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Faculty of Engineering

School of Information
Technology and Engineering

**ELG 3101 / 3501** **Electromagnetic Engineering / Électromagnétisme Appliqué** **Fall / Automne 2007**

**FINAL EXAMINATION (3 hours) / EXAMEN FINAL (3 heures)**

| | | | |
|---|---|---|---|
| **Professor** | H. Schriemer, M.C.E. Yagoub | **Nom/Name :** | ________________________ |
| **Date** | Dec. 19th, 19 déc. 2007 | **St./Étudiant #:** | ________________________ |

- This booklet contains 12 pages, including the cover page.
- DO NOT SEPARATE THE PAGES OF THIS BOOKLET!
- Answer all the questions.
- This is a closed-book exam. Equations are provided on the last page of this booklet.
- Calculators are permitted, but must not be pre-programmed.
- Remember to provide units.
- Write down any assumptions that you make.
- All questions have to be computed analytically with the aid of the relevant expressions.

- Ce livret contient 12 pages, incluant la page de couverture.
- NE PAS SÉPARER LES PAGES DE CE LIVRET !
- Répondre à toutes les questions.
- Cet examen est à livres fermés. Les équations sont fournies à la dernière page de ce livret.
- Les calculatrices sont permises, mais ne doivent pas être pré-programmées.
- Ne pas oublier de préciser les unités.
- Écrire lisiblement toute supposition que vous faites.
- Toutes les autres questions doivent être résolues par calcul analytique en s'aidant des équations appropriées.

| | |
|---|---|
| **Q1/** | **/ 8** |
| **Q2/** | **/ 9** |
| **Q3/** | **/ 9** |
| **Q4/** | **/ 10** |
| **Q5/** | **/ 8** |
| **Q6/** | **/ 11** |

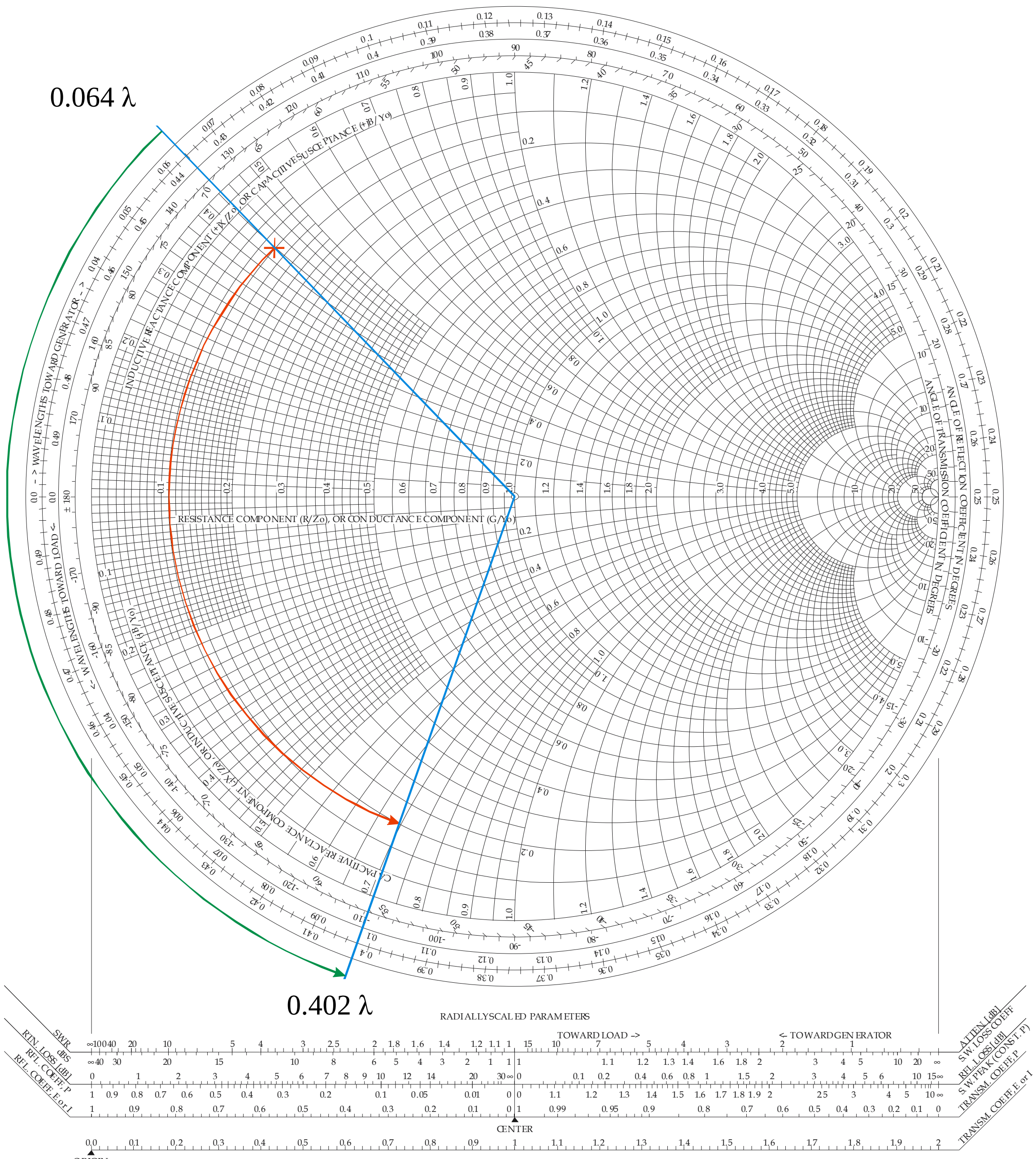
Final Mark / Note finale:

**/55**

# Question 1

The input impedance for a $100\Omega$ lossless transmission line of length $1.162\lambda$ is $(12+j42)\Omega$. Determine the load impedance using the Smith Chart. Explain what you are doing.

Une ligne de transmission sans pertes de $100\Omega$ et de longueur $1.162\lambda$ a une impédance d'entrée de $(12+j42)\Omega$. Déterminer l'impédance de charge avec l'abaque de Smith. Expliquer ce que vous faites.




Given: $l = 1.162\lambda$; $Z_0 = 100\Omega$; $Z_{IN} = 12 + j42\,\Omega$.

Solution: 0.064 λ – 1.162 λ = -1.098 λ. Adding 1.5 λ gives 0.402 λ. $Z_L = Z_0(0.15 - 0.7j) = 15 - 70j\,\Omega$

# Question 2

In a nonmagnetic medium, we have

$\mathbf{E}(z, t) = 2 \cos(\pi 10^8 t - 3z)\, \mathbf{a}_x - \sin(\pi 10^8 t - 3z)\, \mathbf{a}_y$ (V/m).

Dans un milieu non magnétique, nous avons

$\mathbf{E}(z, t) = 2 \cos(\pi 10^8 t - 3z)\, \mathbf{a}_x - \sin(\pi 10^8 t - 3z)\, \mathbf{a}_y$ (V/m).

(a) Determine the wavelength.

Déterminer la longueur d'onde.

$$\cos(\omega t - k z) \quad \Rightarrow \quad k = 3$$

$$k = \frac{2\pi}{\lambda} \quad \Rightarrow \quad \lambda = \frac{2\pi}{3} = 2.09 \text{ m}$$

(b) Determine the dielectric constant of the medium.

Déterminer la constante diélectrique du milieu.

$$k = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_0 \varepsilon_0}\sqrt{\mu_r \varepsilon_r} = \frac{\omega}{c}\sqrt{\mu_r \varepsilon_r}$$

$$\mu_r = 1 \quad \Rightarrow \quad \varepsilon_r = \left(\frac{kc}{\omega}\right)^2 = \left(\frac{(3)(3\times10^8)}{\pi\times10^8}\right)^2 = \left(\frac{9}{\pi}\right)^2 = 8.21$$

(c) Find the **H** field.

Déterminer le champ **H**.

*See next page.*

$$\tilde{\mathbf{E}}(z) = 2\hat{\mathbf{a}}_x \exp(-j3z) - \hat{\mathbf{a}}_y \exp(j\pi/2)\exp(-j3z)$$

$$\tilde{\mathbf{H}}(z) = \eta^{-1}\,\hat{\mathbf{k}} \times \tilde{\mathbf{E}}(z), \quad \text{where} \quad \hat{\mathbf{k}} = \hat{\mathbf{a}}_z$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\sqrt{\frac{\mu_r}{\varepsilon_r}} = \frac{1}{n}\sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{\pi}{9}\sqrt{\frac{4\pi\times10^{-7}}{8.854\times10^{-12}}} = 131.5$$

$$\tilde{\mathbf{H}}(z) = 2(131.5)^{-1}\exp(-j3z)\,\hat{\mathbf{a}}_z \times \hat{\mathbf{a}}_x - (131.5)^{-1}\exp(j\pi/2)\exp(-j3z)\,\hat{\mathbf{a}}_z \times \hat{\mathbf{a}}_y$$

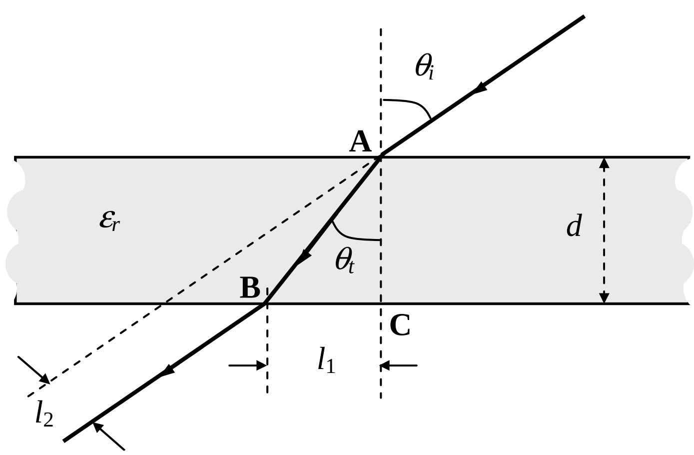
$$= 0.015\exp(-j3z)\,\hat{\mathbf{a}}_y + 0.0076\exp(j\pi/2)\exp(-j3z)\,\hat{\mathbf{a}}_x$$

$$\mathbf{H}(z,t) = 0.015\cos(\pi\times10^8 - 3z)\,\hat{\mathbf{a}}_y + 0.0076\sin(\pi\times10^8 - 3z)\,\hat{\mathbf{a}}_x$$

## Question 3

A plane wave is incident from air obliquely on a transparent sheet of thickness $d$ = 10mm with a relative permittivity of $\varepsilon_r$ = 6.25, at point **A** as shown below. The incidence angle is 30°.

Une onde plane est incidente obliquement de l'air vers une lame transparente d'épaisseur $d$ = 10mm et de permittivité relative $\varepsilon_r$ = 6.25, au point **A** comme montré ci-dessous. L'angle d'incidence est 30°.




(a) Find the transmitted angle $\theta_t$.

Déterminer l'angle transmis $\theta_t$.

$$n_1 \sin\theta_i = n_2 \sin\theta_t \quad \Rightarrow \quad \sin\theta_t = \left(\frac{n_1}{n_2}\right)\sin\theta_i = \left(\frac{1}{\sqrt{6.25}}\right)\sin(30^\circ) = \frac{0.5}{2.5} = 0.2$$

$$\therefore \quad \theta_i = \arcsin(0.2) = 11.5^\circ \text{ or } 0.20 \text{ rad}$$

(b) Find the distance $l_1$ at the point **B** of exit from the sheet.

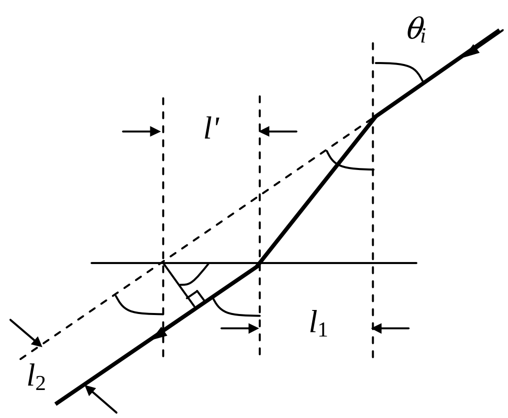
Déterminer la distance $l_1$ au point de sortie **B** de la lame.

$$\tan\theta_t = \frac{l_1}{d} \quad \Rightarrow \quad l_1 = d\tan\theta_t = 10\tan\left(11.5^\circ\right) = 10(0.203) = 2.03\,\text{mm}$$

(c) Find the lateral displacement $l_2$ of the emerging wave.

Déterminer le déplacement latéral $l_2$ de l'onde émergeante.

All noted angles are equal to $\theta_i$




$$\tan\theta_i = \frac{l_{tot}}{d} \quad \Rightarrow \quad l_{tot} = d\tan\theta_i = 10\tan\left(30^\circ\right) = 10(0.577) = 5.77\text{ mm}$$

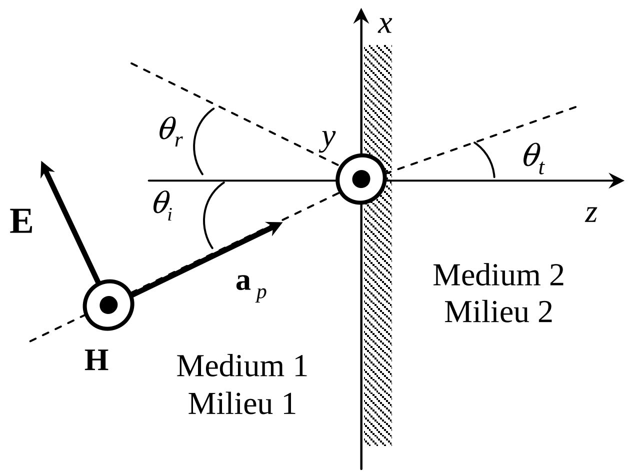
$$l' = l_{tot} - l_1 = 5.77 - 2.03 = 3.74\text{ mm}$$

$$\cos\theta_\text{i} = \frac{l_2}{l'} \quad \Rightarrow \quad l_2 = l'\cos\theta_\text{i} = 3.74\cos\left(30^\circ\right) = 3.24\text{ mm}$$

# Question 4

Consider a uniform plane wave obliquely incident from medium 1 onto medium 2, as shown. The constitutive parameters of medium 1 are $\varepsilon_r = 1.0$ and $\mu_r = 1.0$, while those of medium 2 are $\varepsilon_r = 2.1$ and $\mu_r = 1.0$. The electric field vector incident at the interface is given by **E**($x$, $z$) = 8.66 $\mathbf{a}_x$ - 5 $\mathbf{a}_z$

Soit une onde plane uniforme obliquement incidente d'un milieu 1 à un milieu 2 (voir figure). Les paramètres du milieu 1 sont $\varepsilon_r = 1.0$ et $\mu_r = 1.0$, tandis que pour le milieu 2, $\varepsilon_r = 2.1$ et $\mu_r = 1.0$. Le vecteur du champ électrique incident à l'interface est donnée par **E**($x$, $z$) = 8.66 $\mathbf{a}_x$ - 5 $\mathbf{a}_z$.




(a) Determine the Brewster angle, if it exists. If it does not exist, then explain why it does not exist.

Déterminer l'angle de Brewster, s'il existe. S'il n'existe pas, expliquer pourquoi il n'existe pas.

$$\Gamma_{//} = 0 \quad \Rightarrow \quad \eta_2 \cos\theta_t = \eta_1 \cos\theta_B \quad \Rightarrow \quad \eta_2^2\left\{1-\sin^2\theta_t\right\} = \eta_1^2\left\{1-\sin^2\theta_B\right\}$$

Since $k_1 \sin\theta_B = k_2 \sin\theta_t \quad \therefore \quad \sin^2\theta_t = \left(\frac{k_1}{k_2}\right)^2 \sin^2\theta_B$. Thus $\eta_2^2\left\{1-\left(\frac{k_1}{k_2}\right)^2 \sin^2\theta_B\right\} = \eta_1^2\left\{1-\sin^2\theta_B\right\}$

$$\left(\frac{\eta_2}{\eta_1}\right)^2 - \left(\frac{\eta_2}{\eta_1}\right)^2\left(\frac{k_1}{k_2}\right)^2 \sin^2\theta_B = 1-\sin^2\theta_B . \quad \text{Hence} \quad \sin^2\theta_B = \frac{1-\left(\frac{\eta_2}{\eta_1}\right)^2}{1-\left(\frac{\eta_2}{\eta_1}\right)^2\left(\frac{k_1}{k_2}\right)^2} = \frac{1-\left(\frac{\mu_2}{\varepsilon_1}\right)\left(\frac{\varepsilon_1}{\mu_2}\right)}{1-\left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2}$$

$$\textit{Nonmagnetic}: \quad \sin^2\theta_B = \frac{1-\left(\frac{\varepsilon_1}{\varepsilon_1}\right)}{1-\left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2} = \frac{1}{1+\left(\frac{\varepsilon_1}{\varepsilon_2}\right)} \quad \text{and} \quad \cos^2\theta_B = \frac{\left(\frac{\varepsilon_1}{\varepsilon_2}\right)}{1+\left(\frac{\varepsilon_1}{\varepsilon_2}\right)}$$

$$\text{Therefore} \quad \theta_B = \arctan\left(\sqrt{\varepsilon_2/\varepsilon_1}\right) = \arctan\left(\sqrt{2.1}\right) = 55.4^\circ \quad \text{or} \quad 0.97 \text{ rad}$$

(b) In what direction is the reflected **electric*** field at the interface?

Dans quelle direction est le champ magnétique réfléchi au niveau de l'interface?

$\hat{\mathbf{E}}_r(x,z) = 8.66\,\hat{\mathbf{a}}_x + 5\,\hat{\mathbf{a}}_z$ **Typo in original question*

(c) In what direction is the transmitted **magnetic*** field at the interface?

Dans quelle direction est le champ électrique transmis au niveau de l'interface ?

$\hat{\mathbf{H}}_t(x,z) = \hat{\mathbf{a}}_y$ **Typo in original question*

(d) What is the incident angle $\theta_i$ ?

Quel est l'angle d'incidence $\theta_i$ ?

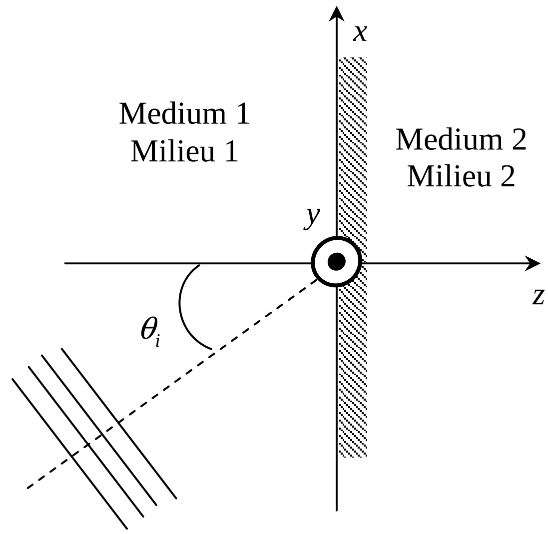
$$\hat{\mathbf{E}}_i(x,z) = \hat{\mathbf{a}}_x \cos\theta_i - \hat{\mathbf{a}}_z \sin\theta_i = \frac{8.66\,\hat{\mathbf{a}}_x - 5\,\hat{\mathbf{a}}_z}{\sqrt{(8.66)^2 + 5^2}} = 0.866\,\hat{\mathbf{a}}_x - 0.5\,\hat{\mathbf{a}}_z$$

$$\cos\theta_i = 0.866 \quad or \quad \sin\theta_i = 0.5 \quad \Rightarrow \quad \theta_i = 30^\circ$$

# Question 5

Consider a uniform plane wave obliquely incident from medium 1 onto medium 2, as shown. The refractive index of medium 1 is $n_1 = 1.0$, while that of medium 2 is $n_2 = 2.0$.

Soit une onde plane uniforme obliquement incidente d'un milieu 1 à un milieu 2 (voir figure). L'indice de réfraction du milieu 1 est $n_1 = 1.0$ tandis que celui du milieu 2 est $n_2 = 2.0$.




(a) What is the critical angle?

Quel est l'angle critique?

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

$$\sin\theta_c = \frac{n_2}{n_1}$$

Since $n_2 > n_1$, no critical angle will exist. This is because $\theta_t < \theta_i$, always.

(b) Explain what happens if the incident angle exceeds the critical angle.

Expliquer ce qui se passe si l'angle d'incidence est supérieur à l'angle critique.

If a situation DOES exist, such that $\theta_i > \theta_c$, then there exists a surface wave propagating along the boundary, with the field amplitudes evanescent into medium 2.

# Question 6

(a) Design an air-filled rectangular waveguide to operate at a fundamental mode of a cutoff frequency of 5 GHz. Assume $a = 2b$.

Concevoir un guide d'onde rectangulaire rempli d'air et opérant à un mode fondamental de fréquence de coupure 5 GHz. On prendra $a = 2b$.

$$f_c^{(m,n)} = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

The fundamental is $TE_{10}$: $f_c^{(10)} = \frac{c}{2a} \quad \Rightarrow \quad a = \frac{c}{2 f_c^{(10)}} = \frac{3\times10^8}{2\left(5\times10^9\right)} = 0.03\ \text{m} \quad \text{or} \quad 3.0\ \text{cm}$

$$\text{Thus:} \quad b = \tfrac{1}{2}a = 1.5\ \text{cm}$$

(b) The cutoff frequency of the $TM_{01}$ mode of the waveguide you designed in (a) is:

[1] 10 GHz.
[2] 5 GHz
**[3] This mode does not propagate in this guide.**
[4] It depends on the generator frequency.

La fréquence de coupure du mode $TM_{01}$ du guide d'onde conçu en (a), est :

[1] 10 GHz.
[2] 5 GHz
**[3] Ce mode ne se propage pas dans ce guide.**
[4] Cela dépend de la fréquence du générateur.

(c) Your answer in (b) is because:

[1] $a = 2b$
**[2] The electric and magnetic fields are zero for the $TM_{01}$ mode.**
[3] Only the electric field is zero but not the magnetic field.
[4] Only the magnetic field is zero but not the electric field.

La fréquence de coupure du mode $TM_{01}$ qui guide d'onde que vous avez conçu en (a), est :

[1] $a = 2b$.
**[2] Les champs électriques et magnétiques sont nuls pour le mode $TM_{01}$.**
[3] Seul le champ électrique est nul (pas le champ magnétique).
[4] Seul le champ magnétique est nul (pas le champ électrique).

(d) Calculate the cutoff frequencies of the first 5-modes of this guide

Calculer les fréquences de coupure des 5 premiers modes de ce guide.

$$f_c^{(m,n)} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$TE_{10}$ $$f_c^{(10)} = \frac{3\times10^{10}}{2}\sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{0}{1.5}\right)^2} = 5.0\times10^9 \text{ Hz} \quad \text{or} \quad 5.0\,\text{GHz}$$

$TE_{01}$ $$f_c^{(01)} = \frac{3\times10^{10}}{2}\sqrt{\left(\frac{0}{3}\right)^2 + \left(\frac{1}{1.5}\right)^2} = 10\times10^9 \text{ Hz} \quad \text{or} \quad 10\,\text{GHz}$$

$TE_{20}$ $$f_c^{(20)} = \frac{3\times10^{10}}{2}\sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{0}{1.5}\right)^2} = 10\times10^9 \text{ Hz} \quad \text{or} \quad 10\,\text{GHz}$$

$TE_{11}$ $$f_c^{(11)} = \frac{3\times10^{10}}{2}\sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{1.5}\right)^2} = 11.2\times10^9 \text{ Hz} \quad \text{or} \quad 11.2\,\text{GHz}$$

$TM_{11}$ $$f_c^{(11)} = \frac{3\times10^{10}}{2}\sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{1.5}\right)^2} = 11.2\times10^9 \text{ Hz} \quad \text{or} \quad 11.2\,\text{GHz}$$

# Equation sheet / Page de formules

$$\Gamma = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_o}{Z_L + Z_o} = |\Gamma|\, e^{j\theta_\Gamma} \qquad S = \frac{|V_{\max}|}{|V_{\min}|} = \frac{1+|\Gamma|}{1-|\Gamma|} \qquad Z(l) = \frac{V(l)}{I(l)} = Z_o \frac{Z_L + Z_o \tanh(\gamma l)}{Z_o + Z_L \tanh(\gamma l)}$$

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$$\nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla\times\mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \qquad \nabla\cdot\mathbf{D} = \rho_v \qquad \nabla\cdot\mathbf{B} = 0$$

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$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{k_1}{k_2} \qquad \eta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$k = \omega\sqrt{\mu\varepsilon} \qquad \mu_0 = 4\pi\times10^{-7}\ \text{H/m} \qquad \varepsilon_0 = 8.854\times10^{-12}\ \text{F/m}$$

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$$\Gamma_\perp = \Gamma_{TE} = \frac{E_o^r}{E_o^i} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} \qquad \tau_\perp = \tau_{TE} = \frac{E_o^{tr}}{E_o^i} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

$$\Gamma_\parallel = \Gamma_{TM} = \frac{E_o^r}{E_o^i} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} \qquad \tau_\parallel = \tau_{TM} = \frac{E_o^{tr}}{E_o^i} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

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$$f_c^{m,n} = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \qquad \frac{1}{\lambda^2} = \frac{1}{\lambda_g^2} + \frac{1}{\lambda_c^2}$$

$$\beta = k\sqrt{1-\left(\frac{f_c^{m,n}}{f}\right)^2} \qquad u_p = \frac{u}{\sqrt{1-\left(\frac{f_c^{m,n}}{f}\right)^2}}$$

$$Z_{TM} = \eta_{TEM}\sqrt{1-\left(\frac{f_c^{m,n}}{f}\right)^2} \qquad Z_{TE} = \frac{\eta_{TEM}}{\sqrt{1-\left(\frac{f_c^{m,n}}{f}\right)^2}}$$