Assignment 2, MAT1348[A], Winter 2021

Due March 12 at 00:00

The assignment is out of 20 marks, there are 9 questions, each will be marked out of 5. The highest 8 questions are summed up and divided by 2 to get a grade out of 20 eventually.

1.

Let $\mathbb{R}^+$ be the set of all positive real numbers. Let $f(x)$ and $g(x)$ be two functions defined on $x \in \mathbb{R}$. We say that $f(x) = O(g(x))$ if

$$\exists M \in \mathbb{R}^+,\ \exists x_0 \in \mathbb{R},\ \forall x \in \mathbb{R},\ (x > x_0) \Rightarrow (|f(x)| \leq M \cdot |g(x)|).$$

Prove that $31x^3 + 41x^2 + 59 = O(x^3)$.

2.

Let $\mathcal{P}(\mathcal{U})$ be the set of all subsets of $\mathcal{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. For each of the following statements indicate clearly whether the statement is true or false and then prove or disprove the statement.

(a) $\forall S, T \in \mathcal{P}(\mathcal{U}), \overline{S \cup T} = \overline{S} \cup \overline{T}$.

(b) $\exists S, T \in \mathcal{P}(\mathcal{U}), \overline{S \cup T} = \overline{S} \cup \overline{T}$

3.

Let $A = \{3k : k \in \mathbb{Z}\}$ and $B = \{15m + 12n : m, n \in \mathbb{Z}\}$. Prove that $A = B$.

4. Let A and B be two sets. Use set operation properties to prove the identity
$\overline{A \oplus B} = \overline{A \cup B} \cup (A \cap B)$.

5. Let $S = \{a, b, c, d\}$. Define relations from S to S, given as subsets of $S \times S$, satisfy the following conditions.

(a) R_1: It is reflexive and symmetric, but not transitive.

(b) R_2: It is reflexive and transitive, but not symmetric.

(c) R_3: It is symmetric and transitive, but not reflexive. This relation, as a subset of $S \times S$, should have at least six elements.

6. Find the number of equivalence relations from S to itself if $S = \{a, b, c, d\}$.

7. (a) Define a bijection from the set $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$ of non-negative integers to the set $\mathbb{N}$ of natural numbers. Prove that the function you defined is surjective and injective.

(b) Use the result in (a) to define a bijection from the set $S = \{x \mid x \in \mathbb{R}, x \geq 0\}$ to set $T = \{x \mid x \in \mathbb{R}, x > 0\}$. Prove that the function you defined is surjective and injective.

8. Prove that $\{14a + 29b : a, b \in \mathbb{Z}\} = \mathbb{Z}$.

9. Let A and B be sets. The *Cartesian product* of A and B is defined to be the set

$$A \times B = \{(a, b) : a \in A \wedge b \in B\}.$$

Prove that $(\mathbb{Z} \times \mathbb{N}) \cap (\mathbb{N} \times \mathbb{Z}) \subsetneq \mathbb{Z} \times \mathbb{Z}$.