Problem Set 1

1. Solve the system

$$\begin{aligned} x_1 - 5x_2 + 4x_3 &= -3 \\ 2x_1 - 7x_2 + 3x_3 &= -2 \\ -2x_1 + x_2 + 7x_3 &= -1 \end{aligned}$$

2. Solve the system

$$\begin{aligned} 2x_1 \qquad\quad - 6x_3 &= -8 \\ x_2 + 2x_3 &= 3 \\ 3x_1 + 6x_2 - 2x_3 &= -4 \end{aligned}$$

3. Determine if the system is consistent. Do not completely solve the system.

$$\begin{aligned} 2x_1 \qquad\qquad\qquad - 4x_4 &= -10 \\ 3x_2 + 3x_3 \qquad\quad &= 0 \\ x_3 + 4x_4 &= -1 \\ -3x_1 + 2x_2 + 3x_3 + x_4 &= 5 \end{aligned}$$

4. Determine the value(s) of h such that the matrix is the augmented matrix of a consistent linear system.

$$\begin{bmatrix} 1 & h & -5 \\ 2 & -8 & 6 \end{bmatrix}$$

5. Row reduce the matrix to reduced echelon form. Circle the pivot positions in the final matrix, and list the pivot columns.

$$\begin{bmatrix} 1 & 2 & 4 & 5 \\ 2 & 4 & 5 & 4 \\ 4 & 5 & 4 & 2 \end{bmatrix}$$

6. Find the general solutions of the system whose augmented matrix is

$$\begin{bmatrix} 1 & -3 & 0 & -5 \\ -3 & 7 & 0 & 9 \end{bmatrix}$$

7. Find the general solutions of the system whose augmented matrix is

$$\begin{bmatrix} 3 & -2 & 4 & 0 \\ 9 & -6 & 12 & 0 \\ 6 & -4 & 8 & 0 \end{bmatrix}$$

8. Suppose a system of linear equations has a 3×5 *augmented* matrix whose fifth column is not a pivot column. Is the system consistent? Why (or why not)?

9. Suppose a 3×5 *coefficient* matrix for a linear system has three pivot columns. Is the system consistent? Why or why not?

10. Determine if $\mathbf{b}$ is a linear combination of $\mathbf{a}_1$, $\mathbf{a}_2$ and $\mathbf{a}_3$.

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \quad \mathbf{a}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \quad \mathbf{a}_3 = \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}.$$

11. Determine if $\mathbf{b}$ is a linear combination of the vectors formed from the columns of the matrix A.

$$A = \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 \\ -7 \\ -3 \end{bmatrix}.$$

12. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix}$, and $\mathbf{y} = \begin{bmatrix} h \\ -3 \\ -5 \end{bmatrix}$. For what value(s) of h is $\mathbf{y}$ in the plane generated by $\mathbf{v}_1$ and $\mathbf{v}_2$?

13. Mark each statement True of False. Justify each answer.

(a) When $\mathbf{u}$ and $\mathbf{v}$ are nonzero vectors, Span$\{\mathbf{u}, \mathbf{v}\}$ contains only the line through $\mathbf{u}$ and the origin, and the line through $\mathbf{v}$ and the origin.

(b) Any list of five real numbers is a vector in $\mathbb{R}^5$.

(c) Asking whether the linear system corresponding to an augmented matrix $[\mathbf{a}_1 \quad \mathbf{a}_2 \quad \mathbf{a}_3 \quad \mathbf{b}]$ has a solution amounts to asking whether $\mathbf{b}$ is in Span$\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$.

(d) The vector $\mathbf{v}$ results when a vector $\mathbf{u} - \mathbf{v}$ is added to the vector $\mathbf{v}$.

(e) The weights $c_1, \ldots, c_p$ in a linear combination $c_1\mathbf{v} + \cdots c_p\mathbf{v}$ cannot all be zero.