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ELG 3106 / 3506 **Electromagnetic Engineering / Électromagnétisme Appliqué** **Fall/Automne 2008**

FINAL EXAMINATION (3 hours) / EXAMEN FINAL (3 heures)

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Date Dec. 6th, 6 déc. 2008 **St./Étudiant #:** ______________________

- This booklet contains 14 pages, including the cover page.
- DO NOT SEPARATE THE PAGES OF THIS BOOKLET!
- Answer all the questions.
- This is a closed-book exam. Equations are provided on the last page of this booklet.
- Calculators are permitted, but must not be pre-programmable.
- Remember to provide units.
- Write down any assumptions that you make.

- Ce livret contient 14 pages, incluant la page de couverture.
- NE PAS SÉPARER LES PAGES DE CE LIVRET !
- Répondre à toutes les questions.
- Cet examen est à livres fermés. Les équations sont fournies à la dernière page de ce livret.
- Les calculatrices sont permises, mais ne doivent pas être pré-programmables.
- Ne pas oublier de préciser les unités.
- Écrire lisiblement toute supposition que vous faites.

Q1/	**/ 9**
Q2/	**/ 13**
Q3/	**/ 10**
Q4/	**/ 13**
Q5/	**/ 10**

Final Mark / Note finale: **/55**

Question 1

The input impedance for a 75Ω lossless transmission line of length 1.25λ is (75 – *j* 75) Ω.

Une ligne de transmission sans pertes de 75Ω et de longueur 1.25λ a une impédance d'entrée de valeur (75 – *j* 75) Ω.

(a) Using the Smith Chart, determine the load impedance. Explain what you are doing.

Déterminer l'impédance de charge en utilisant l'abaque de Smith. Expliquer ce que vous faites.

$Z_{in} = (75 - j75)\,\Omega$, $Z_0 = 75\,\Omega \rightarrow z_{in} = (1 - j)$

Phase is 0.338λ . Subtract the phase accumulation of 1.25λ to return to the load

Phase at load = $0.338\lambda - 1.25\lambda = -0.912\lambda \equiv 0.088\lambda$

Drawing a line from this phase position on the WTG grid through the SWR-circle locates the load at

$$z_L = 0.5 + j0.5$$

Since $Z_0 = 75\,\Omega \quad \therefore \quad Z_L = (37.5 + j37.5)\,\Omega$

(b) Using the Smith Chart, design a short-circuit stub matching network for such load. Find the solution that minimizes the length *l* of the shorted stub. Explain what you are doing.

En utilisant l'abaque de Smith, concevoir pour cette charge un réseau d'adaptation avec un stub en court-circuit. Déterminer la solution qui minimise la longueur *l* du stub. Expliquer ce que vous faites.

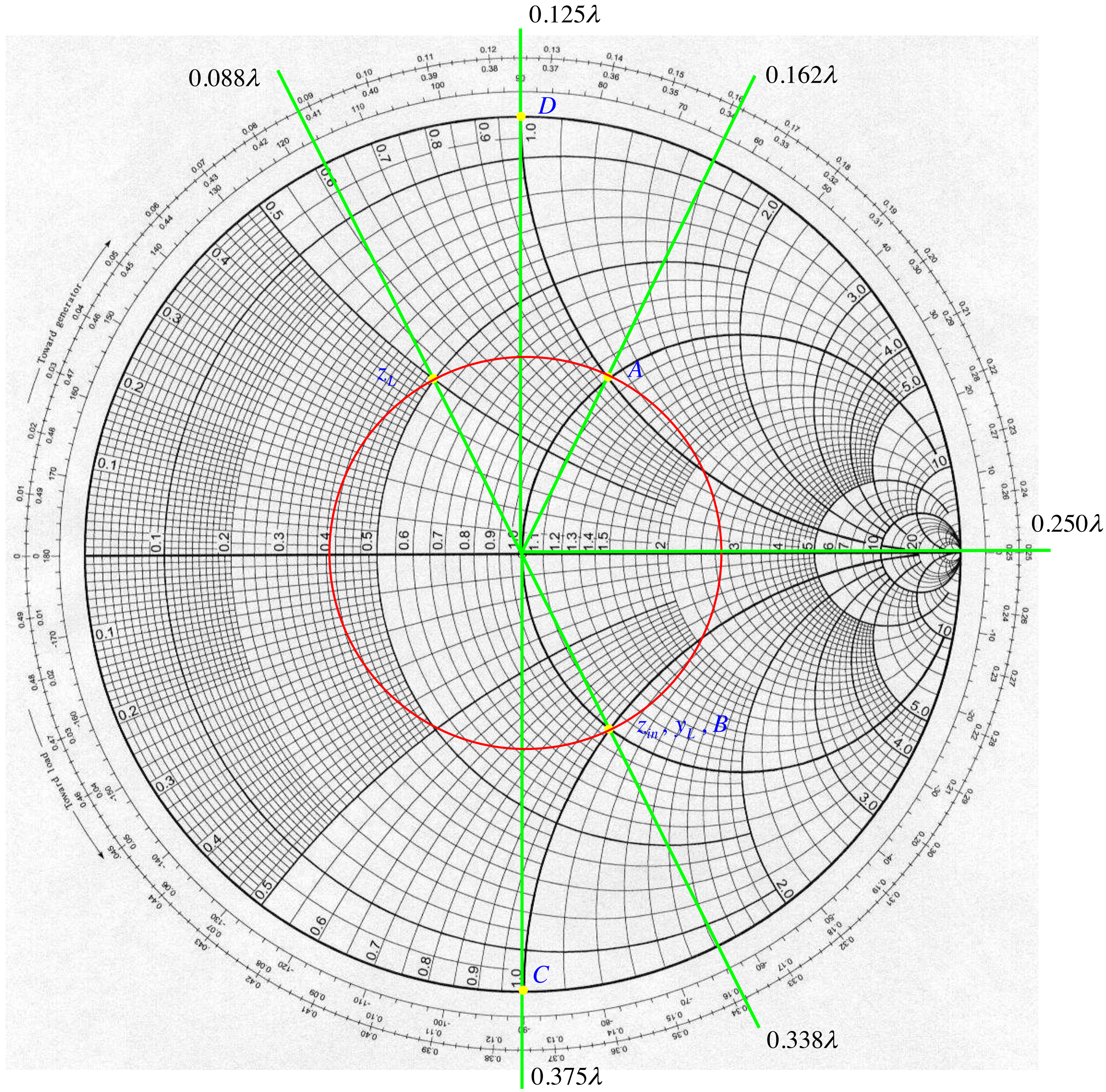
$Z_L = (37.5 + j37.5)\Omega \rightarrow z_L = (0.5 + j0.5) \rightarrow y_L = (1 - j)$

$y_1 = 1 + j$
$y_2 = 1 - j$ CW from y_L to points is distance from load: $d_1 = (0.5 - 0.338 + 0.162)\lambda = 0.324\lambda$
$d_2 = 0\lambda$

Corresponding stub lengths : $y_1' = -j$
$y_2' = +j$

CW from short circuit (impedance at 0, admittance at 0.25) to : $l_1 = (0.375 - 0.25)\lambda = 0.125\lambda$
$l_2 = (0.25 + 0.125)\lambda = 0.375\lambda$

The first solution minimizes the length of the shorted stub.



Question 2

A 10.0 MHz magnetic field travels in a fluid for which the propagation velocity is $1.0\text{x}10^8$ m/sec. Initially, we have $\mathbf{H}(0,0) = 2.0\ \mathbf{a}_x$ A/m. The amplitude drops to 1.0 A/m after the wave travels 5.0 meters in the y direction.

Un champ magnétique se propage à 10.0 MHz dans un fluide avec une vitesse de propagation de $1.0\text{x}10^8$ m/sec. Initialement, nous avons $\mathbf{H}(0,0) = 2.0\ \mathbf{a}_x$ A/m. L'amplitude décroît à 1.0 A/m après que l'onde ait parcourue 5.0 mètres dans la direction des y.

(a) Determine the wavelength, phase constant, and relative permittivity.

Déterminer la longueur d'onde, la constante de phase et la permittivité relative.

$$u_p = 1x10^8 \text{ m/s}; \quad \lambda = \frac{u_p}{f} = 10 \text{ m},$$

Assume a real phase constant: $\beta = \dfrac{2\pi}{\lambda} = 0.2\pi \dfrac{\text{rad}}{\text{m}}$

Assume a real phase velocity: $u_p = \dfrac{c}{\sqrt{\mu_r \varepsilon_r}}$,

where ε_r is the real part of the complex relative permittivity; strictly speaking, since $\varepsilon_c = \varepsilon' - j\varepsilon'' = \varepsilon_0\left(\varepsilon_r - j\varepsilon_r''\right)$, our above assumptions require $\varepsilon''/\varepsilon_r \ll 1$.

We assume a non-magnetic medium, hence $\mu_r = 1$

Therefore $\varepsilon_r = 9$.

(b) What is the attenuation constant?

Quelle est la constante d'atténuation ?

$$\mathbf{H}(0,0) = 2.0\mathbf{a}_x \frac{A}{m}; \quad \therefore H_o = 2.0,\ \phi = 0$$

$$H(y=5) = 1 = H_o e^{-\alpha y} = 2.0e^{-\alpha(5)}; \text{ solving we get } \alpha = 0.14$$

(c) Give the real solution for the magnetic field as a function of position and time.

Donner la solution réelle pour le champ magnétique en fonction de la position et du temps.

$$\omega = 2\pi f = 2\pi\left(10x10^{6}\right) = 20\pi x10^{6}\,\frac{rad}{s}$$

$$\mathbf{H}(y,t) = H_o e^{-\alpha y}\cos\left(\omega t - \beta y + \phi\right)\mathbf{a}_x\,\frac{A}{m}$$

$$\mathbf{H}(y,t) = 2.0e^{-0.14y}\cos\left(20\pi x10^{6}t - 0.2\pi y\right)\mathbf{a}_x\,\frac{A}{m}$$

(d) What is Ampere's law?

Quelle est la loi d'Ampère ?

$$\nabla\times\mathbf{H} = \varepsilon\frac{\partial\mathbf{E}}{\partial t}.$$

(e) What is the magnitude of the electric field?

Quelle est l'amplitude du champ électrique ?

$$\varepsilon \frac{\partial \mathbf{E}}{\partial t} = \begin{vmatrix} \hat{\mathbf{a}}_x & \hat{\mathbf{a}}_y & \hat{\mathbf{a}}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_0 e^{-\gamma y} & 0 & 0 \end{vmatrix} = -H_0 \frac{\partial}{\partial y}\left[\exp(-\gamma y)\right]\hat{\mathbf{a}}_z = \gamma H_0 \exp(-\gamma y)\hat{\mathbf{a}}_z$$

Thus: $\mathbf{E} = \dfrac{\gamma H_0}{j\omega\varepsilon}\exp(-\gamma y)\exp(j\omega t)\hat{\mathbf{a}}_z = \dfrac{-j(\alpha + j\beta)H_0}{\omega\varepsilon}\exp(-\alpha y)\exp(-j\beta y)\exp(j\omega t)\hat{\mathbf{a}}_z$

Since: $\mathbf{E} = E_0 \exp(-\alpha y)\exp\left[j(\omega t - \beta y + \phi)\right]\hat{\mathbf{a}}_z$

Therefore: $E_0 = \dfrac{H_0\sqrt{\alpha^2 + \beta^2}}{\omega\varepsilon} = \dfrac{(2.0)\sqrt{(0.14)^2 + (0.2\pi)^2}}{(20\pi \times 10^6)(8.854 \times 10^{-12})(9)} = 257 \text{ V/m}$

(f) What is the phase difference between the magnetic and electric fields?

Quel est le déphasage entre le champ magnétique et le champ électrique ?

$\phi = \arctan\left(-\alpha/\beta\right) = \arctan(-0.223) = -12.56°$

Therefore : $\Delta\phi = \phi_H - \phi_E = 0 - (-12.56°) = 12.56°$

Question 3

The electric field of a uniform plane wave propagating in free space is given by

Le champ électrique d'une onde plane uniforme se propageant dans un espace libre est donné par

$$\tilde{\mathbf{E}} = (\hat{\mathbf{x}} + j\,\hat{\mathbf{y}})30\exp(-j\pi z/6) \text{ V/m.}$$

(a) Determine the direction of the electric field in the $z = 0$ plane at $t = 0$ns, 5ns, and 10 ns.

Déterminer la direction du champ électrique dans le plan $z = 0$ à $t = 0$ns, 5ns et 10 ns.

$$\begin{aligned}\mathbf{E}(z,t) &= \text{Re}\left\{\tilde{\mathbf{E}}(z)e^{j\omega t}\right\} = \text{Re}\left\{(\hat{\mathbf{x}} + j\,\hat{\mathbf{y}})30\,\text{e}^{-j\pi z/6}\,\text{e}^{j\omega t}\right\} \\ &= \text{Re}\left\{\left(\hat{\mathbf{x}} + e^{j\pi/2}\,\hat{\mathbf{y}}\right)30\,\text{e}^{-j\pi z/6}\,\text{e}^{j\omega t}\right\} \\ &= \hat{\mathbf{x}}30\cos(\omega t - \pi z/6) - \hat{\mathbf{y}}30\sin(\omega t - \pi z/6) \quad \text{V/m}\end{aligned}$$

Since: $\left|\mathbf{E}(z,t)\right| = \sqrt{E_x^2(z,t) + E_y^2(z,t)}$

Since $f = \dfrac{c}{\lambda} = \dfrac{kc}{2\pi} = \dfrac{\left(\pi/6\right)\left(3\times10^8\right)}{2\pi} = 2.5\times10^7$ Hz, therefore $\omega t = 2\pi f t = 5\pi\times10^7 t$ s^{-1}, and

$$\begin{aligned}&(i)\mathbf{E}(0,0) = \hat{\mathbf{x}}30\cos(0) - \hat{\mathbf{y}}30\sin(0) = \hat{\mathbf{x}}30 \quad \text{V/m} \\ &(ii)\mathbf{E}\left(0,5\times10^{-9}\right) = \hat{\mathbf{x}}30\cos\left(\pi/4\right) - \hat{\mathbf{y}}30\sin\left(\pi/4\right) = \hat{\mathbf{x}}\frac{30}{\sqrt{2}} - \hat{\mathbf{y}}\frac{30}{\sqrt{2}} \quad \text{V/m} \\ &(ii)\mathbf{E}\left(0,10\times10^{-9}\right) = \hat{\mathbf{x}}30\cos\left(\pi/2\right) - \hat{\mathbf{y}}30\sin\left(\pi/2\right) = -\hat{\mathbf{y}}30\,\text{V/m}\end{aligned}$$

(b) What is the wave's polarization? Justify your answer.

Quelle est la polarisation de l'onde ? Justifiez votre réponse.

Since $\varphi(z,t) = \arctan\left(\frac{E_y}{E_x}\right) = -(\omega t - \pi z/6)$, we have

$(i)\ \varphi(0,0) = 0$

$(ii)\ \varphi(0, 5\times10^{-9}) = -\pi/4 \equiv -45^\circ$

$(iii)\ \varphi(0, 10\times10^{-9}) = -\pi/2 \equiv -90^\circ$

and

$(i)\ |\mathbf{E}(0,0)| = 30\ \text{V/m}$

$(ii)\mathbf{E}(0, 5\times10^{-9}) = \sqrt{\left(\frac{30}{\sqrt{2}}\right)^2 + \left(\frac{30}{\sqrt{2}}\right)^2} = 30\ \text{V/m}$

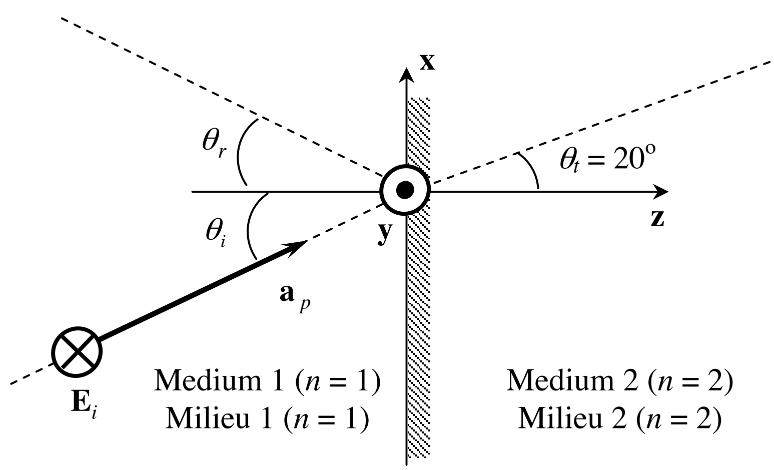
$(ii)\mathbf{E}(0, 10\times10^{-9}) = 30\ \text{V/m}$

So the wave is left-hand circularly (LHC) polarized.

Question 4

Consider a uniform plane wave obliquely incident from medium 1 (air) to a nonmagnetic non conductor medium (medium 2) of index of refraction of $n = 2$, as shown below; the transmitted angle is $\theta_t = 20^\text{o}$. The magnitude of the incident electric field is $E_i = 1$ V/m.

Soit une onde plane uniforme obliquement incidente d'un milieu 1 (air) à un milieu non magnétique non conducteur (milieu 2) d'indice de réfraction $n = 2$ (voir figure ci-dessous). L'angle de transmission est $\theta_t = 20^\text{o}$. L'amplitude du champ électrique incident est $E_i = 1$ V/m.



(a) What is the polarization of the incident wave?

Quelle est la polarisation de l'onde incidente ?

Perpendicular – linearly polarized TE.

The electric field is perpendicular (transverse) to the plane of i ncidence.

(b) Find the incident angle θ_i.

Déterminer l'angle incident θ_i.

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{n_1}{n_2} = \frac{\omega\sqrt{\mu_1\,\varepsilon_1}}{\omega\sqrt{\mu_2\,\varepsilon_2}} = \frac{\sqrt{\mu_{r1}\,\varepsilon_{r1}}}{\sqrt{\mu_{r2}\,\varepsilon_{r2}}} = \frac{1}{2} = 0.5 \quad \rightarrow \quad \sin\theta_i = 2 * 0.342 = 0.684$$

$$\rightarrow \quad \theta_i = 43.157^o$$

(c) Find the reflection coefficient.

Déterminer le coefficient de réflexion.

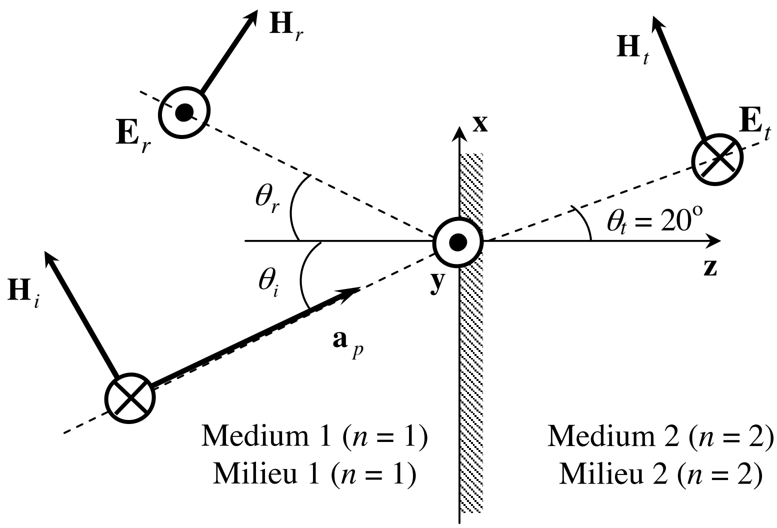
$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = 120\pi\sqrt{\frac{\mu_r}{\varepsilon_r}} \qquad n = \sqrt{\mu_r \ \varepsilon_r}$$

$$\rightarrow \qquad \eta_1 = 120\pi \ \Omega \qquad \text{and} \qquad \eta_2 = \frac{120\pi}{2} = 60\pi \ \Omega$$

$$\Gamma_\perp = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} = \frac{60\pi * 0.7295 - 120\pi * 0.9397}{60\pi * 0.7295 + 120\pi * 0.9397} = \frac{0.7071 - 1}{0.7071 + 1} = -0.44$$

(d) Sketch *properly* the reflected and transmitted field directions on the figure. Justify your answer.

Dessiner *soigneusement* sur la figure les directions des champs réfléchis et transmis. Justifiez votre réponse.



Incident fields: $\mathbf{E}_i$, $\mathbf{H}_i$ and the direction of propagation $\mathbf{a}_p$ should form a direct trihedral.

Reflected fields: with a negative reflection coefficient $\Rightarrow$ $\mathbf{E}_i$ is opposite to $\mathbf{E}_r$ .Furthermore, since the direction of propagation also change (from z > 0 to z < 0), then $\mathbf{H}_r$ should be in the same direction as $\mathbf{H}_i$.

Transmitted fields: $\mathbf{E}_t$, $\mathbf{H}_t$ are in the same direction than $\mathbf{E}_i$, $\mathbf{H}_i$ (respectively) since $\tau = 1 + \Gamma > 0$.

Question 5

(a) Design an air-filled rectangular waveguide to operate at a fundamental mode of a cutoff frequency of 10 GHz. Assume $a = 2b$.

Concevoir un guide d'onde rectangulaire rempli d'air et opérant à un mode fondamental de fréquence de coupure 10 GHz. On prendra $a = 2b$.

$$f_c^{(m,n)} = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

The fundamental is TE_{10}: $$f_c^{(10)} = \frac{c}{2a} \quad \Rightarrow \quad a = \frac{c}{2f_c^{(10)}} = \frac{3\times10^8}{2\left(10\times10^9\right)} = 0.015\text{ m} \quad \text{or} \quad 1.5\text{ cm}$$

Thus: $b = \frac{1}{2}a = 0.75\text{ cm}$

(b) Calculate the cutoff frequencies of the first 5-modes of this guide

Calculer les fréquences de coupure des 5 premiers modes de ce guide.

$$f_c^{(m,n)} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$TE_{10} \quad f_c^{(10)} = \frac{3\times10^{10}}{2}\sqrt{\left(\frac{1}{1.5}\right)^2 + \left(\frac{0}{0.75}\right)^2} = 1.0\times10^{10}\text{ Hz} \quad \text{or} \quad 10\text{ GHz}$$

$$TE_{01} \quad f_c^{(01)} = \frac{3\times10^{10}}{2}\sqrt{\left(\frac{0}{1.5}\right)^2 + \left(\frac{1}{0.75}\right)^2} = 2.0\times10^{10}\text{ Hz} \quad \text{or} \quad 20\text{ GHz}$$

$$TE_{20} \quad f_c^{(20)} = \frac{3\times10^{10}}{2}\sqrt{\left(\frac{2}{1.5}\right)^2 + \left(\frac{0}{0.75}\right)^2} = 2.0\times10^{10}\text{ Hz} \quad \text{or} \quad 20\text{ GHz}$$

$$TE_{11} \quad f_c^{(11)} = \frac{3\times10^{10}}{2}\sqrt{\left(\frac{1}{1.5}\right)^2 + \left(\frac{1}{0.75}\right)^2} = 2.24\times10^{10}\text{ Hz} \quad \text{or} \quad 22.4\text{ GHz}$$

$$TM_{11} \quad f_c^{(11)} = \frac{3\times10^{10}}{2}\sqrt{\left(\frac{1}{1.5}\right)^2 + \left(\frac{1}{0.75}\right)^2} = 2.24\times10^{10}\text{ Hz} \quad \text{or} \quad 22.4\text{ GHz}$$

Extra space / Espace supplémentaire

Equation sheet / Page de formules

$$\Gamma = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_o}{Z_L + Z_o} = |\Gamma|\, e^{j\theta_\Gamma} \qquad S = \frac{|V_{\max}|}{|V_{\min}|} = \frac{1+|\Gamma|}{1-|\Gamma|} \qquad Z(l) = \frac{V(l)}{I(l)} = Z_o\,\frac{Z_L + Z_o \tanh(\gamma l)}{Z_o + Z_L \tanh(\gamma l)}$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \qquad \nabla\times\mathbf{H} = \mathbf{J} + \frac{\partial\mathbf{D}}{\partial t} \qquad \nabla\cdot\mathbf{D} = \rho_v \qquad \nabla\cdot\mathbf{B} = 0$$

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{k_1}{k_2} \qquad \eta = \sqrt{\frac{\mu}{\varepsilon}} \qquad k = \omega\sqrt{\mu\varepsilon}$$

$$\Gamma_\perp = \Gamma_{TE} = \frac{E_o^r}{E_o^i} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} \qquad \tau_\perp = \tau_{TE} = \frac{E_o^{tr}}{E_o^i} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

$$\Gamma_\parallel = \Gamma_{TM} = \frac{E_o^r}{E_o^i} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} \qquad \tau_\parallel = \tau_{TM} = \frac{E_o^{tr}}{E_o^i} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

$$f_c^{m,n} = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(m/a\right)^2 + \left(n/b\right)^2} \qquad \frac{1}{\lambda^2} = \frac{1}{\lambda_g^2} + \frac{1}{\lambda_c^2}$$

$$\beta = k\sqrt{1-\left(f_c^{m,n}/f\right)^2} \qquad u_p = \frac{u}{\sqrt{1-\left(f_c^{m,n}/f\right)^2}}$$

$$Z_{TM} = \eta_{TEM}\sqrt{1-\left(\frac{f_c^{m,n}}{f}\right)^2} \qquad Z_{TE} = \frac{\eta_{TEM}}{\sqrt{1-\left(\frac{f_c^{m,n}}{f}\right)^2}}$$