STA201 - Why Numbers Matter
Midterm Winter 2018
March 7, 2018
110 Minutes

Name (Print): Solutions Student Number: _____

This test contains 7 pages (including this cover page) and 7 problems. Check to see if any pages are missing. Enter all requested information on the top of this page.

This is a closed book test. You are allowed a **non-programmable** calculator during this test

- ☐ **Show all your work**. Answers, correct or not, unsupported by calculations, thought process, or explanation will not earn any marks.
- ☐ **Organize your work** in a reasonably neat and coherent way, in the space provided. Work scattered all over the page that cannot be understood will not earn full marks.
- ☐ **Round final answers** to two decimal places.
- ☐ **Multiple Choice:** Always select the best (most correct) option
- ☐ You may use the back pages to do rough work but **ONLY work written in the provided space will be graded**.
- Breathe, and do your best! If you're stuck, move on to a problem that's more manageable. Allocate your time accordingly to the weight of the problem!

Do not write in the table to the right.

Problem	Points	Score
1	3	
2	5	
3	14	
4	10	
5	12	
6	8	
7	6	
Total:	58	

1. (3 points) Determine whether the following statement is a logical fallacy. If it is, briefly describe and explain the faulty reasoning in the statement. Maximum 3 (proper) sentences.

During the last American election, women either supported Hillary Clinton for president, or they did not believe in women's rights. ✓ yes, logical fallacy ✓✓ describe/explain why

- Incomplete list of consequences
- False dilemma
- There are those who belong to neither group for valid reasons

2. (5 points) State and formulate the truth table for the **contrapositive** of the following statement:

If minority youths benefit from positive representation in media, then movies like Black Panther are a step in the right direction in providing strong role models of colour.

p → q

(NOT q) → (NOT p)

Contrapositive Statement:

If movies like Black Panther are not a step in the right direction in providing strong role models of colour, then minority youths do NOT benefit from positive representation in media.

*negation ✓ * direction ✓

Truth Table:

p	q	NOT q ✓	NOT p ✓	(NOT q) → (NOT p) ✓
T	T	F	F	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

3. (14 points) A $650,000 ten year fixed rate mortgage was taken out at 6.24% APR to be amortized in 30 years.

(a) (3 points) Fill in the table below:

r	$\frac{0.0624}{12} = 0.0052$ ✓
t	$30 \times 12 = 360$ ✓
Term of mortgage	10 yrs or 120 months ✓

Values to be used in (b)

(b) (1 point) Using the values in (a), find the monthly payment amount.

$$\text{monthly amount} = \text{loan} \times \frac{r(1+r)^t}{(1+r)^t - 1}$$

$$= \frac{650{,}000\,(0.0052)\,(1.0052)^{360}}{(1.0052)^{360} - 1}$$

$$= \$3997.94$$ ✓

If students used correct "r", "t" values here, they should get 2/3 in (a) if they filled in differently.

(c) (3 points) Find the total amount paid out over the 30 years assuming APR does not change after the first 10 years. How much interest did you pay on this mortgage?

After 30 years: $30 \times 12 = 360$ payments ✓

Amount paid = (6) $\times 360$ or $3997.94 \times 360 = 1{,}439{,}258.40$ ✓

Loan amount = 650,000

Interest paid = $1439258.4 - 650000$

$= \$789{,}258.4$ ✓

Paid an extra $789,258.4 in interest.

(d) (1 point) **MULTIPLE CHOICE - FILL IN #1 ON BUBBLE SHEET** Instead of making the payment in (b) once a month, you split this amount into two monthly payments: you pay half in the middle of the month, then pay the rest at the end of the month. This would lead to....

(a) More interest being paid, you're paying more frequently

(b) A shorter loan, you'll pay off the loan sooner

(c) No difference, you're paying the same amount each month

(d) A longer loan, you're making smaller payments each time

(e) (6 points) For your *first two monthly payments*, find the amount of each payment that is paid towards the interest, towards the principal, and your equity after the first two payments.

$r = 0.0052$ Original Loan: 650000

Payment Amount	Interest Paid	Principal Repaid	Outstanding Balance (OB)
3997.94	$650000 \times 0.0052 = 3380$ ✓	$3997.94 - 3380 = 617.94$ ✓	$650000 - 617.94 = 649{,}382.06$ ✓
3997.94	$649382.06 \times 0.0052 = 3376.79$ ✓	$3997.94 - 3376.79 = 621.15$ ✓	$649{,}382.06 - 621.15 = 648{,}760.91$ ✓

(last outstanding balance) $\times r$ ↑ Payment − interest paid ↑ last OB − principal paid

Equity = $617.94 + 621.15 = 1239.09$ ✓

4. (10 points) A common bacteria on a kitchen countertop was initially a population of 115. After 12 hours without change to the countertop, it now numbered at 977.

(a) (2 points) What is the percent growth and average in bacteria during this 12 hour period?

% growth: $\frac{977 - 115}{115} \times 100\%$

$= 749.57\%$ ✓

grew by 749.57% over 12 hours.

hourly growth

Avg. hourly growth: $\frac{977 - 115}{12}$

$= 71.83$ ✓

grew by 71.83 per hour on average

(b) (2 points) Fill in the blanks using the options below:

Using ___% growth___ ✓ *will give a larger bacteria count at the end of one week than* ___Avg. growth___ ✓.

PERCENT GROWTH **AVERAGE GROWTH**

(c) (6 points) Find the two populations of the bacteria after two days if the percent growth is maintained and if the average growth is maintained.

% growth

- 749.57% per 12 hours
- 2 days = 4 12hr periods ✓

Amount = (Initial) $\times (1 + 7.4957)^4$ ✓

$= \text{Initial} \times 8.4957^4$

If initial = 115 → 599,093.37 ✓

If initial = 977 → 5,089,688.89

Avg. Growth

- 71.83 per hour
- 2 days = 48 hours ✓

Amount = Initial + 48×71.83 ✓

If initial = 115 → $115 + 48 \times 71.83 = 3562.84$ ✓

If initial = 977 → $977 + 48 \times 71.83 = 4424.84$

(d) (BONUS - 2 points) After how many hours would the initial population of bacteria quadruple? *Population growth is an application of exponential growth!

$4 \times 115 = 460$

$460 = 115(8.4957)^t$ ✓ → $t = 0.65$ ✓ t = # of 12 hour periods

$4 = 8.4957^t$

$\frac{\log_{10} 4}{\log_{10} 8.4957} = t$

∴ After $0.65 \times 12 = 7.78$ hours will initial population quadruple

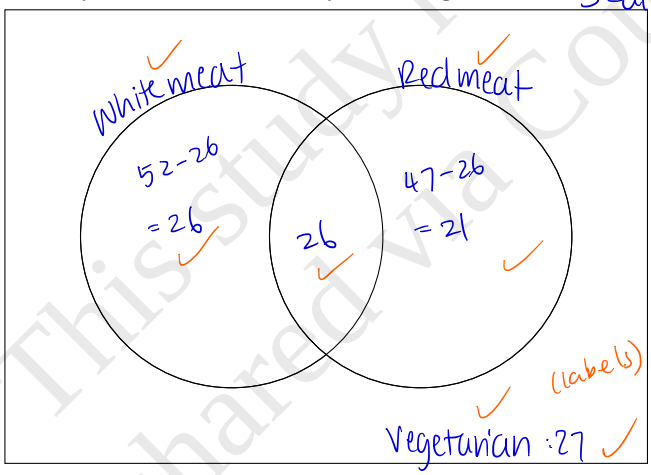
5. (12 points) In a survey of people's eating habits, 100 people were surveyed and the following information was collected:

- 73 of the people responded that they are meat eaters
- Among the meat eaters, 52 reported that they eat white meat (chicken, fish, turkey, etc.)
- Among the meat eaters, 47 reported that they eat red meat (beef, pork, lamb, etc.)

(a) (1 point) How many of the people surveyed are vegetarian (do not eat meat)?

$n(\text{Vegetarian}) = 100 - 73 = 27$ ✓

(b) (9 points) Label and correctly fill in every area in the Venn diagram below. Show any work you did to find any missing values. S = all surveyed



- 73 total meat eaters
- 52+47 = 99 that identify as white or red meat eaters
- Those that eat both will be counted TWICE (or overcounted ONCE)
- 99 − 73 = 26 over counts
- n(white AND red meat) = 26 ✓

(c) (2 points) How many of those surveyed either eat ONLY white meat or eat ONLY red meat?

$n(\text{white meat only or red meat only}) = 26 + 21 = 47$ ✓✓

1 mark if they only did total meat eaters)

or

$n(\text{all meat eaters}) - n(\text{both}) = 73 - 26 = 47$

6. (8 points) The interest earned on unpaid purchases on a HBC MasterCard is calculated using simple interest for each day following the transaction during the billing cycle. The credit card has an annual percentage rate of 29.9% with a billing cycle that ends on the 15th of every month. Interest is calculated using the average daily balance method. You go out for Winterlicious on February 1, 2018 with some friends and pay for your bill of $41.58 (tax and tip included) using your MasterCard. On February 14th, you go out for dinner with a date and charged $86.76 on your HBC MasterCard. Assuming no additional charges to your card...

   (a) (2 points) Your minimum payment due on March 8, 2018 is the largest of $10 or 2% of your balance. Find your minimum payment on March 8. ↳ payment due @ end of grace period

   Jan. 16 – Jan. 31 (16 days) : $0
   Feb. 1 – Feb. 13 (13 days) : $41.58
   Feb. 14 – Feb. 15 (2 days) : $41.58 + 86.76 = $128.34

   minimum payment = max(10, 0.02 × 128.34) ✓
   = max(10, 2.57) = 10 ✓

   (b) (6 points) Find your balance on March 15 if you only made the minimum payment on March 8. Assume the credit card uses average daily balance method to calculate how interest is accrued.

   Hint: Interest is added once at the end of the billing cycle. You'll need to first find your actual balance on February ~~28~~ 15. This is your balance going into March and the balance you'll need to use in finding your average daily balance.

   * Avg daily balance ✓
   * Interest charges ✓
   * Final balances ✓

   For Jan-Feb. Billing cycle:

   End balance = $128.34 + Interest

   Interest = (Avg daily balance) × $\frac{0.299}{365}$ × (16+13+2)

   = $\left(\frac{0\times16 + 13\times41.58 + 2\times128.34}{31}\right) \times \frac{0.299}{365} \times 31 = 25.72 \times \frac{0.299}{365} \times 31 = 0.65$ ✓

   End balance = $128.34 + $0.65 = $128.99 ✓

   Feb. – Mar. Cycle:

   Feb. 16 – Mar. 7 (20 days): $128.99
   Mar. 8 – Mar. 15 (8 days): $128.99 − 10
   = $118.99 ← Balance on card w/o interest

   End balance = (balance) $118.99 + Interest = 118.99 + 2.89 = 121.88

   Interest = $\left(\frac{20\times128.99 + 8\times118.99}{28}\right) \times \frac{0.299}{365} \times 28$ ✓

   = 126.13 × $\frac{0.299}{365}$ × 28 = 2.89 ✓

7. (6 points) **MULTIPLE CHOICE** For each of the questions below, select the most correct answer and **fill in #2-7 on the attached bubble sheet.**

   (a) (1 point) Which of the following will lead to a shorter loan term (assuming all other quantities remain unchanged):

   (a) Increasing the interest rate
   (b) Increasing the principal
   (c) Increasing the compound frequency
   (d) Increasing payment frequency

Relative intensity $= 2 \times 10^{0.1(75)}$
$= 63{,}245{,}553.2$

(b) (1 point) If two 75 decibel speakers are placed side by side, then... Decibel $= 10 \log_{10}(63245553.2)$
$= 78$ dB

(a) The resulting sound output will be a marginal (small) increase in decibels
(b) The relative sound intensity will double
(c) The resulting sound output will more than double in decibels
(d) The resulting sound output will be 150 dB

(c) (1 point) Which of the following will decrease the amount of interest paid on a loan?

(a) Increasing APR
(b) Shorter compounding periods
(c) Decreasing principal
(d) Decreasing payment frequency ← making annual payments instead of monthly - principal decreases slowly → more interest

(d) (1 point) In implication statements (IF-THEN), which of the following pairs of statements is logically equivalent?

(a) Converse and Inverse
(b) Converse and Contrapositive
(c) Implication and Converse
(d) Inverse and Contrapositive

p	q	NOT p	NOT q	$q \to p$ (Converse)	NOT p → NOT q (Inverse)	NOT q → NOT p (Contrapositive)
T	T	F	F	T	T	T
T	F	F	T	T	T	F
F	T	T	F	F	F	T
F	F	T	T	T	T	T

(e) (1 point) If $3 + 2 = 5$, then $9 + 9 = 21$ (p=T, q=F)

(a) TRUE
(b) FALSE

(f) (1 point) If $3 + 2 = 6$, then $7 + 9 = 16$ (p=F, q=T)

(a) TRUE
(b) FALSE

If you need a break (for fun): Each row, column, and group of outlined 9 spaces contain the digits 1-9 exactly once. Following these rules, fill in the empty boxes

2	5	3	8	9	4	1	7	6
1	9	6	2	3	7	5	8	4
8	7	4	1	5	6	9	2	3
3	8	1	9	4	2	6	5	7
9	6	7	3	8	5	4	1	2
4	2	5	7	6	1	8	3	9
6	1	9	5	2	3	7	4	8
7	3	8	4	1	9	2	6	5
5	4	2	6	7	8	3	9	1

7	3	4	9	5	6	2	8	1
1	8	9	4	3	2	7	5	6
2	5	6	7	8	1	3	9	4
4	6	3	2	7	9	8	1	5
5	9	7	1	6	8	4	3	2
8	1	2	3	4	5	6	7	9
3	7	1	5	2	4	9	6	8
6	2	5	8	9	7	1	4	3
9	4	8	6	1	3	5	2	7