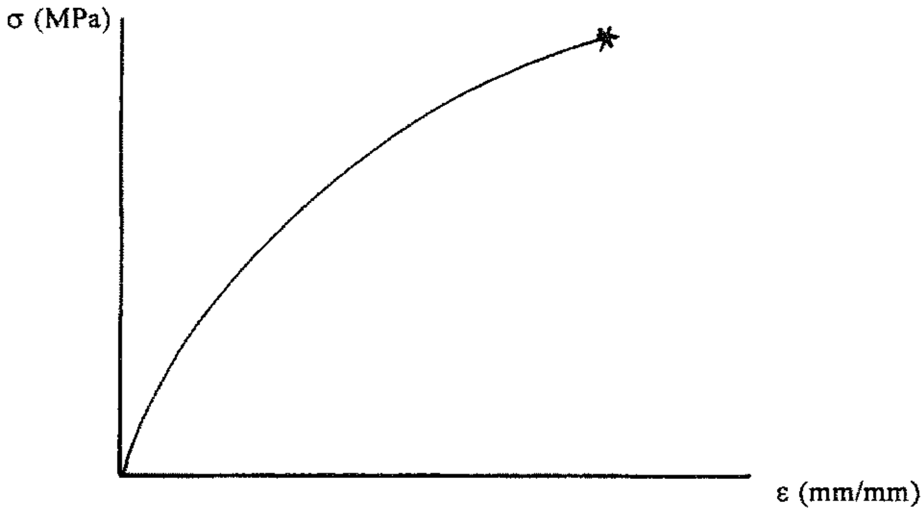


Figure 1.3 Typical Tensile Stress-Strain Diagram for Cast Iron

Because the relationship between stress and strain is a constant for any one material, we are now able to calculate the behaviour of structural systems that consist of axially loaded elements. As we shall see in later chapters, the same stress-strain relationships can also be used to derive the behaviour of systems that include members subjected to bending moment such as beams, or to any combination of axial load, shear, and bending moment. In the next section we will examine, for design purposes, how we model the behaviour of tension ties, short columns and simple trusses.

1.2 ELONGATION AND SIMPLE DEFLECTION

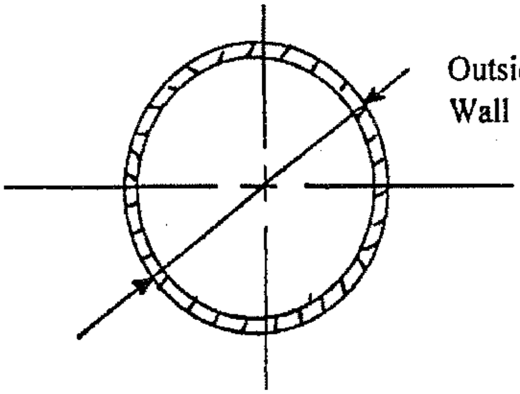
The first type of analysis problem to be considered is the determination of the behaviour of tension and short compression members subject to axial load only. As noted before, the term "behaviour" includes both stresses and deformations. In analysis one knows the dimensions of the tension or compression member, and the type of material as defined by the modulus of elasticity, E. Sample Problem 1.1 illustrates the manner in which the behaviour of an axially loaded member can be established. Note that the term "elongation" is used for both increases and decreases in length.

The elongation of an axially loaded bar with a uniform cross-section can also be determined by using Equation 1.3 by substituting "$\sigma = P / A$" and "$\epsilon = \Delta L / L$" into Equation 1.2 so that:

$$\sigma = \frac{P}{A} = E\frac{\Delta L}{L} = E\,\varepsilon$$, and this can be rewritten as

$$\Delta L = \frac{PL}{AE} \qquad \text{..........Eq. 1.3}$$

SAMPLE PROBLEM 1.1 The cross-section of a steel pipe subjected to an axial tensile load of 150 kN is shown below. The member has a length of 4 metres and the modulus of elasticity for the steel is 200 x 10^3 MPa. The outside diameter of the pipe is 101.6 mm and the wall thickness is 5.74 mm. Determine the axial stress in the material and the elongation of the column.



Solution:

Step 1 Compute the cross-sectional area, A, of the column.

$$A = \frac{\pi}{4}(101.6^2 - 90.12^2) = 1730\,\text{mm}^2$$

Step 2 Compute the magnitude of the uniform stress knowing the area and the load, P.

$$\sigma = \frac{150000\,\text{N}}{1730\,\text{mm}^2} = 86.7\,\text{MPa} \quad \text{Tension}$$

Step 3 Compute the elongation of the column using the relationship $\sigma = E\epsilon$.

$$86.7\frac{\text{N}}{\text{mm}^2} = 200 \times 10^3\,\frac{\text{N}}{\text{mm}^2} \times \frac{\Delta\text{L mm}}{4000\,\text{mm}}$$

ΔL = 1.73 mm

Note how relatively small this displacement is with respect to the size of the structure. Would you notice this displacement if you observed the process from a distance of one to two metres?

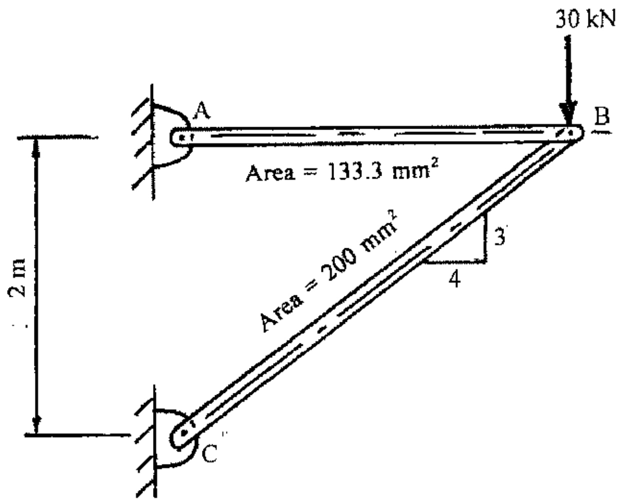
Using either the fundamental relationship given by Equation 1.2 or the derived Equation 1.3, one can compute the elongation caused by an axial load acting on a tension or compression member. Alternatively, any one of the terms in Equation 1.3 can be computed if the others are known. As it will be shown in Section 1.5, Equation 1.1 can also be used to design members subject to tension.

1.3 DEFLECTION OF PIN-CONNECTED TRUSSES

Another type of problem involves the determination of the movement of the joints of an assemblage of axially loaded members such as a pin connected truss. In these problems the calculations of the individual elongations of the members is only the first step in a more extensive calculation to determine the movement of the joints. One of the earliest applications of computers to structural analysis was to replace the tedious hand calculations used in the determination of these movements in relatively complex trusses.

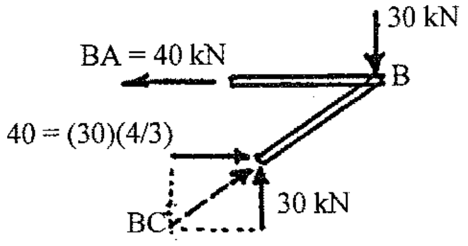
The following simple example using basic geometric principles illustrates the nature of the more complex problem.

SAMPLE PROBLEM 1.2 Determine the movement of the pin at B for the truss shown.



Solution:

Step 1 Compute the force in each member using a free body diagram of joint B and the equations of equilibrium.



$\sum F_y = 0;$ BC = 50 kN Compression

$\sum F_x = 0;$ BA = 40 kN Tension

Step 2 Compute the elongation of each member knowing the properties of the two members.

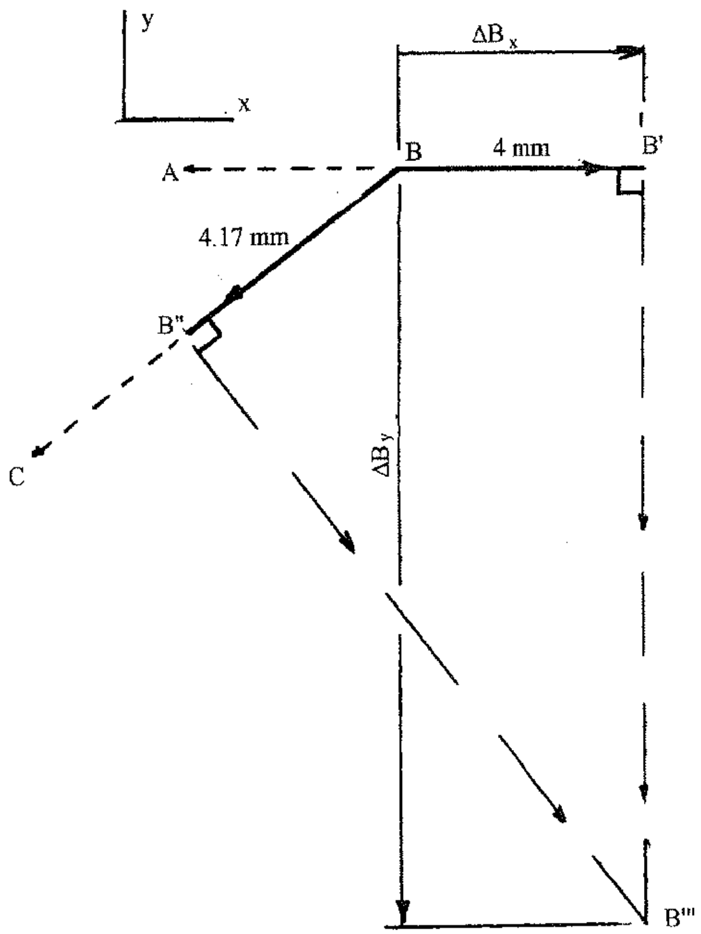
$$\Delta_{AB} = \frac{(40000\text{ N})(2667\text{ mm})}{(133.3\text{ mm}^2)(200\times 10^3\text{ N/mm}^2)} = +4.00\text{ mm}$$

$$\Delta_{BC} = \frac{(-50000\ \mathrm{N})(3333\ \mathrm{mm})}{(200\ \mathrm{mm}^2)(200\times10^3\ \mathrm{N/mm}^2)} = -4.17\ \mathrm{mm}$$

(Note: The units are compatible in the expressions)

Step 3 Consider joint B. Member AB increases in length and excluding any consideration of member CB the point B moves to B'. On the other hand, member CB shortens and point B would move to B" if member AB were ignored. These movements are shown below in the magnification of point B.

Clearly point B cannot move to both B' and B" unless the pin at B fails which is not the case. Therefore a point B''' must be located which is compatible for both members. If we allow AB to rotate about A, then for small displacements B" will move perpendicular to AB. Similarly CB can rotate about C resulting in B" moving perpendicular to CB. The only point which satisfies the two elongations and the two rotations is B'''. Therefore the movement of joint B is given by the two values ΔB_x and ΔB_y which can either be computed using geometry or measured from a graphical solution. Note, as before, that the displacements are indeed small, relative to the size of the structure.



ΔB_x = 4.0 mm
ΔB_y = -12.3 mm

1.4 BUCKLING OF COMPRESSION MEMBERS

As shown in the Section 1.2, if given the material properties and material, it is possible to determine the load vs displacement behaviour of a bar subjected to an axial force, whether it be in tension or compression. But the behaviours of tension and compression members can be very different because compression members can fail by "buckling". This phenomenon is readily observed by applying a compression load to a slender bar such as a metre stick or a hacksaw blade. An initially straight bar or member "buckles" under increased load because it is no longer able to maintain its straightness. This process of becoming bent or "buckled" is considered to be a structural failure.

The axial load can be such that when the bar buckles, and thus is no longer support the load in a nominally straight condition, the load is producing a stress which is below the yield stress of the material. This phenomenon is referred to as elastic buckling. The calculation of the elastic buckling load was first derived by the Swiss mathematican, Leonhard Euler, in 1757. His formula for the buckling load is:

$$P_{Euler} = \frac{\pi^2 EI}{L^2} \quad \text{......................Eq. 1.4}$$

The only term that you have not already encountered is I, second moment of the member cross-sectional area, which is covered in Chapter 10 in the textbook. (Note that "Second Moment of Area" is often referred to as "Moment of Inertia of Area") Columns in this region are often referred to as "long" columns; they return to their original shape after the load is removed, that is, they behave in an elastic manner.

The Euler equation is plotted as Maximum Strength vs. Length in Figure 1.4 as the curve DC and continuing dashed above C. Figure 1.4 is a typical plot of the strength of a prismatic pin-ended column fabricated from steel with properties as shown in Figure 1.2.

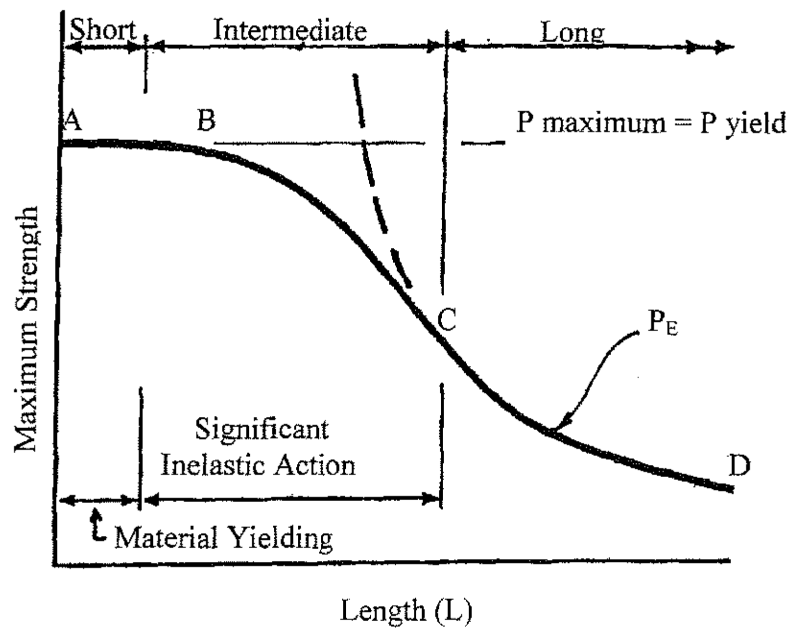


Figure 1.4 Strength of Steel Compression Members with respect to Length

As shown in Figure 1.4, there are three possible types of failure behaviour. The portion from A to B represents failure by yielding or squashing of the material with no associated buckling. Columns that fail in this manner are referred to as "short columns" and the failure is by yielding as it is for a tension member because the yielding occurs at a lower load than the load calculated for buckling.

For many years engineers were convinced that Euler's equation was incorrect because the failure loads that they observed in tests were somewhat lower than those calculated using the Euler formula. In fact the buckling formula is correct for the region C to D but until late in the nineteenth century very few practical compression members were slender enough to have their strength controlled by elastic buckling.

In region B to C the buckling failure is referred to as elasto-plastic because some yielding of the material occurs before a buckling load is reached and as a result permanent deformations remain after the load is removed. The most significant deviation of a "real" column from the "perfect" column is found in this intermediate "elasto-plastic" region. Here in this transition between pure yielding and Euler elastic buckling the load is carried by a combined buckling and yielding behaviour.

The Euler equation assumes a "perfect" or straight column, and "perfect" linear material response. When used in design it is usually modified to account for the effects of geometric and material imperfections. In the course CIV101F Structures, Materials, and Design, it will be assumed for the sake of simplicity that the failure of compression members can be represented by failure by yielding, or failure by elastic buckling.

1.5 DESIGN OF TENSION AND COMPRESSION MEMBERS

The **"analysis"** of a given structure involves the determination of its expected behaviour knowing the loading, the material, the overall geometry, and the dimensions of all of the parts. There is often a unique solution for the internal stresses and deflections for a given structure and loading, and we seek that solution when we analyze the structure. In this chapter we have been concerned with the analysis of axially loaded bars such as ties, struts, columns, and truss members. These are linear "two-force" members. Other chapters will examine the analysis of more complicated systems.

The objective in structural **"design"** is to select a system that can safely resist the anticipated loads. Thus in design there are many feasible solutions but only a few will be suitable and economical. In all cases the final design must provide a reliable level of safety, that is, a low probability of failure. For this aspect of design, a resistance or strength is provided that is larger than the load effects multiplied by a prescribed load factor. Originally, reasonable safety was provided by simply requiring that the actual maximum stresses be suitably less than the yield stress for the material. Today the provision of a reasonable level of safety is achieved by increasing the anticipated or service loads by a "load factor" and the structure is then required to have at least that much capacity or resistance at failure whether it be through yielding or buckling.

Why do we require the capacity or resistance to equal or exceed a maximum expected load multiplied by a load factor and what is a "reasonable" load factor? The "why" is readily answered because it is easy to imagine that the loading might be greater than expected, or the material might not be as strong as specified, or the analysis might only be an approximation, or the designer may err estimating the largest load effect. The magnitude of the load factor is rooted in the experience gained by structural engineers over many centuries. Thus for low-carbon steel tension members the load factor is about 1.7;

the member strength or resistance must exceed the maximum expected load factored by 1.7. Engineers discovered that if they reduced this load factor below about 1.7 then the frequency of failure became excessive. The magnitude of the simple load factor defined here depends upon many variables such as the material, workmanship, the predictability or knowledge of the loading, and the type and seriousness of failure.

TENSION MEMBERS The design of tension members is usually quite straight forward. Normally the material and type of cross-section will be dictated by external factors such as the connections and the configuration of the system.

Example
Figure 1.2 is the typical stress strain behaviour of ordinary structural steel and shows that the yield stress is 300 MPa. The maximum calculated service load on a tension member in a structure is estimated to be 150 kN, and the required load factor is 1.8 for this structure. Determine the required cross-sectional area of the member using this material.

Step 1: Compute the factored load: (150 kN)(1.8) = 270 kN.

Step 2: Using Equation 1.1 compute the required cross-sectional area assuming that the axial stresses due to the factored load will be equal to the yield stress:

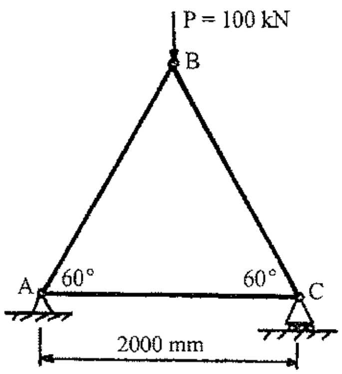
$$\sigma = P/A, \qquad \text{or} \qquad P = \sigma_y A > 270{,}000 \text{ N}, \qquad A \geq 270000 \text{ N} / 300 \text{ N/mm}^2 = 900 \text{ mm}^2$$

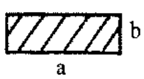
The final step is to select, usually from handbooks, a section with a cross-sectional area that is NOT LESS than 900 mm^2. To compute the elongation of the selected member at the service load one would use the actual cross-sectional area for the chosen member.

COMPRESSION MEMBERS The design of compression members is more complicated because of the additional consideration of the second possible failure mode of buckling. The material and type of cross-section will again normally be dictated by external factors. But even with that pre-knowledge, the design process is frequently iterative because some member parameters must be assumed before a rational selection of member size can be made on a trial basis. Engineering design is usually an iterative process in which one seeks for the best solution while recognizing the competing constraints.

SAMPLE PROBLEM 1.3

Design members **AB**, **BC** and **CA** of the truss shown below. Structural steel is to be used in the design. The yield stress for the steel is 310 MPa, the modulus of elasticity is 200,000 MPa, and the load factor is 2.0. The rectangular cross-section (a = 3b) will be cut from plates whose thickness, b, is only available in 5 mm increments. The lateral constraint system for members **AB** and **BC** is such that the supported length for buckling about the strong axis is 2 metres and that for buckling about the weak axis is 1 metre.

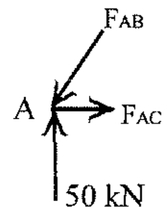




Solution

Step 1: Determine the reaction components and then the member forces $R_A = R_C = 50$ kN ↑

Joint A



$+\uparrow\ \Sigma F_y = 0$ ..... $-F_{AB}\cos 30 + 50 = 0$

$F_{AB} = 57.74$ kN $\therefore$ (C)

$+\rightarrow\ \Sigma F_x = 0$ ..... $F_{AC} - 57.74\cos 60 = 0$

$F_{AC} = 57.74\,(\cos 60) = +28.9$ kN $\therefore$ (T)

Step 2: Design of Members

2.1 Tension member AC

Factored Load = (28.9 kN) (2) = 57.7 kN $\qquad A_{required} = 57.7\text{ kN} \times 10^3 / 310\text{ N/mm}^2 = 186.3\text{ mm}^2$

Compute "b": $(b)(3b) = 186.3\text{ mm}^2 \quad b = 7.9$ mm

Member AC: Use 10 mm x 30 mm

2.2 Compression members: AB will now be designed and member BC will be the same size

2.2.1 Yielding

Factored Load = (57.7 kN) (2) = 115.4 kN $\qquad A_{required} = 115.4 \times 10^3\text{ N} / 310\text{ N/mm}^2 = 372.3\text{ mm}^2$

Compute "b" for yielding: $\qquad (b)(3b) = 372.3\text{ mm}^2 \quad b = 11.1$ mm

But a compression member may also fail by buckling so the size required for buckling must be calculated to determine which mode of failure controls the design.

2.2.2 Buckling

(a) Buckling about the strong "x" axis for which the effective length is 2 m = 2000 mm

$$P_E = 115.4 \times 10^3\text{ N} = \frac{(\pi^2)(200 \times 10^3\text{ N/mm}^2)\dfrac{(b)(3b)^3}{12}}{(2000)^2\text{ mm}^2}$$

Compute "b" for strong axis buckling: b = 17.96 mm

2.2.3 Buckling about weak "y" axis for which the effective length is 1 m

$$P_E = 115400 = \frac{(\pi^2)(200 \times 1000)\dfrac{(3b)(b^3)}{12}}{(1000)^2}$$

Compute "b" for weak axis buckling: b = 21.99 mm

Therefore, weak axis buckling controls the size of members **AB** and **BC**.

Member AB and BC: Use 25 mm x 75 mm

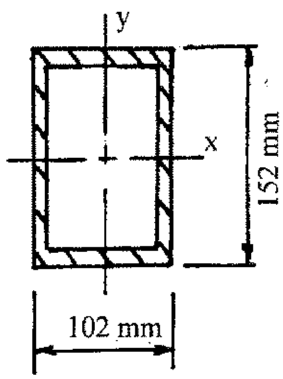
Note: Had the cross-section not been specified, the minimum value for I, second moment of area, could be calculated and an appropriate cross-section selected from tables provided in handbooks.

PROBLEMS

1. A 90 x 90 x 10 steel angle as listed in Section Tables is 3 metres long and supports an axial tension load of 400 kN. The modulus of elasticity for the material is 200 x 10^3 MPa. Determine the stress in the material and the elongation of the bar. (Answers: 235 MPa, 3.53 mm)

2. The hollow steel section shown in Figure 1.P1 is 3.5 metres long and supports a compressive load of 800 kN. The wall thickness is 9.5 mm throughout and E = 200 x 10^3 MPa.
   (a) Determine the average compressive stress in the material and the elongation of the column. (Answers: - 179.2 MPa, - 3.14 mm)

   (b) Determine the load factor if the service load is 800 kN. Assume that the yield stress is 350 MPa. Consider both yielding and buckling. (Answer: 1.43)



I_x = 58 x 10^6 mm^4
I_y = 7.10 x 10^6 mm^4

Figure 1.P1

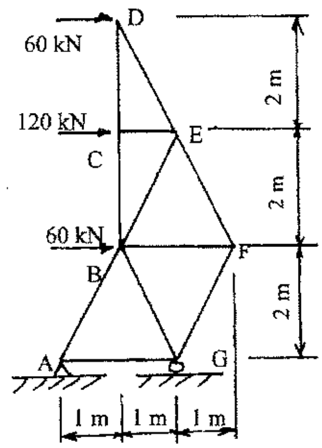
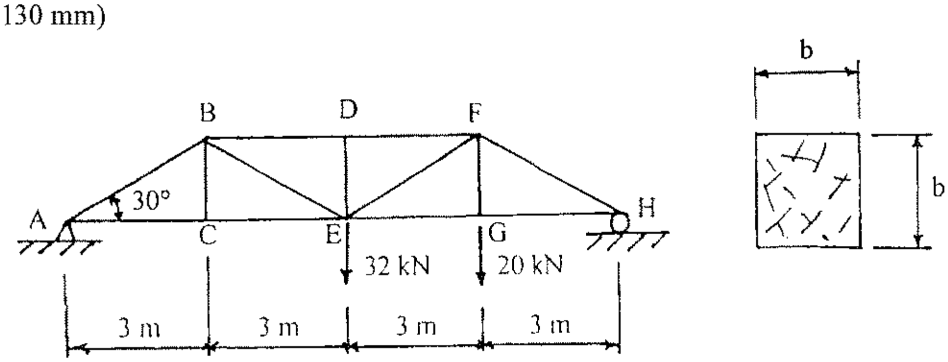


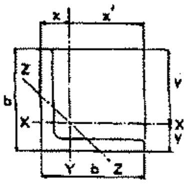
Figure 1.P2

3. Select an equal legged angle for member CB of the truss shown in Figure 1.P2 assuming that the yield stress is 350 MPa and the load factor is 1.8. Determine the actual stress in CB and its elongation. Select a solid square steel cross-section for member EF assuming that the side dimensions are only available in increments of 5 mm. Use a load factor of 2.2, σ_{yield} = 340 MPa, and E = 200 x 10^3 MPa. (Answers: 55 x 55 x 6, 70 mm x 70 mm)

4. The four members AB, BD, DF, and FH or the truss shown in Figure 1.P3 are to be fabricated from the same size square timber sections as shown. Determine the required dimension "b" for the four members assuming that the failure stress for the timber is 40 MPa, the load factor is 2.8, and the modulus of elasticity E = 12 x 10^3 MPa. The side dimensions are only available in increments of 10 mm. (Answer: 130 mm)



ANGLES
Equal Legs



PROPERTIES AND DIMENSIONS

Size	Thickness t	Mass	Dead Load	Area	Axis X-X and Axis Y-Y I	S	r	x or y	Axis Z-Z r
mmxmm	mm	kg/m	kN/m	mm²	10^6mm⁴	10^3mm³	mm	mm	mm
200X200	30	87.1	0.855	11 100	40.3	290	60.3	60.9	39.0
	25	73.6	0.722	9 380	34.8	247	60.9	59.2	39.1
	20	59.7	0.585	7 600	28.8	202	61.6	57.4	39.3
	16	48.2	0.473	6 140	23.7	165	62.1	55.9	39.5
	13	39.5	0.387	5 030	19.7	136	62.6	54.8	39.7
	10	30.6	0.300	3 900	15.5	106	63.0	53.7	39.9
150X150	20	44.0	0.431	5 600	11.6	110	45.5	44.8	29.3
	16	35.7	0.350	4 540	9.63	90.3	46.0	43.4	29.4
	13	29.3	0.287	3 730	8.05	74.7	46.4	42.3	29.6
	10	22.8	0.223	2 900	6.37	58.6	46.9	41.2	29.8
125X125	16	29.4	0.288	3 740	5.41	61.5	38.0	37.1	24.4
	13	24.2	0.237	3 080	4.54	51.1	38.4	36.0	24.5
	10	18.8	0.185	2 400	3.62	40.2	38.8	34.9	24.7
	8	15.2	0.149	1 940	2.96	32.6	39.1	34.2	24.8
100X100	16	23.1	0.227	2 940	2.65	38.3	30.0	30.8	19.5
	13	19.1	0.187	2 430	2.24	31.9	30.4	29.8	19.5
	10	14.9	0.146	1 900	1.80	25.2	30.8	28.7	19.7
	8	12.1	0.118	1 540	1.48	20.6	31.1	28.0	19.8
	6	9.14	0.090	1 160	1.14	15.7	31.3	27.2	19.9
90X90	13	17.0	0.167	2 170	1.60	25.6	27.2	27.2	17.6
	10	13.3	0.131	1 700	1.29	20.2	27.6	26.2	17.6
	8	10.8	0.106	1 380	1.07	16.5	27.8	25.5	17.7
	6	8.20	0.080	1 040	0.826	12.7	28.1	24.7	17.9
75X75	13	14.0	0.137	1 780	0.892	17.3	22.4	23.5	14.6
	10	11.0	0.108	1 400	0.725	13.8	22.8	22.4	14.6
	8	8.92	0.087	1 140	0.602	11.3	23.0	21.7	14.7
	6	6.78	0.066	864	0.469	8.68	23.3	21.0	14.8
	5	5.69	0.056	725	0.398	7.32	23.4	20.6	14.9
65X65	10	9.42	0.092	1 200	0.459	10.2	19.6	19.9	12.7
	8	7.66	0.075	976	0.383	8.36	19.8	19.2	12.7
	6	5.84	0.057	744	0.300	6.44	20.1	18.5	12.8
	5	4.91	0.048	625	0.255	5.45	20.2	18.1	12.9
55X55	10	7.85	0.077	1 000	0.268	7.11	16.4	17.4	10.7
	8	6.41	0.063	816	0.225	5.87	16.6	16.7	10.7
	6	4.90	0.048	624	0.177	4.54	16.9	16.0	10.8
	5	4.12	0.040	525	0.152	3.85	17.0	15.6	10.8
	4	3.33	0.033	424	0.125	3.13	17.1	15.2	10.9
	3	2.52	0.025	321	0.096	2.39	17.3	14.9	11.0
45X45	8	5.15	0.050	656	0.118	3.82	13.4	14.2	8.76
	6	3.96	0.039	504	0.094	2.98	13.7	13.4	8.79
	5	3.34	0.033	425	0.081	2.53	13.8	13.1	8.82
	4	2.70	0.026	344	0.067	2.07	13.9	12.7	8.87
	3	2.05	0.020	261	0.052	1.58	14.1	12.4	8.93
35X35	6	3.01	0.030	384	0.042	1.74	10.5	10.9	6.81
	5	2.55	0.025	325	0.036	1.49	10.6	10.6	6.83
	4	2.07	0.020	264	0.030	1.22	10.7	10.2	6.86
	3	1.58	0.015	201	0.024	0.940	10.8	9.86	6.91
25X25	5	1.77	0.017	225	0.012	0.724	7.39	8.06	4.87
	4	1.44	0.014	184	0.010	0.599	7.50	7.71	4.87

2 LOAD, PRESSURE AND STRESS BLOCKS

2.1 INTRODUCTION

We have already studied two types of loads, concentrated loads and distributed loads. As we have seen, concentrated loads are represented by a single force vector. On the other hand, distributed loads along lines are represented by a combination of lengths and load intensity vectors that have units of force per unit length. As explained in Section 4.10 of the text, the magnitude of the single force that is equivalent to a distributed load is equal to the **AREA** of the load diagram, its direction is the same as the load intensity vectors, and it is located at the centroid of the area of the load diagram.

Two examples of distributed loads along lines and their equivalent single forces are shown in Figure 2.1. The equivalent single forces should be shown superimposed on the original distributed load diagram rather than on a separate diagram. This has been done in these examples by using a broken heavy line to represent the equivalent single force; a convenient alternate format is to use a coloured line.

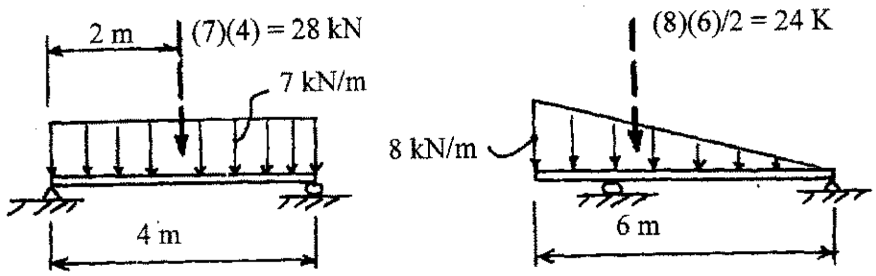


Figure 2.1 Examples of Distributed Loads

Distributed loads can also act over areas such as bridge decks or floors. In these cases the load can be defined by an area and a load intensity vector that has units of force per unit area. For example consider the loading in Figure 2.2 in which the uniform load intensity has a value of 7 kN/m^2 and the area represented by the plate ABCD is 3m by 5m. It is apparent that the magnitude of the total load is given by the load intensity multiplied by the area over which it acts, that is:

Total Load = (7 kN/m^2) × (3 m) × (5 m) = 105 kN

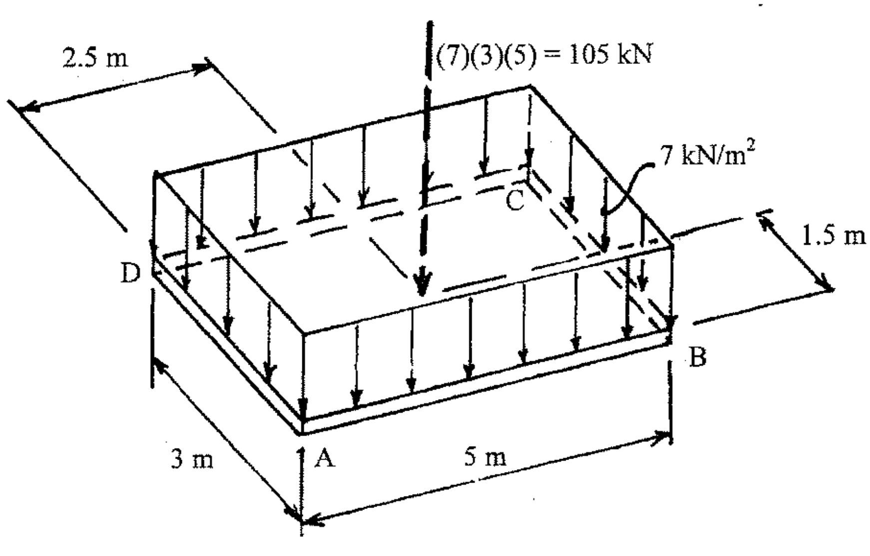


Figure 2.2 Example of a Uniform Load Acting Over an Area

For this type of distributed load the magnitude of the equivalent single force is equal to the **VOLUME** of the load diagram. The equivalent single force acts through the centroid of the load diagram as shown in Figure 2.2, and in the same direction as the original load intensity vectors. As in the previous example, the equivalent single force has been shown superimposed on the load diagram.

Another example of a distributed load acting over an area is shown in Figure 2.3.

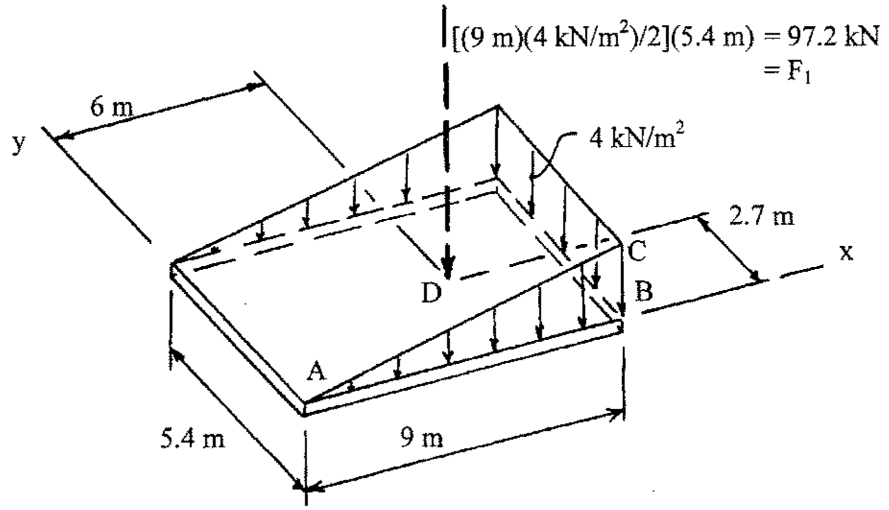


Figure 2.3 Uniformly Varying Load Acting over an Area

In this case the load intensity varies uniformly in the x-direction and the resulting load block is wedge-shaped. The magnitude of the equivalent single force, F_1, is equal to the volume of the block, that is:

$$F_1 = \text{volume} = (\text{Area of triangle ABC})(\text{Length of wedge})$$

$$= \frac{(9\text{ m})(4\text{ kN/m}^2)(5.4\text{ m})}{2} = 97.2\text{ kN}$$

As shown in Figure 2.3, the force F_1 acts through point D that can be established knowing the centroid of the volume or block. Centroids of volumes that have constant cross sections are easily established once the centroid of the cross sectional area itself is known.

Throughout engineering and science, one encounters numerous examples of phenomena being visually represented by areas or volumes. It is an extraordinarily powerful tool for representing many concepts especially for those who have the ability to visualize objects in 3-dimensions. The use of blocks to represent loads acting over areas is one of the simplest examples.

So far we have modelled distributed loads such as those that might act on a floor or bridge deck by means of a volume or block. In these cases, the units for the load will usually be kilonewtons per square metre and those for the area will be in metres. Load intensities can also be expressed in terms of newtons per square metre, N/m^2. One can visualize similar volumes or blocks to describe these distributed loads; the only difference being that the magnitudes of the corresponding volumes or blocks would be in newtons (N) instead of kN. Normally load intensities with such units are referred to as **PRESSURES** and the derived SI units used are called pascals (Pa) rather than N/m^2.

Finally, one can express the load intensities in terms of newtons per square millimetre or megapascals (MPa), and the areas in terms of millimetres. The resulting volumes would again yield the magnitude of the equivalent single force in newtons, (N). As we will discover in the next section, force intensities of this type are usually called "stresses". An example of a stress block is shown in Figure 2.4 along with the corresponding equivalent single force

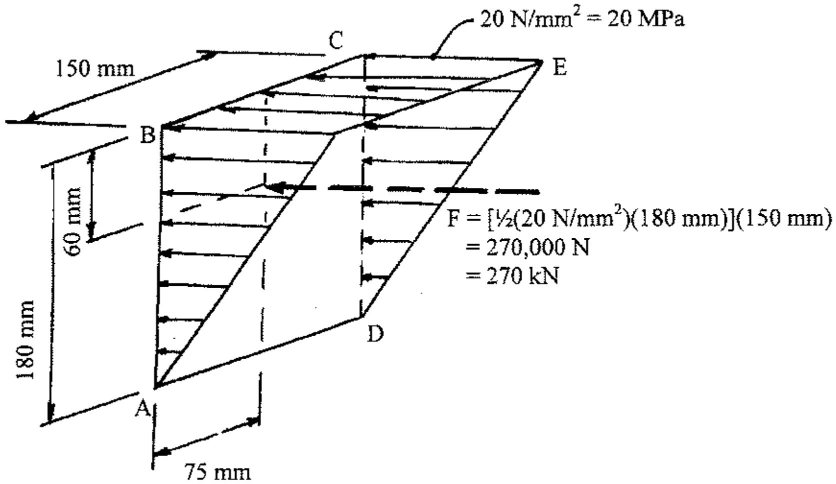


Figure 2.4 Example of a Stress Block

In this case, the area ABCD being acted upon lies in a vertical plane and the force intensity vectors are horizontal rather than vertical as in the previous examples. However, as shown, one can still visualize a block that has a volume equal to the magnitude of the equivalent single force, F.

$$F = (\text{Area of triangle CDE})\ (\text{length of wedge})$$

$$= \frac{(20\ \text{N/mm}^2)(180\ \text{mm})}{2}(150\ \text{mm}) = 270 \times 10^3\ \text{N} = 270\ \text{kN}$$

In all of the above examples, the areas were rectangular, and the load intensities, regardless of their units, were either uniform over the area or uniformly varying in one direction only. One can also have blocks for situations where highly variable force intensities act on irregular areas. Evidently in these cases the use of integration or some other numerical method would likely be required to calculate the corresponding volumes and their locations. But the basic principle remains the same: **a distributed load acting over an area can be visualized as a volume:**

(a) the volume is equal to the magnitude of the equivalent single force, and

(b) the line of action of the equivalent single force passes through the centroid of the volume.

2.2 AXIAL STRESSES IN BARS

In Chapter 1 we learned about the word "stress" as it is used in mechanics, particularly axial stress that is either tension or compression. Figure 2.5 (a) shows a freebody diagram of a bar, member BD, with a rectangular cross section; the bar is in equilibrium. At location Q, along the bar, the single internal force, T, is shown acting on the cross section area GHJK in Figure 2.5(b) and the corresponding stress block acting on the cross section area is shown in Figure 2.5(c). The volume of this stress block must be equal to the internal force of 1800N.

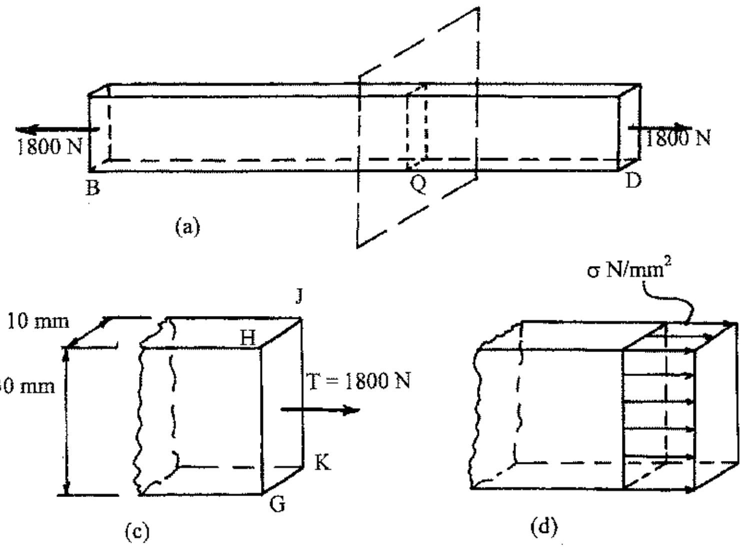


Figure 2.5 Axially Loaded Rectangular Bar

Evidently, a similar stress block could be constructed for the end Q of the freebody QD with the only difference being that the force intensity vectors would act in the opposite direction. **The force intensities that are internal to a solid body or at the interfaces of solid bodies are called stresses.** The stress magnitude at a particular point is the value of the force intensity at that point. Stresses can either be normal to the cutting plane or parallel to it, in the later case they are referred to as shear stresses. As we have seen earlier, the units for stress are the same as are used to measure pressure but the term pressure is usually used to describe force intensities associated with a gas or liquid such as, for example, the pressure at some specified point within a liquid or the pressure of a gas acting on its container.

An example of a more complex stress block is shown in Figure 2.6(a). In this case the magnitude of the equivalent single force is best determined by breaking the stress block into a wedge and a rectangular volume. Thus the magnitude of the equivalent single force, F, is equal to the sum of the two forces, F_1 and F_2, where

$$F_1 = (100\text{ mm})(30\text{ mm})(8\text{ N/mm}^2) = 24000\text{ N}$$

and $$F_2 = \frac{(100\text{ mm})(4\text{ N/mm}^2)}{2}(30\text{ mm}) = 6000\text{ N}$$

Therefore: $F = F_1 + F_2 = 24000\text{ N} + 6000\text{ N} = 30000\text{ N}$

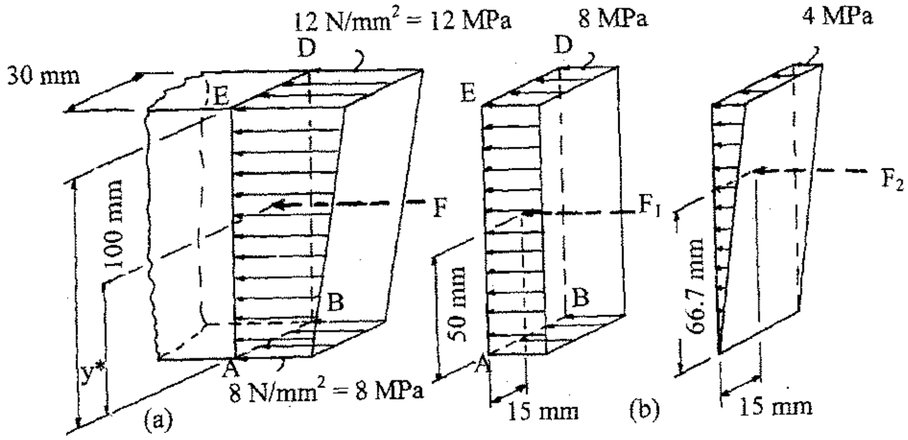


Figure 2.6 Example of a Composite Stress Block

As shown in Figure 2.6(b), the two forces, F_1 and F_2, act through the known centroids of their respective stress blocks or "volumes." If required, the location of the single force, F, can be established from the fact that its moment about any line must be the same as the moment of the two forces, F_1 and F_2 about the same line. Thus referring to Figure 2.6, the distance y* from the line AB can be calculated as follows:

$$\sum M = (y^*)(F) = (50\text{ mm})(F_1) + (66.7\text{ mm})(F_2)$$

$$y^*(30000\text{ N}) = (50\text{ mm})(24000\text{ N}) + (55.7\text{ mm})(6000\text{ N})$$

Solving: $$y^* = \frac{1200000 + 400000)\text{N mm}}{30000\text{ N}} = 53.3\text{ mm}$$

Had moments been taken about line ED, the distance to F would have been computed to be 46.7 mm.

Usually the location of the equivalent single force F for a stress block with multiple components is not established because it is normally not required for subsequent calculations. If the moment of a resultant force is required it is computed using the component forces and their individual moment arms.

As was the case with loads and pressures, stresses can be highly variable and act over irregular areas. To compute the magnitude and moment of the resultant stress blocks one might have to use integral calculus or some other mathematical formulation. Alternatively, the formulas for volumes and centroids of many shapes can be obtained from textbooks.

In conclusion, stresses acting normal to the cross sectional area of a bar are either tension or compression stresses. Stress blocks provide a very convenient manner in which to visualize these stresses and the volume of the stress block is the magnitude of the equivalent single force. Stresses that act parallel to the "cut" surface are called shear stresses. Shear stress blocks can also be constructed although they are not used as commonly.

2.3 STRESS BLOCKS AND INTERNAL BENDING MOMENTS

Let us next consider the case where there are tension and compression stress blocks acting on one internal plane section. As an example, consider Figure 2.7(a) that shows a compression and a tension stress block acting on the area BCFE.

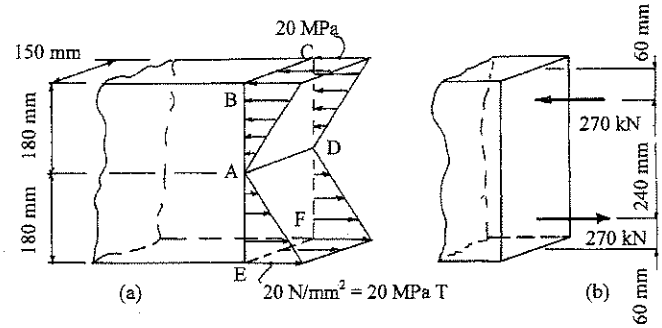


Figure 2.7 Representation of Moment Using Stress Blocks

The compression block at the top is identical to that of Figure 2.4 and its equivalent single force, C, has magnitude:

$$C = \frac{(20\ \mathrm{N/mm^2})(180\ \mathrm{mm})}{2}(150\ \mathrm{mm}) = 270 \times 10^3\ \mathrm{N} = 270\ \mathrm{kN}$$

As can be seen in Figure 2.7(a), the tension stress block at the bottom has the same dimensions as the compression block, the only difference being that the stresses act in the opposite direction, that is, in tension. Consequently the volume of the tension stress block will be the same as for the compression stress block and the equivalent single force, T, will have the same magnitude as C. The two equivalent forces, C and T, that are shown in Figure 2.7(b) represent a couple or moment. Often the equivalent single forces depicted by such stress blocks are shown on separate diagrams rather than superimposed on the stress blocks. In addition, for simplicity, they are usually shown in a simple side view of the beam.

An examination of the situation portrayed in Figure 2.7(b) yields these two important conclusions about stress blocks that represent moments:

(a) **the sum of the two forces, C and T, acting on the cut surface equals zero, and**
(b) **the magnitude of the couple or moment, M, is equal to the magnitude of either force multiplied by the distance between the two forces.**

Thus for the case portrayed in Figure 2.7, the magnitude of the moment M is:

$$M = (270\ \mathrm{kN})\ (240\ \mathrm{mm}) = 64\ 800\ \mathrm{kN\ mm} = 64.8\ \mathrm{kN\ m}$$

Alternatively, the moment M can be computed by taking moments about any line in the cutting plane that is parallel to AD. For example summing moments about line EF and assuming positive moment as defined by the right hand rule yields the same result:

$$M = +\ (270\ \mathrm{kN})(300\ \mathrm{mm}) - (270\ \mathrm{kN})(60\ \mathrm{mm}) = 64\ 800\ \mathrm{kN\ mm} = 64.8\ \mathrm{kN\ m}$$

Sample Problem 2.1 combines a review of the computation of the moment corresponding to two stress blocks acting on a cut section and the determination of the internal bending moment in a beam.

SAMPLE PROBLEM 2.1 The freebody for a beam that supports a partial uniform load of 6 kN/m and a concentrated load P is shown in Figure 2.8(a). The stress blocks due to the internal bending moment at Q on QA are shown in Figure 2.8(b).
Determine:

(a) the total compression force due to the bending moment
(b) the internal bending moment at Q,
and (c) the value of the concentrated load P.

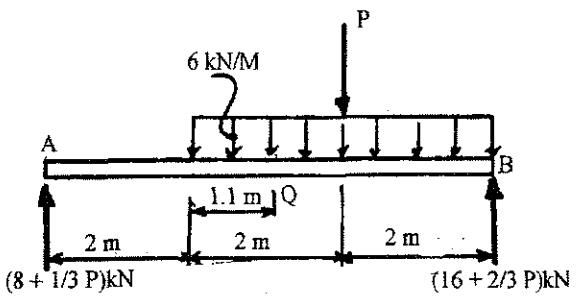


Figure 2.8 (a)

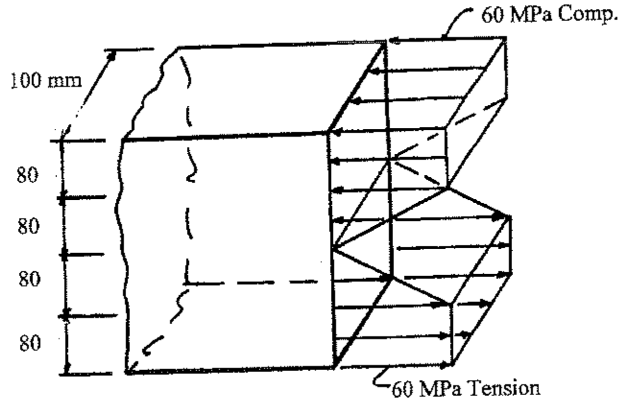


Figure 2.8 (b)

Solution:

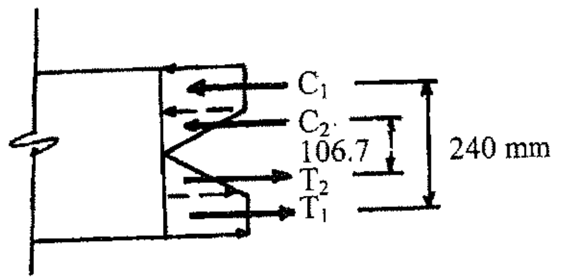
Step 1 The volume of the compression stress block is equal to the compression force. In this case the volume consists of a cube and a wedge. (By symmetry they are equal to the corresponding tension volumes.)

$$C_1 = (60\ \text{N/mm}^2)(80\ \text{mm})(100\ \text{mm}) = 480\ 000\ \text{N} = 480\ \text{kN}$$

$$C_2 = \frac{(60\ \text{N/mm}^2)(80\ \text{mm})(100\ \text{mm})}{2} = 240\ 000\ \text{N} = \underline{240\ \text{kN}}$$

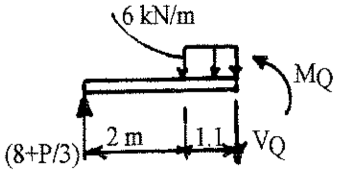
$$\text{Total Compression Force} = 720\ \text{kN}$$

Step 2 The value of the bending moment is computed by summing the moments of the forces C_1, C_2, T_1, and T_2 about a convenient axis. In this case it is possible to take advantage of symmetry and sum the two couples.



$$\Sigma M = (480\ \text{kN})(0.24\ \text{m}) + (240\ \text{kN})(0.1067\ \text{m})$$
$$= 140.8\ \text{kN.m}$$

Step 3 Cut the beam at Q and calculate the internal bending moment at Q using the freebody AQ:



$\Sigma M_Q = 0$

$M_Q + (6)(1.1)(.55) - (8 + P/3)(3.1) = 0$

$M_Q = (21.17 + 1.0333\ P)$ kN.m

Step 4 Equate the moment computed in Step 2 with that computed in Step 3. Solve for P.

$140.8 = 21.17 + 1.0333\ P$ $\qquad$ P = 115.8 kN

Note that the total shear, V_Q, can also be calculated from the given information.

PROBLEMS

2.1 The loading on part of the floor of a warehouse is shown in Figure 2.P1. Determine the magnitude of the equivalent single force and its location. (Answer: 131.6 kN at (2, 2.452) m)

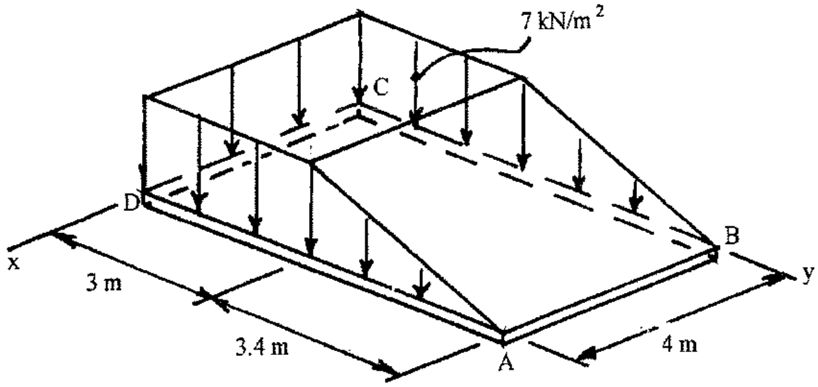


Figure 2.P1

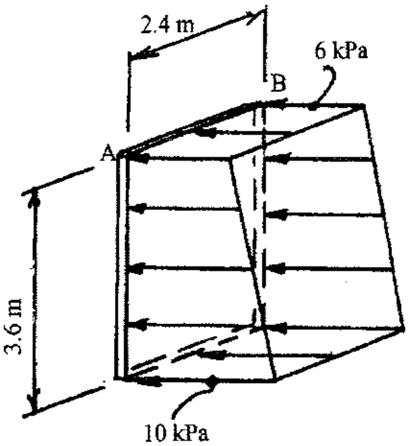


Figure 2.P2

2.2 The pressure distribution on a vertical gate valve in the wall of a reservoir is shown in Figure 2.P2. Determine the equivalent single force acting on the gate and its moment about the line AB. Also locate the equivalent single force. (Answer: 69.12 kN, 134.78 kN.m, 1.95 m below AB)

2.3 Compute the volume of the compression and of the tension stress block shown in Figure 2.P3. What is the value of the corresponding bending moment? (Answer: 130.5 kN, 30.28 kN.m)

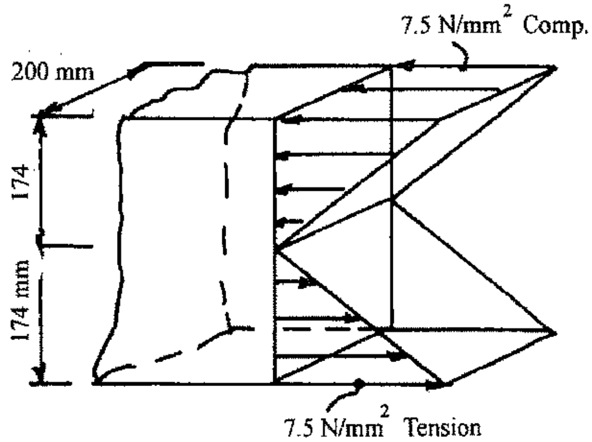


Figure 2.P3 Stresses from bending moment

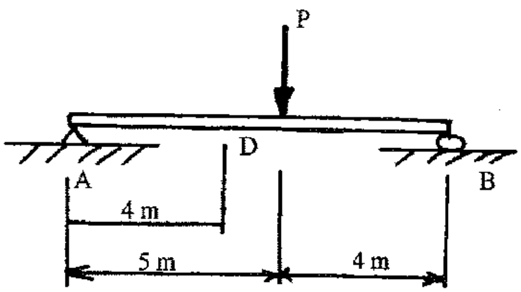
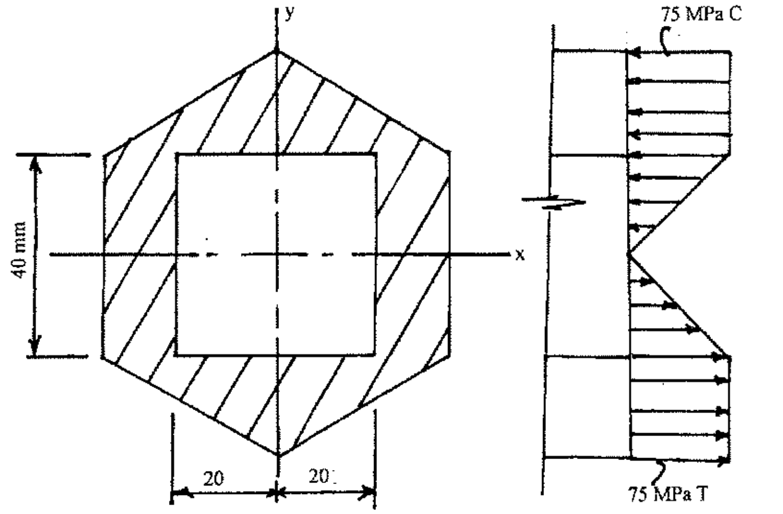


Figure 2.P4 Beam layout

2.4 At location D of the beam shown in Figure 2.P4 the stresses due to bending are as shown in Figure 2.P3. Determine the value of the force P. (Answer: 17.03 kN)

2.5 Shown below in Figure 2.P5 (a) is the cross section of a beam; the stress distribution due to the bending moment M is shown in Figure 2.P5 (b). Determine the volume of the compression stress block, and the bending moment M. The cross section is a regular hexagon with a square, interior hole. (Answers: 73.92 kN, 3.357 kN.m)



(a) Cross section shape (b) Stress distribution

Figure 2.P5 Hexagonal member with concentric square hole

2.6 The cross section of a steel structural tee is shown in Figure 2.P6 (a). The stresses due to a bending moment M at a particular location along the beam are shown in Figure 2.P6 (b). Determine:

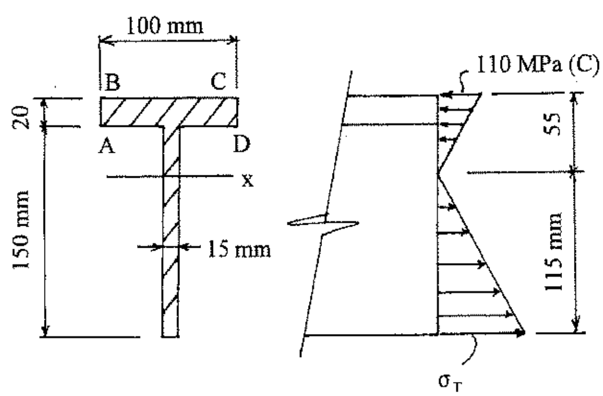
(a) the total compression force acting on the cross section,

(b) the maximum tension stress, σ_T,

(c) the magnitude of the corresponding bending moment M, and

(d) the magnitude of the force acting on ABCD.

(Answers: 198.4 kN, 230 MPa, 23.87 kN·m, 180 kN)



(a) Cross section shape (b) Stress distribution

Figure 2.P6 Structural Tee section in bending

2.7 In a reinforced concrete beam subject to a bending moment only, it is normally assumed that the compression force is taken by part of the concrete and the tension force by the reinforcing steel. The cross section of a reinforced concrete beam is shown below along with the assumed distribution of the compression stresses due to a bending moment M. Determine:

(a) the value of the compression force acting on the cross section
(b) the tension stress in the steel bars
(c) the bending moment M.

(Answers: 715 kN, 252.9 MPa, 284.4 kN.m)

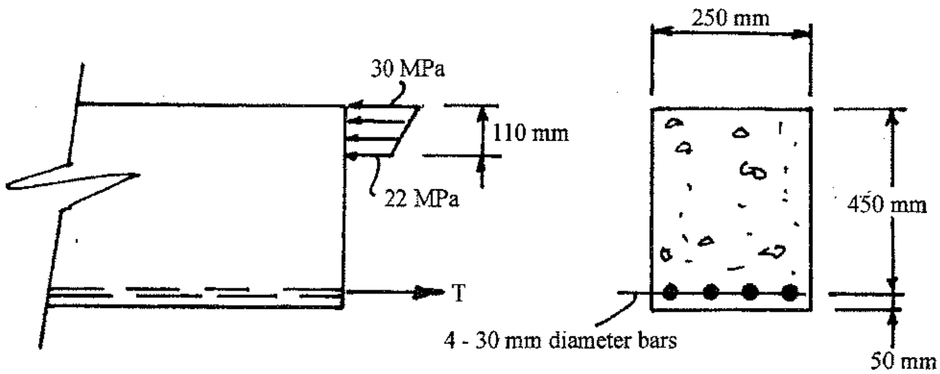


Figure 2.P7 Reinforced concrete beam in subjected to bending moment

3 STRESSES DUE TO BENDING OF BEAMS

3.1 INTRODUCTION

In Chapter 2, we studied stress blocks and the manner in which they can be used to represent bending moment in a beam. As shown there, the compression stress block has the same volume as the tension stress block, and together the two define a couple which has the magnitude of the bending moment. However, the normal problem is to determine the magnitude of the stress resulting from moment at any location in the beam. In this chapter we will derive the relationship that enables the bending stresses to be determined by calculation for beams that respond linearly and elastically to load.

3.2 THE FUNDAMENTAL ASSUMPTIONS

The derivation of the bending stress formula involves these three assumptions:

1. The material is linearly elastic, or stress is proportional to strain.
2. Plane sections before bending remain plane during bending.
3. Bending stresses are independent of the stresses caused by internal axial and shear forces in the beam.

The first assumption was explained in Chapter 1. The second can be explained by considering Figure 3.1:

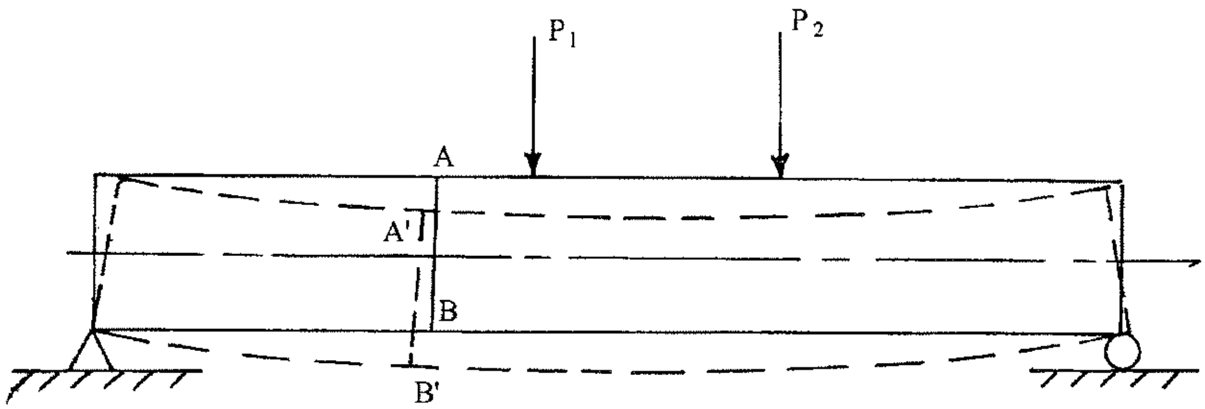


Figure 3.1 Representation of Plane Sections Remaining Plane

A plane passing through the unloaded beam at AB can be shown experimentally to move to A'B' after the beam is loaded. What is important to observe is that it remains a plane even after the load is applied; this behaviour is characteristic of most beams regardless of material. Finally, it has been shown experimentally and theoretically, that the computed bending stresses neglecting the axial and shear forces acting on the cross section are quite accurate, and thus can be used for most engineering purposes. If stresses resulting from bending must be determined to a higher degree of accuracy, then the affect of the axial and shear forces can be incorporated using a much more complex set of calculations.

Before proceeding we need to isolate a "freebody diagram" of a "beam element" by passing two cutting planes separated by a distance "dx" through the freebody of a beam perpendicular to the longitudinal axis as shown in Figure 3.2(a).

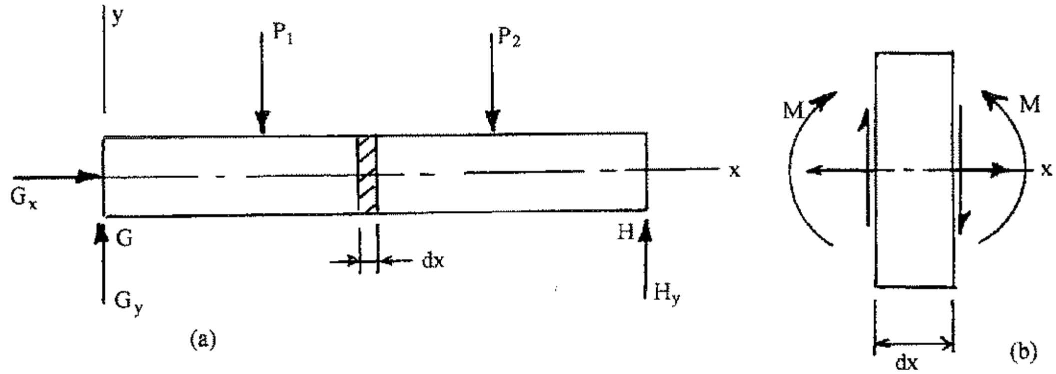


Figure 3.2 Defining a Beam Element in Equilibrium

The beam segment between the two cutting planes is referred to as a beam element. Because the length of a beam element approaches zero, we can assume that the bending moment, M, at the two ends of the beam element are virtually the same. Recall that in a statically determinate system, internal bending moments can be computed using only the freebody of the member and the equations of equilibrium. The free body diagram for a beam element is shown in Figure 3.2 (b).

3.3 DERIVATION OF THE BENDING STRESS FORMULA

Because plane sections remain plane, the beam element in Figure 3.2(b) assumes the shape shown in Figure 3.3(a) after the load is applied to the beam, that is after the bending moments are applied. As can be seen, the bottom longitudinal fibres have lengthened, the upper ones have shortened, and the two ends of the beam element have remained planar. It is convenient to consider that the left side has remained in its original position and thus relatively the plane section on the right side has rotated to the position shown in Figure 3.3(b). It should be emphasized that the two beam elements are, in fact, identical in that the change in length of the longitudinal fibres is identical in both cases.

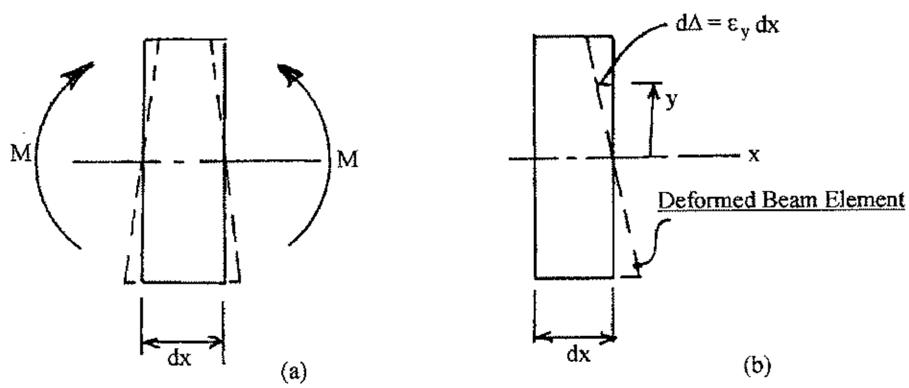


Figure 3.3 Deformation of a Beam Element due to Bending Moment

Considering Figure 3.3(b), the fibre at a distance y from the x-axis has been reduced in length by the amount $d\Delta$ where:

$$d\Delta = \varepsilon_y dk \qquad \text{.....Eq. 3.1}$$

where ε_y = strain in the fibre at distance y from the x-axis

It is apparent from the fact that plane sections have remained plane, that the strain in each fibre is proportional to its distance from the x-axis because the original length of all longitudinal fibres was the same, that is, dx. Thus:

$$\varepsilon \propto y \qquad \text{.....Eq. 3.2}$$

In addition, from the assumption that the material is linearly elastic we know that stress is proportional to strain, that is:

$$\sigma \propto \varepsilon \qquad \text{.....Eq. 3.3}$$

It follows from Equations 3.2 and 3.3 that the stresses due to bending must be proportional to the distance from the x-axis, that is:

$$\sigma \propto y \qquad \text{.....Eq. 3.4}$$

The relationship in Equation 3.4 can be expressed as an equality if a constant of proportionality K is introduced as shown in Equation 3.5:

$$\sigma = Ky \qquad \text{.....Eq. 3.5}$$

Before Equation 3.5 can be used to compute a stress due to bending moment, we must determine the location of the reference axis (location of zero longitudinal strain) from which y is measured, and the value of the constant K. These are obtained from the following two requirements:

1. The total force in the x direction on any cross section along the beam equals zero because the compression and tension forces which define bending moment are equal.

2. The moment of the tension and compression stress blocks is equal to the bending moment.

Figure 3.4(a) shows the bending moment at a particular location along a beam and the corresponding stress blocks are shown in the side/sectional view Figure 3.4(b). The assumed, generalized cross section for the beam is shown in Figure 3.4(c). Note that the stress block has been shown in a manner that corresponds to the sense of the change in length of the fibres, ie. compression acts to shorten and tension acts to lengthen the fibres. Although this is somewhat different to the convention used in Chapter 2, the volumes of the compression and tension stress blocks are identical.

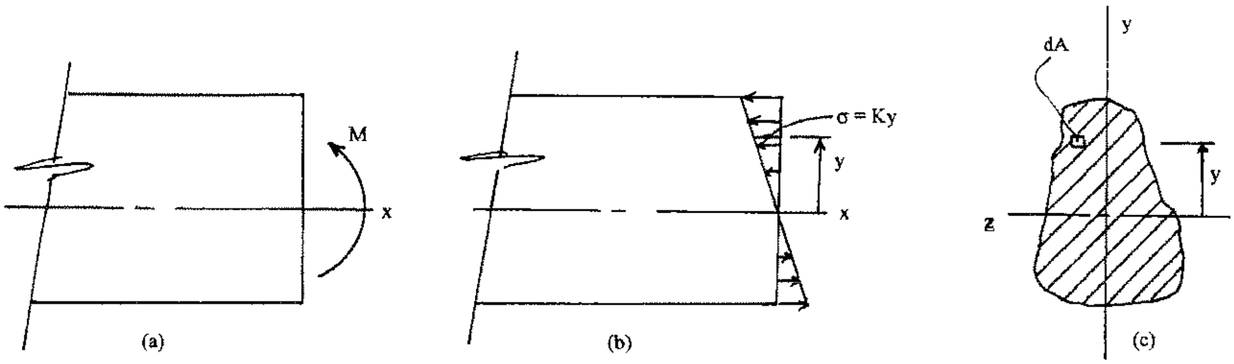


Figure 3.4 Bending Moment, Stress Blocks, and Beam Cross Section

As shown in Figure 3.4(b), the stress at a distance y from the x-axis is given by Equation 3.5 and that stress acts over the elemental area dA shown in Figure 3.4(c). The volume of the stress block associated with dA is a force, dF, where:

$$dF = (\sigma)(dA) = (Ky)(dA) \qquad \text{.....Eq. 3.6}$$

The total force acting on the cross section is the sum of all of the forces, dF, that is:

$$F = \int_A dF = \int_A KydA \qquad \text{....Eq. 3.7}$$

However, the total force equals zero because only a moment is acting on the cross section. Therefore, from Equation 3.7 we can write:

$$K = \int_A ydA = 0 \qquad \text{.....Eq. 3.8}$$

The constant of proportionality, K, will only equal zero for the trivial case where there is no deformation of the beam element, that is, there is no load on the beam. If K is not equal to zero then the integral in Equation 3.8 must equal zero for all loadings and that is only true if the z-axis shown in Figure 3.5(c) is a centroidal axis of the area of the cross section. We can conclude therefore that the x-axis and the z-axis must both pass through the centroid of the cross section. Therefore the distance y in Equation 3.5 is defined with respect to the z-axis.

Having determined the reference axis for the distance y, the constant of proportionality K can be established. The moment, dM, of the stress block associated with dA computed with respect to the z-axis is

$$dM = y(dF) = (y)(\sigma dA) = y[(Ky)dA] = Ky^2dA$$

The applied moment, M, is equal to the sum of the moments of all elemental stress blocks, that is:

$$M = \int_A Ky^2 dA = K \int_A y^2 dA \qquad \text{....Eq. 3.9}$$

The integral in Equation 3.9 defines a property of the cross section and is not related to the bending moment or the material. It is a mathematical property of the area called the "Second Moment of Area" or "Moment of Inertia of Area" and given the symbol I_z. Thus Equation 3.9 can be rewritten to define K as:

$$K = M_z/I_z \qquad \text{....Eq. 3.10}$$

Substituting this result into Equation 3.5 yields:

$$\sigma = \frac{M_z y}{I_z} \qquad \text{....Eq. 3.11}$$

where σ = stress (MPa) at a distance y (mm) from the z-axis

M_z = bending moment (N.mm) about the z-axis

I_z = second moment of area (mm^4) of the cross section with respect to the centroidal z-axis.

Using Equation 3.11, we can compute the stress due to a bending moment at any location in a given beam made from linearly elastic material. As will be shown in the next section, this equation can also be used to design beams.

3.4 DESIGN OF SIMPLY-SUPPORTED PRISMATIC STEEL AND TIMBER BEAMS

It is evident from Equation 3.11 that the maximum stresses due to bending of a prismatic beam will occur:

a. at the locations of the maximum bending moments which can be determined from the moment diagram, and

b. at the locations in the cross section where y is maximum, that is, in the extreme top and bottom fibres.

It will also be assumed in CIV101F that the failure load on a beam occurs when the maximum stress due to the bending moment just reaches the yield stress for the material. Thus the maximum bending moment due to the service loads must be increased by the load factor. Accordingly Equation 3.11 can be re-written as:

$$\sigma_{yield}(\text{MPa}) = \frac{(M_{max})(\text{Load Factor})}{I_z}(y_{max}) \qquad \text{....Eq. 3.12}$$

In this form of the equation, two terms relate solely to the cross sectional area, y_{max} and I_z, and they can be used to compute the section modulus, S, as shown in Equation 3.13.

$$\frac{I_z}{y_{max}} = \text{Section Modulus} = S_z \frac{mm^4}{mm} = mm^3 \qquad \text{....Eq. 3.13}$$

Substituting Equation 3.13 into Equation 3.14 yields:

$$S_{required}(mm^3) = \frac{M_{max}(\text{Load Factor})}{\sigma_{yield}} \qquad \text{....Eq. 3.14}$$

Section moduli for steel and timber sections are tabulated in handbooks. Tables showing the section properties for a representative sample of wide flange beams, hollow structural sections, and standard channels are given on the last three pages of these notes. In the design of beams, the type of cross section and the material are normally chosen in advance; then one selects the beam which has the least cross sectional area from amongst those having the required section modulus. Normally the required cross section is determined from the bending moment. However it is also necessary in practice to check the shear stresses and the deflection. In order to avoid buckling of the compression flange it is also necessary to provide adequate lateral support of the compression flange. In CIV101F the design of beams will be based only on the requirements imposed by the bending moment.

SAMPLE DESIGN PROBLEM 3.1

Assuming adequate lateral support of the compression flange, select a wide flanged beam given that the maximum bending moment due to service loads is 162 kN.m, the yield stress is 280 MPa for both tension and compression, and the load factor is 1.8.

Solution:

Step 1 Determine the required section modulus using Equation 3.14.

$$S_{required}(mm^3) = \frac{M_{max}(\text{load factor})}{\sigma_{yield}} = \frac{162 \times 10^6 \text{ N.mm}(1.8)}{280 \text{ N/mm}^2} = 1041 \times 10^3 \text{ mm}^3$$

Step 2 Identify possible beams from the table of wide flanged beams

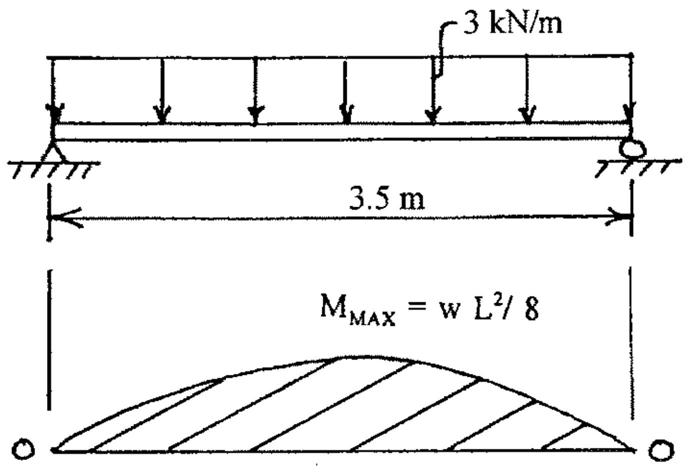
Designation	S_x (10^3 mm^3)	Mass (kg/m)
W406	1060	60
W305	1060	74
W254	1095	89

Step 3 Select the section with the least mass: Use W406 x 60

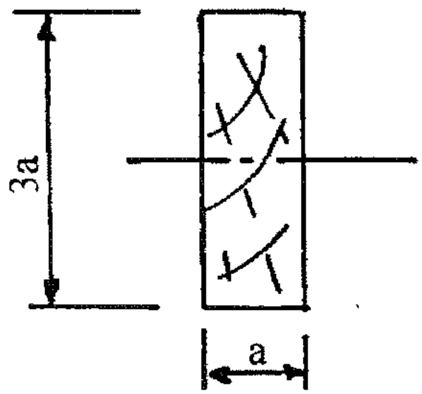
Note: A heavier section might have had to be adopted if there had been a depth restriction. For example, if the depth was limited to 320 mm then the W305 x 74 would have had to be chosen. In practice other checks would include shear and deflection.

SAMPLE DESIGN PROBLEM 3.2

The wooden beam shown below supports a uniformly distributed load of 3 kN/m. The tensile strength of wood is assumed to be 12 MPa and that the compressive strength is higher than the tensile strength. Using a load factor of 2 against failure, design a cross section for which the section height is three times the width. Assume that the beam is laterally supported so that buckling is not a factor.



Moment Diagram



Cross-section

Solution

$$M_{max} = \frac{(3\,\text{kN/m})(3.5\,\text{m})^2}{8} = 4.594\,\text{kN m} \qquad I_x = \frac{(a)(3a)^3}{12} = 2.25\,a^4$$

$$\sigma_{ultimate} = 12\,\text{MPa} = \frac{(M_{max})(\text{Load Factor})y_{max}}{I_x} = \frac{(4.594 \times 10^6\ \text{N mm})(2)(1.5a)}{2.25\,a^4}$$

Compute a = 79.9 mm

Use 80 mm x 240 mm section for bending moment

PROBLEMS

3.1 A simply supported beam which has the cross-section shown in Problem 2.6 is subject to bending moment of 20 kN.m. Determine:

(a) centroidal x-axis

(b) the second moment of area about the centroidal x-axis

(c) the maximum compression and tension stresses due to the bending moment

(Answers: 11.935 x 10^6 mm^4, 92.2 MPa (C))

3.2 The service loads on a steel wide-flange beam are shown in Figure 3.P2. The material has a yield stress of 290 MPa, the load factor is 1.8, and the design criterion is to minimize weight. Neglecting the self-weight of the beam,

(a) determine the cross-section assuming no depth limits (M = 182.2 kN.m, W457 x 89)

(b) determine the cross-section if the depth is not to exceed 400 mm (W305 x 97).

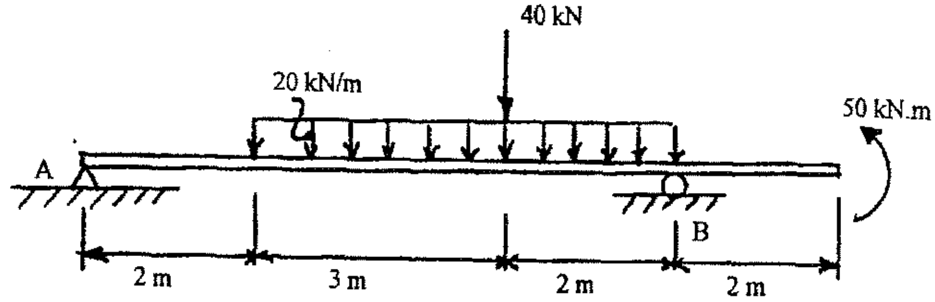


Figure 3.P2

3.3 A welded built-up beam is fabricated from a 203 x 102 x 8 hollow structural section and two steel plates, 10 mm x 300 mm as shown in Figure 3.P3 (a). The loading is shown in Figure 3.P3 (b) and the properties of the hollow structural sections are listed on Page 35. Neglecting the self-weight of the beam, determine the maximum compressive bending stress caused by the loads. Where is it located in the beam?

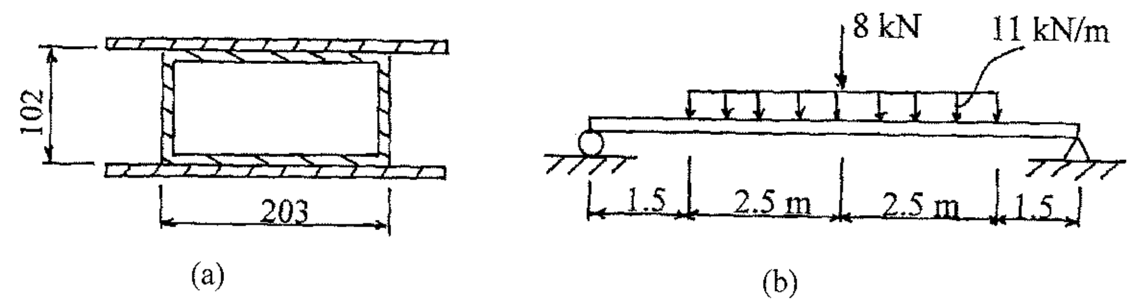
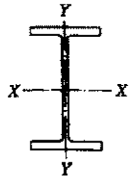


Figure 3.P3

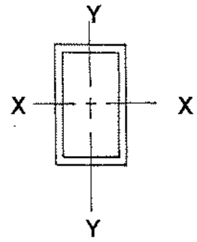


Wide-Flange Beams (SI Units)

Designation*	Area (mm²)	Depth (mm)	FLANGE Width (mm)	FLANGE Thickness (mm)	Web Thickness (mm)	AXIS X–X I (10⁶ mm⁴)	AXIS X–X S (10³ mm³)	AXIS X–X r (mm)	AXIS Y–Y I (10⁶ mm⁴)	AXIS Y–Y S (10³ mm³)	AXIS Y–Y r (mm)
W914 × 342	43610	912	418	32.0	19.3	6245	13715	378	391	1870	94.7
× 238	30325	915	305	25.9	16.5	4060	8880	366	123	805	63.5
W838 × 299	38130	855	400	29.2	18.2	4785	11210	356	312	1560	90.4
× 226	28850	851	294	26.8	16.1	3395	7980	343	114	775	62.7
× 193	24710	840	292	21.7	14.7	2795	6655	335	90.7	620	60.7
W762 × 196	25100	770	268	25.4	15.6	2400	6225	310	81.6	610	57.2
× 161	20450	758	266	19.3	13.8	1860	4900	302	60.8	457	54.6
W686 × 217	27675	695	355	24.8	15.4	2345	6735	290	184	1040	81.5
× 140	17870	684	254	18.9	12.4	1360	3980	277	51.6	406	53.8
W610 × 155	19740	611	324	19.1	12.7	1290	4230	257	108	667	73.9
× 125	15935	612	229	19.6	11.9	985	3210	249	39.3	342	49.5
× 92	11750	603	179	15.0	10.9	645	2145	234	14.4	161	35.1
W533 × 150	19225	543	312	20.3	12.7	1005	3720	229	103	660	73.4
× 124	15675	544	212	21.2	13.1	762	2800	220	33.9	320	46.5
× 92	11805	533	209	15.6	10.2	554	2080	217	23.9	228	45.0
W457 × 144	18365	472	283	22.1	13.6	728	3080	199	83.7	592	67.3
× 113	14385	463	280	17.3	10.8	554	2395	196	63.3	452	66.3
× 89	11355	463	192	17.7	10.5	410	1770	190	20.9	218	42.9
W406 × 149	18970	431	265	25.0	14.9	620	2870	180	77.4	585	64.0
× 100	12710	415	260	16.9	10.0	397	1915	177	49.5	380	62.5
× 60	7615	407	178	12.8	7.7	216	1060	168	12.0	135	39.9
× 39	4950	399	140	8.8	6.4	125	629	159	3.99	57.2	28.4
W356 × 179	22775	368	373	23.9	15.0	574	3115	158	206	1105	95.0
× 122	15550	363	257	21.7	13.0	367	2015	154	61.6	480	63.0
× 64	8130	347	203	13.5	7.7	178	1025	148	18.8	185	48.0
× 45	5710	352	171	9.8	6.9	121	688	146	8.16	95.4	37.8
W305 × 143	18195	323	309	22.9	14.0	347	2145	138	112	728	78.5
× 97	12325	308	305	15.4	9.9	222	1440	134	72.4	477	76.7
× 74	9485	310	205	16.3	9.4	164	1060	132	23.4	228	49.8
× 45	5670	313	166	11.2	6.6	99.1	633	132	8.45	102	38.6
W254 × 89	11355	260	256	17.3	10.7	142	1095	112	48.3	377	65.3
× 67	8580	257	204	15.7	8.9	103	805	110	22.2	218	51.1
× 45	5705	266	148	13.0	7.6	70.8	531	111	6.95	94.2	34.8
× 33	4185	258	146	9.1	6.1	49.1	380	108	4.75	65.1	33.8
W203 × 60	7550	210	205	14.2	9.1	60.8	582	89.7	20.4	200	51.8
× 46	5890	203	203	11.0	7.2	45.8	451	88.1	15.4	152	51.3
× 36	4570	201	165	10.2	6.2	34.5	342	86.7	7.61	92.3	40.9
× 22	2865	206	102	8.0	6.2	20.0	193	83.6	1.42	27.9	22.3
W152 × 37	4735	162	154	11.6	8.1	22.2	274	68.6	7.12	91.9	38.6
× 24	3060	160	102	10.3	6.6	13.4	167	66.0	1.84	36.1	24.6
W127 × 24	3020	127	127	9.1	6.1	8.87	139	54.1	3.13	49.2	32.3
W102 × 19	2470	106	103	8.8	7.1	4.70	89.5	43.7	1.61	31.1	25.4

*W means wide-flange beam, followed by the nominal depth in mm, then the mass in kg per meter of length.

HOLLOW STRUCTURAL SECTIONS
CSA G40.20
Rectangular

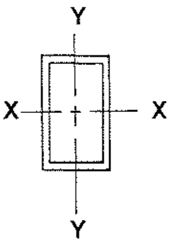


PROPERTIES AND DIMENSIONS

Designation*	Wall Thick-ness	Mass	Dead Load	Area	Axis X-X				Axis Y-Y				Torsional Constant	Shear Constant
					I_x	S_x	r_x	Z_x	I_y	S_y	r_y	Z_y	J	C_{rt}
mm x mm x mm	mm	kg/m	kN/m	mm^2	10^6 mm^4	10^3 mm^3	mm	10^3 mm^3	10^6 mm^4	10^3 mm^3	mm	10^3 mm^3	10^3 mm^4	mm^2
HSS 356x254														
x13	12.70	113	1.11	14 400	253	1 420	132	1 730	150	1 180	102	1 370	304 000	7 740
x9.5	9.53	86.5	0.849	11 000	198	1 120	134	1 340	118	931	104	1 060	235 000	6 050
HSS 305x203														
x13	12.70	93.0	0.912	11 800	147	964	111	1 190	78.2	769	81.2	897	167 000	6 450
x9.5	9.53	71.3	0.700	9 090	116	762	113	926	62.1	611	82.7	701	130 000	5 080
x8.0	7.95	60.1	0.590	7 660	99.4	652	114	787	53.3	525	83.4	596	110 000	4 340
x6.4	6.35	48.6	0.476	6 190	81.5	535	115	640	43.8	431	84.1	486	89 700	3 550
HSS 254x152														
x13	12.70	72.7	0.713	9 260	75.2	592	90.1	747	33.6	442	60.3	522	77 700	5 160
x9.5	9.53	56.1	0.551	7 150	60.4	475	91.9	589	27.2	357	61.7	413	61 400	4 110
x8.0	7.95	47.5	0.466	6 050	52.0	410	92.7	503	23.6	309	62.4	354	52 500	3 530
x6.4	6.35	38.4	0.377	4 900	42.9	338	93.6	411	19.5	256	63.1	290	42 900	2 900
HSS 203x152														
x13	12.70	62.6	0.614	7 970	43.0	423	73.4	528	27.3	359	58.6	432	56 000	3 870
x9.5	9.53	48.5	0.476	6 180	34.8	343	75.1	420	22.3	292	60.0	344	44 400	3 150
x8.0	7.95	41.1	0.403	5 240	30.2	297	75.9	360	19.3	254	60.8	295	38 100	2 730
x6.4	6.35	33.4	0.327	4 250	25.0	246	76.7	295	16.1	211	61.5	243	31 200	2 260
x4.8	4.78	25.5	0.250	3 250	19.5	192	77.5	228	12.6	165	62.2	188	24 100	1 760
HSS 203x102														
x13	12.70	52.4	0.515	6 680	31.3	308	68.4	405	10.2	201	39.1	246	26 700	3 870
x9.5	9.53	40.9	0.401	5 210	25.8	254	70.3	326	8.57	169	40.5	199	21 700	3 150
x8.0	7.95	34.8	0.341	4 430	22.5	221	71.2	281	7.54	148	41.3	172	18 800	2 730
x6.4	6.35	28.3	0.278	3 610	18.8	185	72.2	232	6.35	125	42.0	143	15 600	2 260
x4.8	4.78	21.7	0.213	2 760	14.7	145	73.1	180	5.03	99.0	42.7	111	12 100	1 760
HSS 178x127														
x13	12.70	52.4	0.515	6 680	26.4	297	62.9	378	15.5	244	48.1	298	33 300	3 230
x9.5	9.53	40.9	0.401	5 210	21.7	244	64.6	303	12.8	202	49.6	240	26 800	2 660
x8.0	7.95	34.8	0.341	4 430	19.0	213	65.4	261	11.2	177	50.3	207	23 000	2 320
x6.4	6.35	28.3	0.278	3 610	15.8	178	66.2	216	9.40	148	51.1	171	19 000	1 940
x4.8	4.78	21.7	0.213	2 760	12.4	140	67.1	168	7.41	117	51.8	133	14 700	1 520
HSS 152x102														
x13	12.70	42.3	0.415	5 390	14.7	193	52.2	252	7.67	151	37.7	189	17 500	2 580
x9.5	9.53	33.3	0.327	4 240	12.4	162	54.0	206	6.51	128	39.2	155	14 400	2 180
x8.0	7.95	28.4	0.279	3 620	10.9	143	54.8	179	5.76	113	39.9	135	12 500	1 920
x6.4	6.35	23.2	0.228	2 960	9.19	121	55.7	148	4.88	96.2	40.6	112	10 400	1 610
x4.8	4.78	17.9	0.175	2 280	7.28	95.6	56.5	116	3.89	76.6	41.3	87.8	8 150	1 270

* depth x width x thickness

(continued)



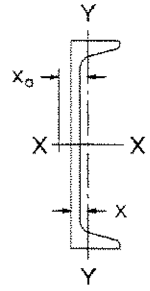
HOLLOW STRUCTURAL SECTIONS
CSA G40.20
Rectangular

PROPERTIES AND DIMENSIONS

Designation*	Wall Thick-ness	Mass	Dead Load	Area	Axis X-X				Axis Y-Y				Torsional Constant	Shear Constant
					I_x	S_x	r_x	Z_x	I_y	S_y	r_y	Z_y	J	C_{rt}
mm x mm x mm	mm	kg/m	kN/m	mm^2	$10^6\ mm^4$	$10^3\ mm^3$	mm	$10^3\ mm^3$	$10^6\ mm^4$	$10^3\ mm^3$	mm	$10^3\ mm^3$	$10^3\ mm^4$	mm^2
HSS 152x76														
x13	12.70	37.3	0.365	4 750	11.5	152	49.3	207	3.71	97.3	28.0	124	9 960	2 580
x9.5	9.53	29.5	0.290	3 760	9.89	130	51.3	171	3.24	85.0	29.4	104	8 450	2 180
x8.0	7.95	25.3	0.248	3 220	8.79	115	52.3	149	2.91	76.3	30.1	91.2	7 450	1 920
x6.4	6.35	20.7	0.203	2 640	7.47	98.0	53.2	125	2.50	65.5	30.8	76.6	6 270	1 610
x4.8	4.78	16.0	0.157	2 040	5.96	78.2	54.1	98.1	2.02	52.9	31.5	60.5	4 950	1 270
HSS 127x76														
x9.5	9.53	25.7	0.252	3 280	6.13	96.5	43.3	126	2.70	70.8	28.7	87.8	6 500	1 690
x8.0	7.95	22.1	0.217	2 820	5.49	86.5	44.2	111	2.44	63.9	29.4	77.4	5 750	1 510
x6.4	6.35	18.2	0.178	2 320	4.70	74.1	45.1	93.4	2.10	55.2	30.1	65.3	4 860	1 290
x4.8	4.78	14.1	0.138	1 790	3.78	59.6	45.9	73.8	1.71	44.8	30.8	51.8	3 850	1 030
HSS 102x76														
x9.5	9.53	21.9	0.215	2 790	3.42	67.4	35.0	87.9	2.16	56.6	27.8	71.6	4 630	1 210
x8.0	7.95	18.9	0.186	2 410	3.10	61.1	35.9	77.9	1.96	51.5	28.5	63.6	4 120	1 110
x6.4	6.35	15.6	0.153	1 990	2.69	52.9	36.7	66.0	1.71	44.8	29.3	54.0	3 500	968
x4.8	4.78	12.2	0.119	1 550	2.18	43.0	37.5	52.6	1.39	36.6	30.0	43.1	2 780	789
x3.2	3.18	8.35	0.082	1 060	1.57	30.8	38.4	37.0	1.01	26.4	30.7	30.4	1 950	565
HSS 102x51														
x9.5	9.53	18.1	0.178	2 310	2.39	47.1	32.2	65.6	0.762	30.0	18.2	39.2	2 070	1 210
x8.0	7.95	15.8	0.155	2 010	2.21	43.6	33.2	59.0	0.714	28.1	18.9	35.6	1 910	1 110
x6.4	6.35	13.1	0.129	1 670	1.95	38.5	34.2	50.7	0.640	25.2	19.6	30.8	1 670	968
x4.8	4.78	10.3	0.101	1 310	1.61	31.8	35.1	40.8	0.537	21.1	20.3	25.0	1 360	789
x3.2	3.18	7.09	0.070	903	1.17	23.1	36.1	29.0	0.397	15.6	21.0	17.9	976	565
HSS 89x64														
x8.0	7.95	15.8	0.155	2 010	1.88	42.2	30.6	55.1	1.09	34.4	23.3	43.3	2 410	908
x6.4	6.35	13.1	0.129	1 670	1.65	37.1	31.4	47.2	0.968	30.5	24.1	37.3	2 080	806
x4.8	4.78	10.3	0.101	1 310	1.36	30.6	32.3	38.0	0.803	25.3	24.8	30.1	1 680	667
x3.2	3.18	7.09	0.070	903	0.990	22.3	33.1	27.0	0.588	18.5	25.5	21.4	1 190	485
HSS 76x51														
x6.4	6.35	10.6	0.104	1 350	0.919	24.1	26.1	31.5	0.479	18.9	18.9	23.6	1 100	645
x4.8	4.78	8.35	0.082	1 060	0.775	20.3	27.0	25.8	0.408	16.1	19.6	19.4	903	546
x3.2	3.18	5.82	0.057	741	0.575	15.1	27.8	18.6	0.306	12.0	20.3	14.0	652	404
HSS 51x25														
x4.8	4.78	4.54	0.045	578	0.150	5.89	16.1	8.21	0.047 6	3.75	9.08	4.91	129	303
x3.2	3.18	3.28	0.032	418	0.122	4.81	17.1	6.34	0.040 0	3.15	9.78	3.85	104	242

* depth x width x thickness
(concluded)

STANDARD CHANNELS (C SHAPES)



PROPERTIES

Designation	Dead Load	Area	Axis X-X			Axis Y-Y				Shear Centre	Torsional Constant	Warping Constant
			I_x	S_x	r_x	I_y	S_y	r_y	x	x_o	J	C_w
	kN/m	mm^2	10^6 mm^4	10^3 mm^3	mm	10^6 mm^4	10^3 mm^3	mm	mm	mm	10^3 mm^4	10^9 mm^6
C380												
x74*	0.730	9 480	168	881	133	4.60	62.4	22.0	20.3	34.9	1 100	131
x60*	0.583	7 570	145	760	138	3.84	55.5	22.5	19.8	39.1	603	109
x50*	0.495	6 430	131	687	143	3.39	51.4	23.0	20.0	42.6	421	95.2
C310												
x45	0.438	5 690	67.3	442	109	2.12	33.6	19.3	17.0	32.4	360	39.9
x37	0.363	4 720	59.9	393	113	1.85	30.9	19.8	17.1	35.9	222	34.6
x31	0.302	3 920	53.5	351	117	1.59	28.1	20.1	17.6	39.3	152	29.3
C250												
x45	0.437	5 670	42.8	337	86.9	1.60	26.8	16.8	16.3	25.3	508	20.5
x37	0.365	4 750	37.9	299	89.4	1.40	24.3	17.1	15.7	28.1	289	18.2
x30	0.291	3 780	32.7	257	93.0	1.16	21.5	17.5	15.4	31.3	153	15.0
x23	0.222	2 880	27.8	219	98.2	0.920	18.8	17.9	15.9	35.7	86.4	11.7
C230												
x30*	0.292	3 800	25.5	222	81.9	1.01	19.3	16.3	14.8	27.7	179	10.5
x22	0.219	2 840	21.3	186	86.6	0.805	16.8	16.8	15.0	32.3	86.6	8.33
x20	0.195	2 530	19.8	173	88.6	0.715	15.6	16.8	15.2	33.7	69.5	7.35
C200												
x28	0.274	3 560	18.2	180	71.6	0.825	16.6	15.2	14.4	25.2	182	6.67
x21	0.200	2 600	14.9	147	75.8	0.627	13.9	15.5	14.0	29.1	77.0	5.04
x17	0.167	2 170	13.5	133	78.7	0.543	12.8	15.8	14.5	32.0	53.8	4.34
C180												
x22	0.214	2 780	11.3	127	63.7	0.568	12.8	14.3	13.5	24.6	110	3.47
x18	0.178	2 310	10.0	113	65.9	0.476	11.4	14.3	13.2	26.5	66.8	2.90
x15	0.142	1 850	8.86	99.6	69.3	0.404	10.3	14.8	13.8	30.3	41.4	2.46
C150												
x19	0.188	2 450	7.11	93.6	53.9	0.425	10.3	13.2	12.9	22.3	98.9	1.84
x16	0.152	1 980	6.21	81.8	56.1	0.351	9.13	13.3	12.6	24.6	53.4	1.53
x12	0.118	1 530	5.36	70.5	59.1	0.278	7.93	13.5	12.9	27.7	30.6	1.21
C130												
x13	0.130	1 690	3.66	57.6	46.5	0.252	7.20	12.2	12.0	22.3	45.0	0.746
x10	0.097	1 260	3.09	48.6	49.5	0.195	6.14	12.4	12.3	26.1	22.5	0.579
C100												
x11	0.106	1 370	1.91	37.4	37.3	0.174	5.52	11.3	11.5	20.9	34.1	0.320
x9	0.092	1 190	1.77	34.6	38.5	0.158	5.18	11.5	11.6	23.0	23.1	0.293
x8	0.079	1 020	1.61	31.6	39.7	0.132	4.65	11.4	11.6	24.2	16.6	0.246
x7	0.069	892	1.53	30.0	41.4	0.122	4.45	11.7	12.6	27.3	13.3	0.233
C75												
x9	0.087	1 120	0.847	22.3	27.4	0.123	4.31	10.5	11.5	19.4	29.7	0.118
x7	0.072	934	0.749	19.7	28.3	0.095 9	3.67	10.1	10.9	20.3	17.5	0.093 4
x6	0.059	763	0.670	17.6	29.6	0.077 2	3.21	10.1	11.0	22.3	10.9	0.076 8
x5	0.053	693	0.635	16.7	30.3	0.068 3	2.98	9.93	11.1	23.1	9.19	0.068 9