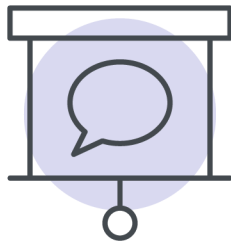


OneClass

CP164

**FINAL EXAM
STUDY GUIDE**

Fall 2018



Lecture Notes

```

"""
-----
food.py
Food class definition.
-----
Author:  David Brown
ID:      999999999
Email:   dbrown@wlu.ca
__updated__ = "2018-03-12"
-----
"""

class Food:
    """
    Defines an object for a single food: name, origin, vegetarian, calories.
    """
    # Constants
    ORIGIN = ("Canadian", "Chinese", "Indian", "Ethiopian",
              "Mexican", "Greek", "Japanese", "Italian", "American",
              "Scottish", "New Zealand", "English")

    @staticmethod
    def origins():
        """
        -----
        Creates a string list of food origins in the format:
            0 Canadian
            1 Chinese
            2 Indian
            ...
        Use: s = Food.origins()
        Use: print(Food.origins())
        -----
        Postconditions:
            returns
            string - A numbered list of valid food origins.
        -----
        """
        string = ""

        for i in range(len(Food.ORIGIN)):
            string += """:{2d} {}
        """.format(i, Food.ORIGIN[i])
        return string

    def __init__(self, name, origin, is_vegetarian, calories):
        """
        -----
        Initialize a food object.
        Use: f = Food( name, origin, is_vegetarian, calories )
        -----
        Preconditions:
            name - food name (str)
            origin - food origin (int)
            is_vegetarian - whether food is vegetarian (boolean)
            calories - caloric content of food (int > 0)
        Postconditions:
            food values are set.
        -----
        """
        assert origin in range(len(Food.ORIGIN)), "Invalid origin ID"
        assert is_vegetarian in (True, False, None), "Must be True or False"
        assert calories is None or calories >= 0, "Calories must be >= 0"

```

```

        self.name = name
        self.origin = origin
        self.is_vegetarian = is_vegetarian
        self.calories = calories
        return

def __str__(self):
    """
    -----
    Creates a formatted string of food data.
    Use: print(f)
    Use: s = str(f)
    -----
    Postconditions:
        returns:
            string - the formatted contents of food (str)
    -----
    """
    if self.calories is None:
        # is a key
        string = "{}, {}".format(self.name, Food.ORIGIN[self.origin])
    else:
        # full data set
        string = """Name:         {}
Origin:        {}
Vegetarian:    {}
Calories:      {:,d}""".format(self.name, Food.ORIGIN[self.origin],
self.is_vegetarian, self.calories)
    return string

def __eq__(self, rs):
    """
    -----
    Compares this food against another food for equality.
    Use: f == rs
    -----
    Preconditions:
        rs - [right side] food to compare to (Food)
    Postconditions:
        returns:
            result - True if name and origin match, False otherwise (boolean)
    -----
    """
    result = (self.name.lower(), self.origin) == (
        rs.name.lower(), rs.origin)
    return result

def __lt__(self, rs):
    """
    -----
    Determines if this food comes before another.
    Use: f < rs
    -----
    Preconditions:
        rs - [right side] food to compare to (Food)
    Postconditions:
        returns:
            result - True if food precedes rs, False otherwise (boolean)
    -----
    """
    result = (self.name.lower(), self.origin) < \
        (rs.name.lower(), rs.origin)
    return result

```

```

def __le__(self, rs):
    """
    -----
    Determines if this food precedes or is or equal to another.
    Use: f <= rs
    -----
    Preconditions:
        rs - [right side] food to compare to (Food)
    Postconditions:
        returns:
            result - True if this food precedes or is equal to rs,
                    False otherwise (boolean)
    -----
    """
    result = self < rs or self == rs
    return result

def write(self, file_variable):
    """
    -----
    Writes a single line of food data to an open file.
    Use: f.write( file_variable )
    -----
    Preconditions:
        file_variable - an open file of food data (file)
    Postconditions:
        The contents of food are written as a string in the format
        name|origin|is_vegetarian to file_variable.
    -----
    """
    print("{}|{}|{}|{}"
          .format(self.name, self.origin, self.is_vegetarian,
self.calories),
          file=file_variable)
    return

def key(self):
    """
    -----
    Creates a formatted string of food key data.
    Use: key = f.key()
    -----
    Postconditions:
        returns:
            the formatted contents of food key (str)
    -----
    """
    return "{}, {}".format(self.name, self.origin)

def __hash__(self):
    """
    -----
    Generates a hash value from a food name.
    Use: h = hash(f)
    -----
    Postconditions:
        returns
            value - the total of the characters in the name string (int > 0)
    -----
    """
    value = 0

    for c in self.name:

```

```
        value = value + ord(c)
    return value
```

```
"""
-----
food_utilities.py
Utilities for working with a Food object.
-----
Author:  David Brown
ID:      999999999
Email:   dbrown@wlu.ca
__updated__ = "2018-01-21"
-----
"""

from food import Food

def get_food():
    """
    -----
    Creates a food object by requesting data from a user.
    Use: f = get_food()
    -----
    Postconditions:
        returns
        food - a completed food object (Food).
    -----
    """
    name = input("Name: ")
    print("Origin")
    print(Food.origins())
    origin = int(input(": "))
    s = input("Vegetarian (Y/N): ")
    # Accept a range of values for vegetarian
    is_vegetarian = s.upper() in ('Y', 'TRUE', '1', 'T', 'YES')
    calories = int(input("Calories: "))
    food = Food(name, origin, is_vegetarian, calories)
    return food

def read_food(line):
    """
    -----
    Creates and returns a food object from a line of string data.
    Use: f = read_food(line)
    -----
    Preconditions:
        line - a vertical bar-delimited line of food data in the format
              name|origin|is_vegetarian|calories (str)
    Postconditions:
        returns
        food - contains the data from line (Food)
    -----
    """
    data = line.strip().split("|")
    food = Food(data[0], int(data[1]), data[2] == "True", int(data[3]))
    return food

def read_foods(file_variable):
    """
    -----
    Reads a file of food strings into a list of Food objects.
    Use: foods = read_foods(file_variable)
    -----
    Preconditions:
        file_variable - a file of food data (file)
    """
```

```
Postconditions:
    returns
    foods - a list of food objects (list of food)
-----
"""
file_variable.seek(0)
foods = []

for line in file_variable:
    food = read_food(line)
    foods.append(food)
return foods

def write_foods(file_variable, foods):
    """
    -----
    Writes a list of food objects to a file.
    Use: write_foods(file_variable, foods)
    -----
    Preconditions:
        file_variable - an open file of food data (file)
        foods - a list of Food objects (list of Food)
    Postconditions:
        file_variable contains the objects in foods as strings in the format
        name|origin|is_vegetarian|calories
    -----
    """
    for food in foods:
        food.write(file_variable)
    return

def get_vegetarian(foods):
    """
    -----
    Creates a list of vegetarian foods.
    Use: v = get_vegetarian(foods)
    -----
    Preconditions:
        foods - a list of Food objects (list of Food)
    Postconditions:
        returns
        veggies - Food objects from foods that are vegetarian (list of Food)
    -----
    """
    veggies = []

    for food in foods:
        if food.is_vegetarian:
            veggies.append(food)
    return veggies

def by_origin(foods, origin):
    """
    -----
    Creates a list of foods by origin.
    Use: v = by_origin(foods, origin)
    -----
    Preconditions:
        foods - a list of Food objects (list of Food)
        origin - a food origin (int)
    Postconditions:
```

```

        returns
        origins - Food objects from foods that are of a particular origin (list
of Food)
-----
"""
assert origin in range(len(Food.ORIGIN))

origins = []

for food in foods:
    if food.origin == origin:
        origins.append(food)
return origins

def average_calories(foods):
    """
    -----
    Determines the average calories in a list of foods.
    Use: avg = average_calories(foods)
    -----
    Preconditions:
        foods - a list of Food objects (list of Food)
    Postconditions:
        returns
        avg - average calories in all Food objects of foods (int)
    -----
    """
    total = 0
    count = len(foods)

    for food in foods:
        total += food.calories

    if count > 0:
        avg = total // count
    else:
        avg = 0
    return avg

def calories_by_origin(foods, origin):
    """
    -----
    Determines the average calories in a list of foods.
    Use: a = calories_by_origin(foods, origin)
    -----
    Preconditions:
        foods - a list of Food objects (list of Food)
        origin - the origin of the Food objects to find (int)
    Postconditions:
        returns
        avg - average calories for all Foods of the requested origin (int)
    -----
    """
    assert origin in range(len(Food.ORIGIN))

    total = 0
    count = 0

    for food in foods:
        if food.origin == origin:
            total += food.calories
            count += 1

```

```

    if count > 0:
        avg = total // count
    else:
        avg = 0
    return avg

def food_table(foods):
    """
    -----
    Prints a formatted table of foods, sorted by name.
    Use: food_table(foods)
    -----
    Preconditions:
        foods - a list of Food objects (list of Food)
    Postconditions:
        prints
        a table of the foods sorted by name
    -----
    """
    foods.sort()
    print("{:35s} {:12s} {:10s} {:8s}".format(
        "Food", "Origin", "Vegetarian", "Calories"))
    print("{} {} {} {}".format("-" * 35, "-" * 12, "-" * 10, "-" * 8))

    for food in foods:
        print("{:35s} {:12s} {:>10} {:8,d}".format(
            food.name, Food.ORIGIN[food.origin], str(food.is_vegetarian),
            food.calories))
    return

def food_search(foods, origin, max_cals, is_veg):
    """
    -----
    Searches for foods that fit certain conditions.
    Use: results = food_search(foods, origin, max_cals, is_veg)
    -----
    Preconditions:
        foods - a list of Food objects (list of Food)
        origin - the origin of the food; if -1, accept any origin (int)
        max_cals - the maximum calories for the food; if 0, accept any calories
value (int)
        is_veg - whether the food is vegetarian or not; if False accept any food
(boolean)
    Postconditions:
        returns
        result - a list of foods that fit the conditions (list of Food)
        foods parameter must be unchanged
    -----
    """
    assert origin in range(-1, len(Food.ORIGIN))

    result = []

    for food in foods:
        if (origin == -1 or food.origin == origin) and (max_cals == 0 or
            food.calories <= max_cals) and (not is_veg or food.is_vegetarian):
            result.append(food)
    return result

def food_test():

```

```
fv = open('foods.txt', 'r', encoding='utf-8')
foods = read_foods(fv)
fv.close()

print("Length: {}".format(len(foods)))
r = food_search(foods, -1, 300, True)
print("Length: {}".format(len(r)))

for f in r:
    print(f)
return
```

```

"""
-----
sorts_linked.py
Linked versions of various sorts. Implemented on linked Deques.
-----
Author:  David Brown
ID:     999999999
Email:  dbrown@wlu.ca
__updated__ = "2017-08-25"
-----
"""

from math import log
from list_linked import List

class Sorts:
    """
    -----
    Defines a number of linked sort operations.
    Uses class attribute 'swaps' to determine how many times
    elements are swapped by the class.
    Use: print(Sorts.swaps)
    Use: Sorts.swaps = 0
    -----
    """
    swaps = 0 # Tracks swaps performed.

    # The Sorts

    @staticmethod
    def selection_sort(a):
        """
        -----
        Sorts a linked list using the Selection Sort algorithm.
        Finds maximum value each pass.
        Use: selection_sort(a)
        -----
        Preconditions:
            a - a linked list of comparable elements (List)
        Postconditions:
            Contents of a are sorted.
        -----
        """
        # Split the list into the sorted (_front) and unsorted parts.
        unsorted = a._front
        a._front = None
        # Go through each node in the unsorted list and find the max value
        # to insert at the front of the sorted list.
        while unsorted is not None:
            max_prev = None
            max_node = unsorted
            previous = unsorted
            current = max_node._next

            while current is not None:
                if current._data > max_node._data:
                    max_prev = previous
                    max_node = current
                    previous = current
                current = current._next
            # Remove the max node from the list.
            Sorts.swaps += 1

            if max_prev is None:

```

```

        unsorted = max_node._next
    else:
        max_prev._next = max_node._next
    # Move the next max node to the front of the sorted list.
    max_node._next = a._front
    a._front = max_node
    return

@staticmethod
def bubble_sort(a):
    """
    -----
    Sorts a linked list using the Bubble Sort algorithm.
    Use: bubble_sort(a)
    -----
    Preconditions:
        a - a linked list of comparable elements (?)
    Postconditions:
        Contents of a are sorted.
    -----
    """
    done = False
    last = None

    while not done:
        # if no elements have been swapped, then the list is sorted
        done = True
        # Get the front of the list.
        previous = None
        current = a._front
        swapped = a._front

        while current is not last and current._next is not None:

            if current._data > current._next._data:
                # If you swapped you need another pass.
                done = False
                # The pair current, current._next is out of order.
                Sorts.swaps += 1
                a._swap(previous, current)
                # Keep track of last node swapped
                swapped = current
                # current is unchanged - update previous
                if previous is None:
                    previous = a._front
                else:
                    previous = previous._next
            else:
                # Move to next node.
                previous = current
                current = current._next

        last = swapped
    # done == True iff no pair of keys was swapped on the last pass.
    return

@staticmethod
def comb_sort(a):
    """
    -----
    Sorts an List using the Comb Sort algorithm.
    Use: comb_sort(a)
    -----
    Preconditions:
        a - a linked list of comparable elements (?)
    """

```

```

Postconditions:
    Contents of a are sorted.
-----
"""
n = len(a)

if n > 0:
    gap = n
    done = False

    while gap > 1 or not done:
        done = True
        previous = None
        current = a._front
        gap = int(gap / 1.3)

        if gap < 1:
            gap = 1

        i = 0
        prev_far = current
        far = current._next
        # Move to the far node for comparison.
        while i < gap - 1 and far is not None:
            prev_far = far
            far = far._next
            i += 1

        while current is not None and far is not None:
            if current._data > far._data:
                Sorts.swaps += 1
                a._swap(previous, prev_far)
                done = False
            # Increment all nodes.
            prev_far = far
            far = far._next
            previous = current
            current = current._next

    return

@staticmethod
def insertion_sort(a):
    """
    -----
    Sorts a linked list using the Insertion Sort algorithm.
    Use: insertion_sort(a)
    -----
    Preconditions:
        a - a linked list of comparable elements (?)
    Postconditions:
        Contents of a are sorted.
    -----
    """
    # Split the list into the sorted (_front) and unsorted parts.
    unsorted = a._front
    a._front = None

    # Go through each node in the unsorted list and insert it into the
    # proper position in the sorted list.
    while unsorted is not None:
        # Isolate the first node of the unsorted list.
        node = unsorted
        unsorted = unsorted._next
        # Find the proper place for the new node in the sorted so far list.

```

```

# (Very similar to priorityQueue insertion.)
previous = None
current = a._front

while current is not None and current._data < node._data:
    previous = current
    current = current._next

# Insert node into the proper place in the sorted list.
Sorts.swaps += 1

if previous is None:
    node._next = a._front
    a._front = node
else:
    node._next = current
    previous._next = node
return

@staticmethod
def merge_sort(a):
    """
    -----
    Sorts a linked list using the Merge Sort algorithm.
    Use: merge_sort(a)
    -----
    Preconditions:
        a - a linked list of comparable elements (List)
    Postconditions:
        Contents of a are sorted.
    -----
    """
    a._front = Sorts._merge_sort_aux(a._front)
    return

@staticmethod
def _merge_sort_aux(current):
    """
    -----
    Divides a linked list into halves, sorts each half, then
    merges the halves back together.
    Use: current = _merge_sort_aux(current)
    -----
    Preconditions:
        current - pointer to a List node (_ListNode)
    Postconditions:
        returns:
        current - pointer to sorted List segment (_ListNode)
    -----
    """
    # Split the list only if there are at least two elements.
    if current is not None and current._next is not None:
        # Generate the left and right lists.
        left, right = Sorts._merge_split(current)
        # Put the left list in order.
        left = Sorts._merge_sort_aux(left)
        # Put the right list in order.
        right = Sorts._merge_sort_aux(right)
        # Merge the left and right lists.
        current = Sorts._merge(left, right)
    return current

@staticmethod
def _merge_split(current):

```

```

"""
-----
Split the list by putting alternating nodes into left and right lists.
Use: left, right = _merge_split(current)
-----
Preconditions:
    current - pointer to a List node (_ListNode)
Postconditions:
    returns:
    left - pointer to left List segment (_ListNode)
    right - pointer to right list segment (_ListNode)
-----
"""
#
left = None
right = None
toggle = 'l'

while current is not None:
    next_node = current._next

    if toggle == 'l':
        current._next = left
        left = current
        toggle = 'r'
    else:
        current._next = right
        right = current
        toggle = 'l'

    current = next_node
return left, right

@staticmethod
def _merge(left, right):
    """
    -----
    Merges two parts of a linked list together.
    Use: new = _merge(left, right)
    -----
    Preconditions:
        left - pointer to a List node (_ListNode)
        right - pointer to a List node (_ListNode)
    Postconditions:
        returns:
        new - pointer to sorted merge of left and right (_ListNode)
    -----
    """
    # Initialize the new list.
    if left._data <= right._data:
        new = left
        left = left._next
    else:
        new = right
        right = right._next

    # Create a pointer to traverse the new list.
    current = new

    # Traverse both lists appending larger value to the end of the list.
    while left is not None and right is not None:

        if left._data <= right._data:
            current._next = left

```

```

        current = current._next
        left = left._next
    else:
        current._next = right
        current = current._next
        right = right._next

# Append the remaining list.
if left is not None:
    current._next = left
elif right is not None:
    current._next = right
return new

@staticmethod
def quick_sort(a):
    """
    -----
    Sorts a linked list using the Quick Sort algorithm.
    Use: quick_sort(a)
    -----
    Preconditions:
        a - a linked list of comparable elements (?)
    Postconditions:
        Contents of a are sorted.
    -----
    """
    a._front = Sorts._quick_sort_aux(a._front)
    return

@staticmethod
def _quick_sort_aux(current):
    """
    -----
    Divides a linked list into halves, sorts each half, then
    concatenates the halves back together.
    Use: current = _quick_sort_aux(current)
    -----
    Preconditions:
        current - pointer to a List node (_ListNode)
    Postconditions:
        returns:
            current - pointer to a sorted List (_ListNode)
    -----
    """
    # to sort the sublist a[p:q] of linked list a into ascending order
    if current is not None:
        # partitions a[first:last] into a[first:pivot] and a[pivot+1:last]
        lesser, equals, greater = Sorts._partition(current)
        lesser = Sorts._quick_sort_aux(lesser)
        greater = Sorts._quick_sort_aux(greater)
        current = Sorts._append(lesser, equals)
        current = Sorts._append(current, greater)
    return current

@staticmethod
def _partition(current):
    """
    -----
    Divides a linked list into three parts.
    Use: l, e, g = _partition(current)
    -----
    Preconditions:
        current - pointer to a List node containing pivot value (_ListNode)
    """

```

```

Postconditions:
    returns:
        lesser - a list of values less than the pivot value (_ListNode)
        greater - a list of values greater than the pivot value (_ListNode)
        equals - a list of values equal to the pivot value (_ListNode)
-----
"""
pivotValue = current._data
lesser = None
greater = None
equals = None

while current is not None:
    next_node = current._next

    if pivotValue > current._data:
        current._next = lesser
        lesser = current
    elif pivotValue < current._data:
        current._next = greater
        greater = current
    else:
        current._next = equals
        equals = current

    current = next_node
return lesser, equals, greater

@staticmethod
def _append(head, tail):
    """
    -----
    Combines two lists together in order.
    Use: current = _append(head, tail)
    -----
    Preconditions:
        head - pointer to a List node of the first list (_ListNode)
        tail - pointer to a List node of the second list (_ListNode)
    Postconditions:
        returns:
            head - pointer to the combined nodes in order (_ListNode)
    -----
    """
    current = head
    previous = None

    while current is not None:
        previous = current
        current = current._next

    if previous is None:
        head = tail
    else:
        previous._next = tail
    return head

@staticmethod
def radix_sort(a):
    """
    -----
    Performs a base 10 radix sort.
    Use: radix_sort(a)
    -----
    Preconditions:

```

```

    a - a List of base 10 integers (List)
Postconditions:
    Contents of a are sorted.
-----
"""
if len(a) > 0:
    # Determine the maximum digit in the numbers in a.
    max_val = a.max()
    passes = int(log(max_val, 10) + 1)
    # Create the empty buckets.
    buckets = []

    for _ in range(10):
        buckets.append(List())

    for digit in range(passes):
        # Calculate the digit extraction numerator and denominator.
        d = 10 ** digit
        n = d * 10

        while not a.empty():
            # Move the list nodes to the appropriate bucket.
            # Extract the individual digit.
            index = a._front._data % n // d
            # Move the front node to the proper bucket.
            buckets[index]._move_front(a)

        for index in range(len(buckets) - 1, -1, -1):
            # Move the nodes back to the original list.
            # (Start with right-most bucket).
            while not buckets[index].empty():
                a._move_front(buckets[index])

    return

# Sort Utilities

@staticmethod
def to_array(a):
    """
    -----
    Copies list values to a Python list.
    Use: values = to_array(a)
    -----
    Preconditions:
        a - a linked list of comparable elements (?)
    Postconditions:
        returns
        values - the contents of a in a Python list.= (list of ?)
    -----
    """
    values = []
    current = a._front

    while current is not None:
        values.append(current._data)
        current = current._next
    return values

@staticmethod
def sort_test(a):
    """
    -----
    Determines whether a linked list is is_sorted or not.
    Use: sort_test(a)

```

```
-----  
Preconditions:  
  a - a linked list of comparable elements (?)  
Postconditions:  
  returns:  
    is_sorted - True if contents of a are sorted, False otherwise.  
-----  
"""  
is_sorted = True  
current = a._front  
  
while is_sorted and current is not None and \\  
      current._next is not None:  
  
    if current._data <= current._next._data:  
        current = current._next  
    else:  
        is_sorted = False  
return is_sorted
```

```
"""
-----
sorts_array.py
Array versions of various sorts.
-----
Author:  David Brown
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__updated__ = "2017-11-21"
-----
"""

# Imports
from math import log
from bst_linked import BST

class Sorts:
    """
    -----
    Defines a number of array-based sort operations.
    Uses class attribute 'swaps' to determine how many times
    elements are swapped by the class.
    Use: print(Sorts.swaps)
    Use: Sorts.swaps = 0
    -----
    """
    swaps = 0 # Tracks swaps performed.

    # The Sorts

    @staticmethod
    def selection_sort(a):
        """
        -----
        Sorts an array using the Selection Sort algorithm.
        Use: Sorts.selection_sort(a)
        -----
        Preconditions:
            a - a Python list of comparable elements (?)
        Postconditions:
            Contents of a are sorted.
        -----
        """
        n = len(a)

        for i in range(n):
            # Walk through entire array
            m = i
            j = i + 1

            for j in range(i + 1, n):
                # Find smallest value in unsorted part of array

                if a[m] > a[j]:
                    # Track smallest value so far
                    m = j

            if m != i:
                # swap elements only if necessary
                Sorts._swap(a, m, i)
        return

    @staticmethod
    def insertion_sort(a):
```

```

"""
-----
Sorts an array using the Insertion Sort algorithm.
Use: Sorts.insertion_sort(a)
-----
Preconditions:
    a - an array of comparable elements (?)
Postconditions:
    Contents of a are sorted.
-----
"""
n = len(a)

for i in range(1, n):
    # Walk through entire array
    # Move each key bigger than save in a[1:n-1] one space to right.
    save = a[i]
    # Count as a swap
    Sorts.swaps += 1
    j = i - 1

    while j >= 0 and a[j] > save:
        Sorts._shift(a, j + 1, j)
        j -= 1
    # Move save into hole opened up by moving previous keys to right.
    a[j + 1] = save
return

@staticmethod
def bubble_sort(a):
    """
    -----
    Sorts an array using the Bubble Sort algorithm.
    Use: Sorts.bubble_sort(a)
    -----
    Preconditions:
        a - an array of comparable elements (?)
    Postconditions:
        Contents of a are sorted.
    -----
    """
    done = False
    last = len(a) - 1

    while not done:
        # assume done is true
        done = True
        last_swapped = 0
        i = 0

        while i < last:
            if a[i] > a[i + 1]:
                # Save the list index swapped.
                last_swapped = i
                # The pair (a[i], a[i+1]) is out of order.
                # Exchange a[i] and a[i + 1] to put them in sorted order.
                Sorts._swap(a, i, i + 1)
                # If you swapped you need another pass.
                done = False
            i += 1
        last = last_swapped
        # Decreases 'last' because everything after last_swapped is already
        # in order. done == False iff no pair of keys swapped on last pass.
    return

```

```

@staticmethod
def bst_sort(a):
    """
    -----
    Sorts an array using the Tree Sort algorithm.
    Use: Sorts.bst_sort(a)
    -----
    Preconditions:
        a - an array of comparable elements (?)
    Postconditions:
        Contents of a are sorted.
    -----
    """
    bst = BST()

    for v in a:
        bst.insert(v)

    a[:] = bst.inorder()
    return

@staticmethod
def shell_sort(a):
    """
    -----
    Sorts an array using the Shell Sort algorithm.
    Use: Sorts.shell_sort(a)
    -----
    Preconditions:
        a - an array of comparable elements (?)
    Postconditions:
        Contents of a are sorted.
    -----
    """
    n = len(a)
    increment = n // 3

    while increment > 0:
        i = increment

        while i < n:
            j = i
            temp = a[i]

            while j >= increment and a[j - increment] > temp:
                Sorts._shift(a, j, j - increment)
                j = j - increment
            a[j] = temp
            i += 1

        increment = increment // 3
    return

@staticmethod
def merge_sort(a):
    """
    -----
    Sorts an array using the Merge Sort algorithm.
    Use: Sorts.merge_sort(a)
    -----
    Preconditions:
        a - an array of comparable elements (?)
    Postconditions:

```

```

        Contents of a are sorted.
    """
    Sorts._merge_sort_aux(a, 0, len(a) - 1)
    return

@staticmethod
def _merge_sort_aux(a, first, last):
    """
    -----
    Divides an array into halves, sorts each half, then
    merges the halves back together.
    Use: Sorts._merge_sort_aux(a, first, last)
    -----
    Preconditions:
        a - an array of comparable elements (?)
        first - beginning of subarray of a (int)
        last - end of subarray of a (int)
    Postconditions:
        Contents of a from first to last are sorted.
    -----
    """

    if first < last:
        middle = (last - first) // 2 + first
        Sorts._merge_sort_aux(a, first, middle)
        Sorts._merge_sort_aux(a, middle + 1, last)
        Sorts._merge(a, first, middle, last)
    return

@staticmethod
def _merge(a, first, middle, last):
    """
    -----
    Merges two parts of an array together.
    Use: Sorts._merge(a, first, middle, last)
    -----
    Preconditions:
        a - an array of comparable elements (?)
        first - beginning of subarray of a (int)
        middle - middle of subarray of a (int)
        last - end of subarray of a (int)
    Postconditions:
        Contents of a are sorted.
    -----
    """
    temp = []
    i = first
    j = middle + 1

    while i <= middle and j <= last:

        if a[i] <= a[j]:
            # put leftmost element of left array into temp array
            temp.append(a[i])
            i += 1
        else:
            # put leftmost element of right array into temp array
            temp.append(a[j])
            j += 1

    # copy any remaining elements from the left half
    while i <= middle:
        temp.append(a[i])

```

```

        i += 1

# copy any remaining elements from the right half
while j <= last:
    temp.append(a[j])
    j += 1

# copy the temporary array back to the passed array
i = 0

while i < len(temp):
    a[first + i] = temp[i]
    i += 1
return

@staticmethod
def quick_sort(a):
    """
    -----
    Sorts an array using the Quick Sort algorithm.
    Use: Sorts.quick_sort(a)
    -----
    Preconditions:
        a - an array of comparable elements (?)
    Postconditions:
        Contents of a are sorted.
    -----
    """
    Sorts._quick_sort_aux(a, 0, len(a) - 1)
    return

@staticmethod
def _quick_sort_aux(a, first, last):
    """
    -----
    Divides an array into halves, sorts each half, then
    concatenates the halves back together.
    Use: Sorts._quick_sort_aux(a, first, last)
    -----
    Preconditions:
        a - an array of comparable elements (?)
        first - beginning of subarray of a (int)
        last - end of subarray of a (int)
    Postconditions:
        Contents of a from first to last are sorted.
    -----
    """
    # to sort the subarray a[p:q] of array a into ascending order
    if first < last:
        # partitions a[first:last] into a[first:pivot] and a[pivot+1:last]
        i = Sorts._partition(a, first, last)
        Sorts._quick_sort_aux(a, first, i - 1)
        Sorts._quick_sort_aux(a, i + 1, last)
    return

@staticmethod
def _partition(a, first, last):
    """
    -----
    Divides an array into two parts.
    Use: Sorts._partition(a, first, last)
    -----
    Preconditions:
        a - an array of comparable elements (?)
    """

```

```

        first - beginning of subarray of a (int)
        last - last of subarray of a (int)
Postconditions:
    Contents of a from first to last are sorted.
-----
"""
pivot_index = first
low = first
high = last - 1 # After we remove pivot it will be one smaller
# print("{} - {} - {}".format(a[first:pivot_index], a[pivot_index],
#                               a[pivot_index + 1:last]))
pivot_value = a[pivot_index]
a[pivot_index] = a[last]

while low <= high:
    # print("{} - {} - {}".format(a[first:pivot_index],
    #                               a[pivot_index], a[pivot_index + 1:last]))
    while low <= high and a[low] < pivot_value:
        low = low + 1
    while low <= high and a[high] >= pivot_value:
        high = high - 1
    if low <= high:
        Sorts._swap(a, low, high)
Sorts._shift(a, last, low)
# a[last] = a[low]
a[low] = pivot_value
# print("{} - {} - {}".format(a[first:pivot_index], a[pivot_index],
#                               a[pivot_index + 1:last]))
# Return the new pivot position.
return low

@staticmethod
def heap_sort(a):
    """
    -----
    Sorts an array using the Heap Sort algorithm.
    Use: Sorts.heap_sort(a)
    -----
    Preconditions:
        a - an array of comparable elements (?)
    Postconditions:
        Contents of a are sorted.
    -----
    """
    first = 0
    last = len(a) - 1
    Sorts._build_heap(a, first, last)
    i = last

    while i > first:
        Sorts._swap(a, i, first)
        Sorts._reheap(a, first, i - 1)
        i -= 1
    return

@staticmethod
def _build_heap(a, first, last):
    """
    -----
    Creates a heap.
    Use: Sorts._build_heap(a, first, last)
    -----
    Preconditions:
        a - an array of comparable elements (list)
    """

```

```

        first - first element in array to process (int)
        last - last element in array to process (int)
Postconditions:
    Contents of a from first to last are sorted.
-----
"""
i = last // 2

while i >= first:
    Sorts._reheap(a, i, last)
    i -= 1
return

@staticmethod
def _reheap(a, first, last):
    """
    -----
    Establishes heap property in a.
    Use: Sorts._reheap(a, first, last)
    -----
    Preconditions:
        a - an array of comparable elements (?)
        first - first element in array to process (int)
        last - last element in array to process (int)
    Postconditions:
        Contents of a from first to last are heaped.
    -----
    """
    done = False

    while 2 * first + 1 <= last and not done:
        k = 2 * first + 1

        if k < last and a[k] < a[k + 1]:
            k += 1

        if a[first] >= a[k]:
            done = True
        else:
            Sorts._swap(a, first, k)
            first = k
    return

@staticmethod
def comb_sort(a):
    """
    -----
    Sorts an array using the Comb Sort algorithm.
    Use: Sorts.comb_sort(a)
    -----
    Preconditions:
        a - an array of comparable elements (?)
    Postconditions:
        Contents of a are sorted.
    -----
    """
    n = len(a)
    gap = n
    done = False

    while gap > 1 or not done:
        done = True
        gap = int(gap / 1.3)

```

```

    if gap < 1:
        gap = 1

    i = 0
    j = gap

    while j < n:
        if a[i] > a[j]:
            Sorts._swap(a, i, j)
            done = False
        i += 1
        j += 1
    return

@staticmethod
def cocktail_sort(a):
    """
    -----
    Sorts an array using the Cocktail Sort algorithm.
    Use: Sorts.cocktail_sort(a)
    -----
    Preconditions:
        a - an array of comparable elements (?)
    Postconditions:
        Contents of a are sorted.
    -----
    """
    n = len(a)
    first = 0
    last = n - 1

    while first < last:
        # Initialize last_swapped to the beginning of the array.
        # Stops bottom loop from executing if no swaps done by top loop.
        last_swapped = 0
        i = first

        while i < last:
            if a[i] > a[i + 1]:
                # test whether the two elements are in the correct order
                Sorts._swap(a, i, i + 1)
                last_swapped = i
            i += 1
        last = last_swapped

        # Initialize first_swapped to the end of the array.
        # Stops top loop from executing if no swaps done by bottom loop.
        first_swapped = n - 1
        i = last

        while i > first:
            if a[i] < a[i - 1]:
                # test whether the two elements are in the correct order
                Sorts._swap(a, i, i - 1)
                first_swapped = i
            i -= 1
        first = first_swapped
    return

@staticmethod
def gnome_sort(a):
    """
    -----
    Sorts an array using the Gnome Sort algorithm.
    -----

```

```

Use: gnome_sort(a)
-----
Preconditions:
    a - an array of comparable elements (list)
Postconditions:
    Contents of a are sorted.
-----
"""
pos = 0
n = len(a)

while pos < n - 1:

    if a[pos] <= a[pos + 1]:
        # Compared elements are in order
        pos += 1
    else:
        # Compared elements must be swapped
        Sorts._swap(a, pos, pos + 1)

        if pos > 0:
            # Go back to compare newly-swapped element
            pos -= 1
        else:
            # index 0 contains smallest element so far
            pos += 1
return

@staticmethod
def gnome_sort2(a):
    """
    -----
    Sorts an array using the Gnome Sort algorithm.
    Use: gnome_sort(a)
    -----
    Preconditions:
        a - an array of comparable elements (?)
    Postconditions:
        Contents of a are sorted.
    -----
    """
    n = len(a)
    i = 0

    while i < n - 1:
        pos = i

        while pos >= 0 and a[pos] > a[pos + 1]:
            # Walk backwards through elements, swapping if necessary
            Sorts._swap(a, pos, pos + 1)
            pos -= 1
        i += 1
    return

@staticmethod
def gnome_r(a):
    """
    -----
    Sorts an array using the Gnome Sort algorithm.
    Use: gnome_sort(a)
    -----
    Preconditions:
        a - an array of comparable elements (?)
    Postconditions:

```

```

        Contents of a are sorted.
    """
    Sorts.gnome_r_aux(a, 0)
    return

@staticmethod
def gnome_r_aux(a, pos):
    """
    -----
    Sorts an array using the Gnome Sort algorithm.
    Use: gnome_sort(a)
    -----
    Preconditions:
        a - an array of comparable elements (?)
        pos - index of first value to compare (int)
    Postconditions:
        Contents of a up to index p are sorted.
    -----
    """
    if pos < len(a) - 1:

        if a[pos] <= a[pos + 1]:
            Sorts.gnome_r_aux(a, pos + 1)
        else:
            Sorts._swap(a, pos, pos + 1)

            if pos > 0:
                Sorts.gnome_r_aux(a, pos - 1)
            else:
                Sorts.gnome_r_aux(a, pos + 1)

    return

@staticmethod
def binary_insert_sort(a):
    """
    -----
    Sorts an array using the Binary Insertion Sort algorithm.
    Use: Sorts.binary_insert_sort(a)
    -----
    Preconditions:
        a - an array of comparable elements (list)
    Postconditions:
        Contents of a are sorted.
    -----
    """
    n = len(a)
    i = 1

    while i < n:
        ins = Sorts._bin_srch_i(a, 0, i, a[i])
        # Move key bigger than save in a[1:n-1] one space to the right.
        if ins < i:
            save = a[i]
            j = i - 1

            while j > ins - 1:
                Sorts._shift(a, j + 1, j)
                # a[j + 1] = a[j]
                j -= 1
            a[ins] = save
        i += 1
    return

```

```

@staticmethod
def _bin_srch_r(a, low, high, value):
    """
    -----
    Sorts an array using the Binary Insertion Sort algorithm.
    Use: Sorts._bin_srch_r(a)
    Recursive algorithm.
    -----
    Preconditions:
        a - an array of comparable elements (list)
        low - starting point of a to search for value (int)
        high - end point of a to search for value (int)
        value - value to search for position in a (?)
    Postconditions:
        returns
        mid - the insert point for value in a between low and high
    -----
    """
    if low == high:
        mid = low
    else:
        mid = low + ((high - low) // 2)

        if value > a[mid]:
            mid = Sorts._bin_srch_r(a, mid + 1, high, value)
        elif value < a[mid]:
            mid = Sorts._bin_srch_r(a, low, mid, value)
    return mid

@staticmethod
def _bin_srch_i(a, low, high, value):
    """
    -----
    Sorts an array using the Binary Insertion Sort algorithm.
    Use: Sorts._bin_srch_r(a)
    Iterative algorithm.
    -----
    Preconditions:
        a - an array of comparable elements (list)
        low - starting point of a to search for value (int)
        high - end point of a to search for value (int)
        value - value to search for position in a (?)
    Postconditions:
        returns
        mid - the insert point for value in a between low and high
    -----
    """
    found = False
    mid = (low + high) // 2

    while low < high and not found:
        # Find the middle of the current subarray.
        if value > a[mid]:
            # Search the right subarray.
            low = mid + 1
            mid = (low + high) // 2
        elif value < a[mid]:
            # Search the left subarray.
            high = mid
            mid = (low + high) // 2
        else:
            found = True
    return mid

```

```
@staticmethod
def radix_sort(a):
    """
    -----
    Performs a base 10 radix sort.
    Use: Sorts.radix_sort(a)
    -----
    Preconditions:
        a - an array of base 10 integers (list)
    Postconditions:
        Contents of a are sorted.
    -----
    """
    if len(a) > 0:
        # Find the largest number in a.
        max_val = max(a)
        # Find the number of digits in the largest number.
        passes = int(log(max_val, 10) + 1)
        # Create the empty buckets.
        buckets = []

        for _ in range(10):
            buckets.append([])

        for digit in range(passes):
            # Calculate the digit extraction numerator and denominator.
            d = 10 ** digit
            n = d * 10

            for v in a:
                # Assign the array values to the appropriate bucket.
                # Extract the individual digit.
                index = v % n // d
                # Add the number to the proper bucket.
                buckets[index].append(v)

            # Put the values back into the original array.
            index = 0

            for bucket in buckets:
                while bucket != []:
                    a[index] = bucket.pop(0)
                    index += 1

        return

@staticmethod
def counting_sort(a):

    if len(a) > 0:
        m = max(a) + 1
        b = [0] * m

        for i in range(m):
            n = a.count(i)
            b[i] = n

        a[:] = []
        i = 0

        for v in b:
            for _ in range(v):
                a.append(i)
            i += 1

        return
```

```

@staticmethod
def counting_sort_2(a):
    m = max(a) + 1
    b = [0] * m

    while a != []:
        i = a.pop()
        b[i] += 1

    for i in range(len(b)):
        for _ in range(b[i]):
            a.append(i)
    return

@staticmethod
def counting_sort_3(a):
    m = max(a) + 1
    b = [0] * m

    for v in a:
        b[v] += 1

    i = 0

    for j in range(len(b)):
        for _ in range(b[j]):
            a[i] = j
            i += 1
    return

# Sort Utilities

@staticmethod
def sort_test(a):
    """
    -----
    Determines whether an array is is_sorted or not.
    Use: sort_test(a)
    -----
    Preconditions:
        a - an array of comparable elements (?)
    Postconditions:
        returns
            is_sorted - True if contents of a are sorted,
                       False otherwise.
    -----
    """
    is_sorted = True
    n = len(a)
    i = 0

    while is_sorted and i < n - 1:
        if a[i] <= a[i + 1]:
            i += 1
        else:
            is_sorted = False
    return is_sorted

@staticmethod
def _swap(a, i, j):
    """
    -----

```

Swaps the data contents of two array elements.

Updates 'swaps'.

Use: `Sorts._swap(a, i, j)`

Preconditions:

 a - an array of comparable elements (?)

 i - index of one value (int $0 \leq i < \text{len}(a)$)

 j - index of another value (int $0 \leq j < \text{len}(a)$)

Postconditions:

 Values in `a[i]` and `a[j]` are swapped.

"""

`Sorts.swaps += 1`

`temp = a[i]`

`a[i] = a[j]`

`a[j] = temp`

`return`

`@staticmethod`

`def _shift(a, i, j):`

"""

Shifts the contents of `a[j]` to `a[i]`.

Updates 'swaps' - worth 1/3 of `_swap`

Use: `Sorts._shift(a, i, j)`

Preconditions:

 a - an array of comparable elements (?)

 i - index of one value (int $0 \leq i < \text{len}(a)$)

 j - index of another value (int $0 \leq j < \text{len}(a)$)

Postconditions:

 Value in `a[j]` is copied to `a[i]`.

"""

`Sorts.swaps += 0.34`

`a[i] = a[j]`

`return`

```
"""
-----
list_array.py
Array version of the list ADT.
-----
Author: David Brown
ID: 999999999
Email: dbrown@wlu.ca
__updated__ = "2018-03-21"
-----
"""
from copy import deepcopy

class List:

    def __init__(self):
        """
        -----
        Initializes an empty list.
        Use: l = List()
        -----
        Postconditions:
            Initializes an empty list.
        -----
        """
        self._values = []
        return

    def empty(self):
        """
        -----
        Determines if the list is empty.
        Use: b = l.empty()
        -----
        Postconditions:
            returns
            True if the list is empty, False otherwise.
        -----
        """
        return len(self._values) == 0

    def is_empty(self):
        """
        -----
        Determines if the list is empty.
        Use: b = l.empty()
        -----
        Postconditions:
            returns
            True if the list is empty, False otherwise.
        -----
        """
        return len(self._values) == 0

    def __len__(self):
        """
        -----
        Returns the size of the list.
        Use: n = len(l)
        -----
        Postconditions:
            Returns the number of values in the list.
        -----
        """
```

```

    """
    return len(self._values)

def insert(self, i, value):
    """
    -----
    Inserts a copy of value into the list at index i.
    Use: l.insert(i, value)
    -----
    Preconditions:
        i - index value (int)
        value - a data element (?)
    Postconditions:
        a copy of value is added to index i, all other values are pushed
right
        If i outside of range of length of list, appended to end
    -----
    """
    self._values = self._values[:i] + [deepcopy(value)] + self._values[i:]
    return

def _linear_search(self, key):
    """
    -----
    Searches for the first occurrence of key in the list.
    Private helper method - used only by other ADT methods.
    Use: i = self._linear_search(key)
    -----
    Preconditions:
        key - a partial data element (?)
    Postconditions:
        returns
            i - the index of key in the list, -1 if key is not found (int)
    -----
    """
    n = len(self._values)
    i = 0

    while i < n and self._values[i] != key:
        i += 1

    if i == n:
        i = -1
    return i

def remove(self, key):
    """
    -----
    Finds, removes, and returns the first value in list that matches key.
    Use: value = l.remove(key)
    -----
    Preconditions:
        key - a partial data element (?)
    Postconditions:
        returns
            value - the full value matching key, otherwise None (?)
    -----
    """
    assert len(self._values) > 0, "Cannot remove from an empty list"

    # Key argument exists - search list for key.
    i = self._linear_search(key)

    if i > -1:

```

```

        value = self._values.pop(i)
    else:
        value = None
    return value

def find(self, key):
    """
    -----
    Finds and returns a copy of the first value in list that matches key.
    Use: value = l.find(key)
    -----
    Preconditions:
        key - a partial data element (?)
    Postconditions:
        returns
            value - a copy of the full value matching key, otherwise None (?)
    -----
    """
    i = self._linear_search(key)

    if i > -1:
        value = deepcopy(self._values[i])
    else:
        value = None
    return value

def peek(self):
    """
    -----
    Returns a copy of the first value in list.
    Use: value = l.peek()
    -----
    Postconditions:
        returns
            value - a copy of the first value in the list (?)
    -----
    """
    assert len(self._values) > 0, "Cannot peek at an empty list"

    value = deepcopy(self._values[0])
    return value

def index(self, key):
    """
    -----
    Finds location of a value by key in list.
    Use: n = l.index(key)
    -----
    Preconditions:
        key - a partial data element (?)
    Postconditions:
        returns
            i - the index of the location of key in the list, -1 if
                key is not in the list. (int)
    -----
    """
    i = self._linear_search(key)
    return i

def _valid_index(self, i):
    """
    -----
    Private helper method to validate an index value.
    Python index values can be positive or negative and range from
    """

```

```

        -len(list) to len(list) - 1
    Use: assert self._valid_index(i)
    -----
    Preconditions:
        i - an index value (int)
    Postconditions:
        returns
            True if i is a valid index, False otherwise.
    -----
    """
    n = len(self._values)
    return -n <= i < n

def __getitem__(self, i):
    """
    -----
    Returns a copy of the nth element of the list.
    Use: value = l[i]
    -----
    Preconditions:
        i - index of the element to access (int)
    Postconditions:
        returns
            value - the i-th element of list (?)
    -----
    """
    assert self._valid_index(i), "Invalid index value"

    value = deepcopy(self._values[i])
    return value

def __setitem__(self, i, value):
    """
    -----
    Places a copy of value into the list at position n.
    Use: l[i] = value
    -----
    Preconditions:
        i - index of the element to access (int)
        value - a data value (?)
    Postconditions:
        The i-th element of list contains a copy of value. The existing
        value at i is overwritten.
    -----
    """
    assert self._valid_index(i), "Invalid index value"

    self._values[i] = deepcopy(value)
    return

def __contains__(self, key):
    """
    -----
    Determines if the list contains key.
    Use: b = key in l
    -----
    Preconditions:
        key - a partial data element (?)
    Postconditions:
        returns
            True if the list contains key, False otherwise. (boolean)
    -----
    """
    i = self._linear_search(key)

```

```

        return i > -1
def max(self):
    """
    -----
    Finds the maximum value in list.
    Use: value = l.max()
    -----
    Postconditions:
        returns
        value - a copy of the maximum value in the list (?)
    -----
    """
    assert len(self._values) > 0, "Cannot find maximum of an empty list"

    max_index = 0
    i = 1
    n = len(self._values)

    while i < n:
        if self._values[i] > self._values[max_index]:
            max_index = i
        i += 1
    value = deepcopy(self._values[max_index])
    return value

def min(self):
    """
    -----
    Finds the minimum value in list.
    Use: value = l.min()
    -----
    Postconditions:
        returns
        value - a copy of the minimum value in the list (?)
    -----
    """
    assert len(self._values) > 0, "Cannot find minimum of an empty list"

    min_index = 0
    i = 1
    n = len(self._values)

    while i < n:
        if self._values[i] < self._values[min_index]:
            min_index = i
        i += 1
    value = deepcopy(self._values[min_index])
    return value

def count(self, key):
    """
    -----
    Finds the number of times key appears in list.
    Use: n = l.count(key)
    -----
    Preconditions:
        key - a partial data element (?)
    Postconditions:
        returns
        number - number of times key appears in list (int)
    -----
    """
    assert len(self._values) > 0, "Cannot count keys in an empty list"

```

```

number = 0
i = 1
n = len(self._values)

while i < n:
    if self._values[i] == key:
        number += 1
    i += 1
return number

def reverse(self):
    """
    -----
    Reverses the order of the elements in list.
    -----
    Postconditions:
        The contents of list are reversed in order with respect
        to their order before the operation was called.
    -----
    """
    n = len(self._values)
    mid = n // 2
    i = 0

    while i < mid:
        j = n - i - 1
        self._values[i], self._values[j] = self._values[j], self._values[i]
        i += 1
    return

def append(self, value):
    """
    -----
    Appends a copy of value to the end of the list.
    Use: l.append(value)
    -----
    Preconditions:
        value - a data element (?)
    Postconditions:
        a copy of value is added to the end of the list.
    -----
    """
    self._values.append(deepcopy(value))
    return

def clean(self):
    """
    -----
    Removes duplicates from the list.
    Use: l.clean()
    -----
    Postconditions:
        The list contains one and only one of each value formerly present
        in the list. The first occurrence of each value is preserved.
    -----
    """
    i = 0

    while i < len(self._values):
        # Check current value against the rest of the values.
        j = i + 1

        while j < len(self._values):

```

```

        if self._values[i] == self._values[j]:
            # Delete the extra value.
            self._values.pop(j)
        else:
            j += 1
        i += 1
    return

def pop(self, *args):
    """
    -----
    Finds, removes, and returns the value in list whose index matches i.
    Use: value = l.pop()
    Use: value = l.pop(i)
    -----
    Preconditions:
        args - an array of arguments (tuple of int)
        args[0], if it exists, is the index i
    Postconditions:
        returns
        value - if args exists, the value at position args[0], otherwise the
last          value in the list, value is removed from the list (?)
    -----
    """
    assert len(self._values) > 0, "Cannot pop from an empty list"
    assert len(args) <= 1, "No more than 1 argument allowed"

    if len(args) == 1:
        # pop the element at position i
        i = args[0]
        value = self._values.pop(i)
    else:
        # pop the last element
        value = self._values.pop()
    return value

def _swap(self, i, j):
    """
    -----
    Swaps the position of two elements in the data list.
    Private helper operations called only from within class.
    -----
    Preconditions:
        i - index of one element to switch (int, 0 <= i < len(self._values))
        j - index of other element to switch (int, 0 <= j <
len(self._values))
    Postconditions:
        The element originally at position i is now at position j,
        and visa versa.
    -----
    """
    assert self._valid_index(i), "Invalid index i"
    assert self._valid_index(j), "Invalid index j"

    temp = self._values[i]
    self._values[i] = self._values[j]
    self._values[j] = temp
    return

def _linear_search_r(self, key):
    """
    -----
    Searches for the first occurrence of key in the list.
    
```

```

Private helper method - used only by other ADT methods.
Use: i = self._linear_search_r(key)
-----
Preconditions:
    key - a partial data element (?)
Postconditions:
    returns
        i - the index of key in the list, -1 if key is not found (int)
-----
"""
i = self._linear_search_r_aux(key, 0, len(self._values))
return i

def _linear_search_r_aux(self, key, i, n):
    """
    -----
    Searches for the first occurrence of key in the list.
    Private helper method - used only by other ADT methods.
    Use: i = self._linear_search(key)
    -----
    Preconditions:
        key - a partial data element (?)
    Postconditions:
        returns
            i - the index of key in the list, -1 if key is not found (int)
    -----
    """
    if i == n:
        i = -1
    elif self._values[i] != key:
        i = self._linear_search_r_aux(key, i + 1, n)
    return i

def insert_front(self, value):
    """
    -----
    Inserts a copy of value into the front of the list.
    Use: l.insert_front(value)
    -----
    Preconditions:
        value - a data element. (?)
    Postconditions:
        value is added to the front of the list.
    -----
    """
    self._values.insert(0, deepcopy(value))
    return

def remove_front(self):
    """
    -----
    Removes the first node in the list.
    Use: value = l.remove_front()
    -----
    Postconditions:
        returns
            value - the first value in the list (?)
    -----
    """
    assert len(self._values) > 0, "Cannot remove from an empty list"

    value = self._values.pop(0)
    return value

```

```

def remove_many(self, key):
    """
    -----
    Finds and removes all values in the list that match key.
    Use: l.remove_many(key)
    -----
    Preconditions:
        key - a data element (?)
    Postconditions:
        Removes all values matching key.
    -----
    """
    i = 0

    while i < len(self._values):
        if self._values[i] == key:
            self._values.pop(i)
        else:
            i = i + 1
    return

def identical(self, rs):
    """
    -----
    Determines whether two lists are identical, i.e. same values appear
    in the same locations in both lists. (iterative version)
    Use: b = l.identical(rs)
    -----
    Preconditions:
        rs - another list (List)
    Postconditions:
        returns
        is_identical - True if this list contains the same values as rs
        in the same order, otherwise False. (boolean)
    -----
    """
    is_identical = False
    n = len(self._values)

    # Lists must be the same length to be identical.
    if len(rs) == n:
        i = 0

        # Compare each element, stop when the elements are not the same
        # or the end of the list is reached.
        while i < n and self._values[i] == rs._values[i]:
            i += 1

        if i == n:
            is_identical = True
    return is_identical

def identical_r(self, rs):
    """
    -----
    Determines whether two lists are identical, i.e. same values appear
    in the same locations in both lists. (recursive version)
    Use: b = l.identical_r(rs)
    -----
    Preconditions:
        rs - another list (List)
    Postconditions:
        returns
        is_identical - True if this list contains the same values as rs
    """

```

```

        in the same order, otherwise False. (boolean)
-----
"""
if len(rs) == len(self._values):
    is_identical = self._identical_r_aux(self._values, rs._values)
else:
    is_identical = False
return is_identical

def _identical_r_aux(self, data1, data2):
    """
    -----
    Recursive auxiliary function for identical_r.
    Use: b = self._identical_r_aux(self._values[?:], rs._values[?:])
    -----
    Preconditions:
        data1 - the current Python list (list)
        data2 - the rs Python list (list)
    Postconditions:
        returns
        is_identical - True if this list contains the same values as rs
        in the same order, otherwise False. (boolean)
    -----
    """
    if len(data1) == 0:
        is_identical = True
    elif data1[0] == data2[0]:
        is_identical = self._identical_r_aux(data1[1:], data2[1:])
    else:
        is_identical = False
    return is_identical

def intersection(self, rs):
    """
    -----
    Returns a list that contains only values that appear in both
    the current List and rs.
    Use: l2 = l1.intersection(rs)
    -----
    Preconditions:
        rs - another List (List)
    Postconditions:
        returns
        new_list - A List that contains only the values found in both
        the current List and rs. Values do not repeat. (List)
    -----
    """
    new_list = List()
    m = len(self._values)
    n = len(rs._values)
    i = 0

    while i < m:
        # Walk through every value in self
        key = self._values[i]
        j = 0

        # Look in new_list for key
        while j < len(new_list._values) and key != new_list._values[j]:
            j += 1

        if j == len(new_list._values):
            # key is not in new_list
            k = 0

```

```

        # look in rs for key
        while k < n and key != rs._values[k]:
            k += 1

        if k < n:
            # key is in rs - add key to new_list
            new_list._values.append(key)
        i += 1
    return new_list

def union(self, rs):
    """
    -----
    Returns a list that contains all values in both
    the current List and rs.
    Use: nl = l.union(rs)
    -----
    Preconditions:
        rs - another list (List)
    Postconditions:
        returns
            new_list - contains all values found in both the current
            List and rs. Values do not repeat. (List)
    -----
    """
    new_list = List()
    i = 0
    n = len(self._values)

    while i < n:
        v = self._values[i]
        # Walk through current list.
        if new_list._linear_search(v) == -1:
            # Value is not in new list.
            new_list._values.append(v)
        i += 1

    i = 0
    n = len(rs._values)

    while i < n:
        v = rs._values[i]
        # Walk through other list.
        if new_list._linear_search(v) == -1:
            # Value is not in new list.
            new_list._values.append(v)
        i += 1
    return new_list

def split_alt(self):
    """
    -----
    Split a List into two parts. even contains the even indexed
    elements, odd contains the odd indexed elements.
    Order of even and odd is not significant. (iterative version)
    Use: even, odd = l.split_alt()
    -----
    Postconditions:
        returns
            even - the even indexed elements of the list (List)
            odd - the odd indexed elements of the list (List)
            The List is empty.
    -----
    """

```

```

"""
even = List()
odd = List()
i = 0

while len(self._values) > 0:

    if i % 2 == 0:
        even._values.append(self._values.pop(0))
    else:
        odd._values.append(self._values.pop(0))
    i += 1
return even, odd

def split(self):
    """
    -----
    Splits list into two parts. ls contains the first half,
    rs the second half. Current list becomes empty.
    Use: ls, rs = l.split()
    -----
    Postconditions:
        returns
        ls - a new List with >= 50% of the original List (List)
        rs - a new List with <= 50% of the original List (List)
    -----
    """
    ls = List()
    rs = List()
    # Split
    middle = len(self._values) // 2 + len(self._values) % 2
    ls._values = self._values[:middle]
    rs._values = self._values[middle:]
    self._values = []
    return ls, rs

def split_apply(self, func):
    """
    -----
    Splits list into two parts. ls contains all the values
    where the result of calling func(value) is True,
    rs contains the remaining values.
    Use: ls, rs = l.split_apply(func)
    -----
    Preconditions:
        func - a function that given a value in the list returns
        True for some condition, otherwise returns False.
    Postconditions:
        returns
        ls - a new List with values where func(value) is True (List)
        rs - a new List with values where func(value) is False (List)
        self is empty. Order of values in new lists is maintained.
    -----
    """
    ls = List()
    rs = List()
    # Split
    while len(self._values) > 0:
        value = self._values.pop()

        if(func(value)):
            ls.insert_front(value)
        else:
            rs.insert_front(value)

```

```

        return ls, rs

def copy(self):
    """
    -----
    Duplicates the current list to a new list in the same order.
    Use: new_list = l.copy()
    -----
    Postconditions:
        returns
        new_list - a copy of self (List)
    -----
    """
    new_list = List()
    new_list._values = deepcopy(self._values)
    return new_list

def combine(self, s2):
    """
    -----
    Combines contents of two lists into a third.
    Use: new_list = l1.combine(s2)
    -----
    Preconditions:
        s2- an array-based List (List)
    Postconditions:
        returns
        new_list - the contents of the current List and s2
                    are interlaced into new_list - current List and s2
                    are empty (List)
    -----
    """
    new_list = List()

    while len(self._values) > 0 and len(s2._values) > 0:
        new_list._values.append(self._values.pop(0))
        new_list._values.append(s2._values.pop(0))

    while len(self._values) > 0:
        new_list._values.append(self._values.pop(0))

    while len(s2._values) > 0:
        new_list._values.append(s2._values.pop(0))

    return new_list

def __iter__(self):
    """
    USE FOR TESTING ONLY
    -----
    Generates a Python iterator. Iterates through the list
    from front to rear.
    Use: for v in q:
    -----
    Postconditions:
        returns
        value - the next value in the list (?)
    -----
    """
    for value in self._values:
        yield value

```

```
"""
-----
bst_linked.py
Linked version of the BST ADT.
-----
Author:  David Brown
ID:      999999999
Email:   dbrown@wlu.ca
__updated__ = "2018-03-21"
-----
"""

# Imports
from copy import deepcopy
from stack_array import Stack

class _BSTNode:

    def __init__(self, value):
        """
        -----
        Creates a node containing a copy of value.
        Use: node = _BSTNode(value)
        -----
        Preconditions:
            value - data for the node (?)
        Postconditions:
            Initializes a BST node containing value. Child pointers are None,
            height is 1.
        -----
        """
        self._data = deepcopy(value)
        self._left = None
        self._right = None
        self._height = 1
        return

    def _update_height(self):
        """
        -----
        Updates the height of the current node.
        Use: node._update_height()
        -----
        Postconditions:
            _height is 1 plus the maximum of the node's (up to) two children.
        -----
        """
        if self._left is None:
            left_height = 0
        else:
            left_height = self._left._height

        if self._right is None:
            right_height = 0
        else:
            right_height = self._right._height

        self._height = max(left_height, right_height) + 1
        return

    def __str__(self):
        """
        USE FOR TESTING ONLY
        -----
        """
```

```
Returns node height and value as a string - for debugging.
-----
"""
return "h: {}, v: {}".format(self._height, self._data)
```

```
class BST:
```

```
def __init__(self):
```

```
-----
"""
Initializes an empty BST.
Use: bst = BST()
-----
Postconditions:
    Initializes an empty bst.
-----
"""
```

```
self._root = None
self._count = 0
return
```

```
def empty(self):
```

```
-----
"""
Determines if bst is empty.
Use: b = bst.empty()
-----
Postconditions:
    returns
    True if bst is empty, False otherwise.
-----
"""
```

```
return self._root is None
```

```
def is_empty(self):
```

```
-----
"""
Determines if bst is empty.
Use: b = bst.empty()
-----
Postconditions:
    returns
    True if bst is empty, False otherwise.
-----
"""
```

```
return self._root is None
```

```
def __len__(self):
```

```
-----
"""
Returns the number of nodes in the BST.
Use: n = len(bst)
-----
Postconditions:
    returns
    the number of nodes in the BST.
-----
"""
```

```
return self._count
```

```
def insert(self, value):
```

```
-----
"""
```

```

Inserts a copy of value into the bst.
Use: b = bst.insert(value)
-----
Preconditions:
    value - data to be inserted into the bst (?)
Postconditions:
    returns
        inserted - True if value is inserted into the BST,
        False otherwise. Values may appear only once in a tree. (boolean)
-----
"""
self._root, inserted = self._insert_aux(self._root, value)
return inserted

def _insert_aux(self, node, value):
    """
    -----
    Inserts a copy of _data into node.
    Private recursive operation called only by insert.
    Use: node, inserted = self._insert_aux(node, value)
    -----
    Preconditions:
        node - a bst node (_BSTNode)
        value - data to be inserted into the node (?)
    Postconditions:
        returns
            node - the current node (_BSTNode)
            inserted - True if value is inserted into the BST,
            False otherwise. Values may appear only once in a tree. (boolean)
    -----
    """
    if node is None:
        # Base case: add a new node containing the value.
        node = _BSTNode(value)
        self._count += 1
        inserted = True
    elif node._data > value:
        # General case: check the left subtree.
        node._left, inserted = self._insert_aux(node._left, value)
    elif node._data < value:
        # General case: check the right subtree.
        node._right, inserted = self._insert_aux(node._right, value)
    else:
        # Base case: value is already in the BST.
        inserted = False

    if inserted:
        # Update the node height if any of its children have been changed.
        node._update_height()
    return node, inserted

def retrieve(self, key):
    """
    -----
    Retrieves a copy of a value matching key in a BST. (Iterative)
    Use: v = bst.retrieve(key)
    -----
    Preconditions:
        key - data to search for (?)
    Postconditions:
        returns
            value - value in the node containing key, otherwise None (?)
    -----
    """

```

```

node = self._root
value = None

while node is not None and value is None:

    if node._data > key:
        node = node._left
    elif node._data < key:
        node = node._right
    elif node._data == key:
        # for comparison counting
        value = deepcopy(node._data)
return value

def remove(self, key):
    """
    -----
    Removes a node with a value matching key from the bst.
    Returns the value matched.
    Use: value = bst.remove(key)
    -----
    Preconditions:
        key - data to search for (?)
    Postconditions:
        returns
        value - value matching key if found,
        otherwise returns None. Update structure of bst as required.
    -----
    """
    assert self._root is not None, "Cannot remove from an empty BST"

    self._root, value = self._remove_aux(self._root, key)
    return value

def _remove_aux(self, node, key):
    """
    -----
    Attempts to find a value matching key in a BST node. Deletes the node
    if found and returns the sub-tree root.
    Private recursive operation called only by remove.
    Use: node, value = self._remove_aux(node, key)
    -----
    Preconditions:
        node - a bst node to search for key (_BSTNode)
        key - data to search for (?)
    Postconditions:
        returns
        node - the current node or its replacement (_BSTNode)
        value - value in node containing key, None otherwise.
    -----
    """
    if node is None:
        # Base Case: the key is not in the tree.
        value = None
    elif key < node._data:
        # Search the left subtree.
        node._left, value = self._remove_aux(node._left, key)
    elif key > node._data:
        # Search the right subtree.
        node._right, value = self._remove_aux(node._right, key)
    else:
        # Value has been found.
        value = node._data
        self._count -= 1

```

```

# Replace this node with another node.
if node._left is None and node._right is None:
    # node has no children.
    node = None
elif node._left is None:
    # node has no left child.
    node = node._right
elif node._right is None:
    # node has no right child.
    node = node._left
else:
    # Node has two children
    if node._left._right is None:
        # left child is replacement node
        repl_node = node._left
    else:
        # find replacement node in right subtree of left node
        repl_node = self._delete_node_left(node._left)
        repl_node._left = node._left

    repl_node._right = node._right
    node = repl_node

if node is not None and value is not None:
    # If the value was found, update the ancestor heights.
    node._update_height()
return node, value

def _delete_node_left(self, parent):
    """
    -----
    Finds a replacement node for a node to be removed from the tree.
    Private operation called only by _remove_aux.
    Use: repl_node = self._delete_node_left(node, node._right)
    -----
    Preconditions:
        parent - node to search for largest value (_BSTNode)
    Postconditions:
        returns
            repl_node - the node that replaces the deleted node. This node
            is the node with the maximum value in the deleted node's left
            subtree (_BSTNode)
    -----
    """
    child = parent._right

    if child._right is None:
        # child has largest value in left subtree
        repl_node = child
        # move child's left tree up
        parent._right = child._left
    else:
        repl_node = self._delete_node_left(child)

    # Recursively update all parent node heights
    parent._update_height()
    return repl_node

def remove_root(self):
    """
    -----
    Removes the root node and returns its value.
    Use: value = bst._remove_root()
    -----

```

```

Postconditions:
    returns
    value - value in root.
-----
"""
assert self._root is not None, "Cannot remove the room of an empty BST"

# Value has been found.
value = self._root._data
self._count -= 1
# Replace the root with another node.

if self._root._left is None and self._root._right is None:
    # root has no children.
    self._root = None
elif self._root._left is None:
    # root has no left child.
    self._root = self._root._right
elif self._root._right is None:
    # root has no right child.
    self._root = self._root._left
else:
    # root has two children
    if self._root._left._right is None:
        # left child is replacement node
        repl_node = self._root._left
    else:
        # find replacement node in right subtree of left node
        repl_node = self._delete_node_left(self._root._left)
        repl_node._left = self._root._left

    repl_node._right = self._root._right
    self._root = repl_node

if self._root is not None:
    # Update the root height.
    self._root._update_height()
return value

def __contains__(self, key):
    """
    -----
    Determines if the bst contains key.
    Use: b = key in bst
    -----
    Preconditions:
        key - a comparable data element (?)
    Postconditions:
        returns
        True if the bst contains key, False otherwise.
    -----
    """
    value = self.retrieve(key)
    return value is not None

def height(self):
    """
    -----
    Returns the maximum height of a BST, i.e. the length of the
    largest path from root to a leaf node in the tree.
    Use: h = bst.height()
    -----
    Postconditions:
        returns

```

```

        maximum height of bst (int)
-----
"""
if self._root is None:
    h = 0
else:
    h = self._root._height
return h

def identical(self, rs):
    """
    -----
    Determines whether two BSTs are identical.
    Use: b = bst.identical(rs)
    -----
    Preconditions:
        rs - another bst (BST)
    Postconditions:
        returns
        is_identical - True if this bst contains the same values
        in the same order as rs, otherwise returns False (boolean)
    -----
    """
    if self._count != rs._count:
        is_identical = False
    else:
        is_identical = self._identical_aux(self._root, rs._root)
    return is_identical

def _identical_aux(self, node1, node2):
    """
    -----
    Determines whether two subtrees are identical.
    Use: b = self._identical_aux(node1, node2)
    -----
    Preconditions:
        node1 - node of the current BST (_BSTNode)
        node2 - node of the rs BST (_BSTNode)
    Postconditions:
        returns
        result - True if this subtree contains the same values as rs
        subtree in the same order, otherwise returns False (boolean)
    -----
    """
    if node1 is None and node2 is None:
        # Reached a bottom of the tree.
        result = True
    elif node1 is not None and node2 is not None \
        and node1._data == node2._data and node1._height ==
node2._height:
        result = self._identical_aux(node1._left, node2._left) \
            and self._identical_aux(node1._right, node2._right)
    else:
        result = False
    return result

def parent_i(self, key):
    """
    -----
    Returns the value of the parent node of a key node in a bst.
    -----
    Preconditions:
        key - a key value (?)
    Postconditions:

```

```

        returns
        value - a copy of the value in a node that is the parent of the
        key node, None if the key is not found. (?)
    """
    -----
    assert self._root is not None, "Cannot locate a parent in an empty BST"

    # Find the node containing the key.
    node = self._root
    parent = None
    found = False

    while node is not None and found is False:

        if key < node._data:
            parent = node
            node = node._left
        elif key > node._data:
            parent = node
            node = node._right
        else:
            found = True

    if parent is None or not found:
        value = None
    else:
        value = deepcopy(parent._data)
    return value

def parent_r(self, key):
    """
    -----
    Returns the value of the parent node in a bst given a key.
    -----
    Preconditions:
        key - a key value (?)
    Postconditions:
        returns
        value - a copy of the value in a node that is the parent of the
        key node, None if the key is not found.
    """
    -----
    assert self._root is not None, "Cannot locate a parent in an empty BST"

    # Find the node containing the key _data.
    return self._parent_aux(self._root, key, None)

def _parent_aux(self, node, key, parent):
    """
    -----
    Returns the _data of the parent node in a bst given a _data.
    Private recursive operation called only by parent_r.
    Use: v = self._parent_aux(node, key, parent of node)
    -----
    Preconditions:
        node - a BST node
        key - a key _data
        parent - the parent node of the current node
    Postconditions:
        returns
        value - the value of the parent node, None if it has no parent (?)
    """
    -----
    if node is None:

```

```

        # Base Case: the key is not in the tree.
        value = None
    elif key < node._data:
        # General Case: Search the left subtree.
        value = self._parent_aux(node._left, key, node)
    elif key > node._data:
        # General Case: Search the right subtree.
        value = self._parent_aux(node._right, key, node)
    elif parent is None:
        # Base Case: Value has been found, but without a parent node.
        value = None
    else:
        # Base Case: Value has been found. Return the parent value.
        value = deepcopy(parent._data)
    return value

def max(self):
    """
    -----
    Finds the maximum value in BST. (Iterative algorithm)
    Use: value = bst.max()
    -----
    Postconditions:
        returns
        value - a copy of the maximum value in the BST (?)
    -----
    """
    assert self._root is not None, "Cannot find maximum of an empty BST"
    # Find the node containing the largest _data.
    # (It is the right-most node.)
    node = self._root

    while node._right is not None:
        node = node._right

    value = deepcopy(node._data)
    return value

def max_r(self):
    """
    -----
    Returns the largest value in a bst. (Recursive algorithm)
    Use: value = bst.max_r()
    -----
    Postconditions:
        returns
        value - a copy of the maximum value in the BST (?)
    -----
    """
    assert self._root is not None, "Cannot find maximum of an empty BST"

    value = self._max_aux(self._root)
    return value

def _max_aux(self, node):
    """
    -----
    Returns the largest value in a BST node. (Recursive algorithm)
    Use: v = self._max_aux(node)
    -----
    Preconditions:
        node - valid linked BST node (_BSTNode)
    Postconditions:
        returns
    """

```

```

        value - a copy of the largest value in the node subtree (?)
    """
    -----
    """
    # Find the node containing the largest _data.
    # (It is the right-most node.)
    if node._right is None:
        value = deepcopy(node._data)
    else:
        value = self._max_aux(node._right)
    return value

def min(self):
    """
    -----
    Finds the minimum value in BST. (Iterative algorithm)
    Use: value = bst.min()
    -----
    Postconditions:
        returns
        value - a copy of the minimum value in the BST (?)
    -----
    """
    assert self._root is not None, "Cannot find minimum of an empty BST"
    # Find the node containing the smallest _data.
    # (It is the left-most node.)
    node = self._root

    while node._left is not None:
        node = node._left

    value = deepcopy(node._data)
    return value

def min_r(self):
    """
    -----
    Returns the minimum value in a bst. (Recursive algorithm)
    Use: value = bst.min_r()
    -----
    Postconditions:
        returns
        value - a copy of the minimum value in the BST (?)
    -----
    """
    assert self._root is not None, "Cannot find minimum of an empty BST"

    value = self._min_aux(self._root)
    return value

def _min_aux(self, node):
    """
    -----
    Returns the minimum value in a BST node. (Recursive algorithm)
    Use: v = self._min_aux(node)
    -----
    Preconditions:
        node - valid linked BST node (_BSTNode)
    Postconditions:
        returns
        value - a copy of the minimum value in the node subtree (?)
    -----
    """
    # Find the node containing the minimum _data.
    # (It is the left-most node.)

```

```

    if node._left is None:
        value = deepcopy(node._data)
    else:
        value = self._min_aux(node._left)
    return value

def leaf_count(self):
    """
    -----
    Returns the number of leaves (nodes with no children) in bst.
    Use: n = bst.leaf_count()
    (Recursive algorithm)
    -----
    Postconditions:
        returns
        count - number of nodes with no children in bst (int)
    -----
    """
    count = self._leaf_count_aux(self._root)
    return count

def _leaf_count_aux(self, node):
    """
    -----
    Returns the number of leaves (nodes with no children) in bst.
    Use: n = bst.leaf_count()
    (Recursive algorithm)
    -----
    Preconditions:
        node - a BST node (_BSTNode)
    Postconditions:
        returns
        count - number of nodes with no children below node (int)
    -----
    """
    if node is None:
        count = 0
    elif node._left is None and node._right is None:
        # Base case: node has no children.
        count = 1
    else:
        count = self._leaf_count_aux(node._left) + \
            self._leaf_count_aux(node._right)
    return count

def two_child_count(self):
    """
    -----
    Returns the number of the three types of nodes in a BST.
    Use: zero, one, two = bst.node_counts()
    -----
    Postconditions:
        returns
        zero - number of nodes with zero children (int)
        one - number of nodes with one child (int)
        two - number of nodes with two children (int)
    -----
    """
    return self._two_child_count_aux(self._root)

def _two_child_count_aux(self, node):
    """
    -----
    Returns the number of types of nodes in a BST node.
    """

```

```

-----
Preconditions:
    node - a BST node (_BSTNode)
Postconditions:
    returns
    zero - number of nodes with zero children (int)
    one - number of nodes with one child (int)
    two - number of nodes with two children (int)
-----
"""
if node is None:
    # Base case: node is empty.
    count = 0
elif node._left is not None and node._right is not None:
    # General case: node has two children.
    count = 1 + self._two_child_count_aux(node._left) + \
        self._two_child_count_aux(node._right)
else:
    # General case: node has one child.
    count = self._two_child_count_aux(node._left) + \
        self._two_child_count_aux(node._right)
return count

def one_child_count(self):
    """
    -----
    Returns the number of the three types of nodes in a BST.
    Use: zero, one, two = bst.node_counts()
    -----
    Postconditions:
        returns
        zero - number of nodes with zero children (int)
        one - number of nodes with one child (int)
        two - number of nodes with two children (int)
    -----
    """
    return self._one_child_count_aux(self._root)

def _one_child_count_aux(self, node):
    """
    -----
    Returns the number of types of nodes in a BST node.
    -----
    Preconditions:
        node - a BST node (_BSTNode)
    Postconditions:
        returns
        zero - number of nodes with zero children (int)
        one - number of nodes with one child (int)
        two - number of nodes with two children (int)
    -----
    """
    if node is None:
        # Base case: empty node..
        count = 0
    elif node._left is None and node._right is not None:
        # General case: node has one child.
        count = 1 + self._one_child_count_aux(node._right)
    elif node._left is not None and node._right is None:
        # General case: node has one child.
        count = 1 + self._one_child_count_aux(node._left)
    else:
        # General case: node has two children.
        count = self._one_child_count_aux(node._left) + \

```

```

        self._one_child_count_aux(node._right)
    return count

def node_counts(self):
    """
    -----
    Returns the number of the three types of nodes in a BST.
    Use: zero, one, two = bst.node_counts()
    -----
    Postconditions:
        returns
        zero - number of nodes with zero children (int)
        one - number of nodes with one child (int)
        two - number of nodes with two children (int)
    -----
    """
    zero, one, two = self._node_counts_aux(self._root)
    return zero, one, two

def _node_counts_aux(self, node):
    """
    -----
    Returns the number of types of nodes in a BST node.
    -----
    Preconditions:
        node - a BST node (_BSTNode)
    Postconditions:
        returns
        zero - number of nodes with zero children (int)
        one - number of nodes with one child (int)
        two - number of nodes with two children (int)
    -----
    """
    if node is None:
        # Base case: no node.
        one = 0
        two = 0
        zero = 0
    elif node._left is None and node._right is None:
        # Base case: node has no children.
        one = 0
        two = 0
        zero = 1
    elif node._left is not None and node._right is None:
        # General case: node has a single left child.
        zero, one, two = self._node_counts_aux(node._left)
        one += 1
    elif node._left is None and node._right is not None:
        # General case: node has a single right child.
        zero, one, two = self._node_counts_aux(node._right)
        one += 1
    else:
        # General case: node has two children. Get node counts
        # from both children.
        zero_l, one_l, two_l = self._node_counts_aux(node._left)
        zero_r, one_r, two_r = self._node_counts_aux(node._right)
        zero = zero_l + zero_r
        one = one_l + one_r
        # Count the current node as one with two children.
        two = two_l + two_r + 1
    return zero, one, two

def total_depth(self):
    """

```

```

-----
Returns the total depth of a bst.
-----
Postconditions:
    returns
        the total depth count - i.e. the sum of all the node depths
        in the tree (int)
-----
"""
return self._total_depth_aux(self._root)

def _total_depth_aux(self, node):
    """
    -----
    Returns the total depth of a bst.
    -----
    Preconditions:
        node - a BST node (_BSTNode)
    Postconditions:
        returns
            the total depth count - i.e. the sum of all the node depths
            in the tree (int)
    -----
    """
    if node is None:
        total_depth = 0
    else:
        total_depth = node._height + self._total_depth_aux(node._left) + \
            self._total_depth_aux(node._right)
    return total_depth

def mirror(self):
    """
    -----
    Creates a mirror version of a BST. All nodes are swapped with nodes on
    the other side the tree. Nodes may take the place of an empty spot.
    The resulting tree is a mirror image of the original tree. (Note that
    the mirrored tree is not a BST.) The original BST is unchanged.
    Use: tree = bst.mirror()
    -----
    Postconditions:
        returns
            tree - a mirror version of subtree of node.
    -----
    """
    tree = BST()
    tree._root = self._mirror_aux(self._root)
    return tree

def _mirror_aux(self, node):
    """
    -----
    Creates a mirror version of a BST. All nodes are swapped with nodes on
    the other side the tree. Nodes may take the place of an empty spot.
    The resulting tree is a mirror image of the original tree. (Note that
    the mirrored BST is no longer a BST itself.)
    Use: self._mirror_aux(node)
    -----
    Preconditions:
        node - a binary tree node (_BSTNode)
    Postconditions:
        returns
            tree - a mirror version of subtree of node.
    -----
    """

```

```

"""
if node is not None:
    new_node = _BSTNode(node._data)
    new_node._right = self._mirror_aux(node._left)
    new_node._left = self._mirror_aux(node._right)
else:
    new_node = None
return new_node

def balanced(self):
    """
    -----
    Returns whether a bst is balanced, i.e. the difference in
    height between all the bst's node's left and right subtrees is <= 1.
    Use: b = bst.balanced()
    -----
    Postconditions:
        returns
        is_balanced - True if the bst is balanced, False otherwise (boolean)
    -----
    """
    is_balanced = self._balanced_aux(self._root)
    return is_balanced

def _balanced_aux(self, node):
    """
    -----
    Determines whether the BST is is_balanced.
    Private operation called only by _is_valid_aux.
    Use: b = self._balanced_aux(node)
    -----
    Preconditions:
        node - the node to check the balance of (_BSTNode)
    Postconditions:
        returns
        balanced - True if node is is balanced, False otherwise (boolean)
    -----
    """
    if node is None or node._height == 1:
        # Base case: node is empty or a leaf, so no children.
        is_balanced = True
    elif abs(self._node_height(node._left) -
             self._node_height(node._right)) > 1:
        # Base case: left or right subtree is too deep.
        is_balanced = False
    else:
        # General case: check the children of node.
        is_balanced = self._balanced_aux(node._left) and \
            self._balanced_aux(node._right)
    return is_balanced

def _node_height(self, node):
    """
    -----
    Helper function to determine the height of node - handles empty node.
    Private operation called only by _is_valid_aux.
    Use: h = self._node_height(node)
    -----
    Preconditions:
        node - the node to get the height of (_BSTNode)
    Postconditions:
        returns
        height - 0 if node is None, node._height otherwise (int)
    -----
    """

```

```

"""
if node is None:
    height = 0
else:
    height = node._height
return height

def retrieve_r(self, key):
    """
    -----
    Retrieves a _data in a BST. (Recursive)
    Use: v = bst.retrieve(key)
    -----
    Preconditions:
        key - data to search for (?)
    Postconditions:
        returns
        value - If bst contains key, returns value, else returns None.
    -----
    """
    # Find the node containing the key _data.
    value = self._retrieve_r_aux(self._root, key)
    return value

def _retrieve_r_aux(self, current, key):
    """
    -----
    Retrieves a _data in a BST.
    -----
    Preconditions:
        current - a bst node (_BSTNode)
        key - data to search for (?)
    Postconditions:
        returns
        value - contains key, else returns None (?)
    -----
    """
    if current is None:
        # Base case: at bottom of tree and key not found.
        value = None
    else:
        if key < current._data:
            value = self._retrieve_r_aux(current._left, key)
        elif key > current._data:
            value = self._retrieve_r_aux(current._right, key)
        else:
            value = deepcopy(current._data)
    return value

def average_depth(self):
    """
    -----
    Returns the average depth of a bst.
    -----
    Postconditions:
        returns
        avg-depth - total depth count divided by the number of nodes
                    in the tree (int)
    -----
    """
    count = self._count
    total_depth = self.total_depth()
    avg_depth = total_depth / count
    return avg_depth

```

```

def valid(self):
    """
    -----
    Determines if a tree is a valid BST, i.e. the values in all left nodes
    are smaller than their parent, and the values in all right nodes are
    larger than their parent, and height of any node is 1 + max height of
    its children.
    Use: b = bst.valid()
    -----
    Postconditions:
        returns
        is_valid - True if tree is a BST, False otherwise (boolean)
    -----
    """
    is_valid = self._valid_aux(self._root)
    return is_valid

def _valid_aux(self, node):
    """
    -----
    Determines if a subtree is a valid BST.
    -----
    Preconditions:
        node - a binary tree node (_BSTNode)
    Postconditions:
        returns
        is_valid - True if node is root of a valid BST, False otherwise
    (boolean)
    -----
    """
    if node is None:
        is_valid = True
    elif node._left is not None and node._left._data >= node._data \
         or node._right is not None and node._right._data <= node._data:
        # print("BST Violation at value: {}".format(node._data))
        is_valid = False
    elif node._height <= self._node_height(node._left) \
         or node._height <= self._node_height(node._right):
        # print("Height Violation at value: {}".format(node._data))
        is_valid = False
    else:
        is_valid = self._valid_aux(node._left) and \
                  self._valid_aux(node._right)
    return is_valid

def update(self, value, update):
    """
    -----
    Updates a value in a bst by applying a function to it.
    Use: bst.update(value, func)
    -----
    Preconditions:
        value - a comparable part of a data element (?)
        update - an update function compatible with value (function)
    Postconditions:
        returns
        updated - True if value is in bst and is updated, False if
        value is not in bst, but adds value to bst in that case.
        (Iterative algorithm.)
    -----
    """
    parent = None
    current = self._root

```

```

updated = False

while current is not None:
    # Find the location for the new node.
    parent = current

    if value < current._data:
        location = BST._LEFT
        current = current._left
    elif value > current._data:
        location = BST._RIGHT
        current = current._right
    else:
        # value is already in bst.
        current._data = update(current._data)
        updated = True
        location = None
        current = None

if parent is None:
    self._root = _BSTNode(value)
elif location == BST._LEFT:
    parent._left = _BSTNode(value)
elif location == BST._RIGHT:
    parent._right = _BSTNode(value)
return updated

def update_r(self, key, update):
    """
    -----
    Updates a value in a bst by applying a function to it.
    Use: bst.update(value, func)
    -----
    Preconditions:
        value - a comparable part of a data element (?)
        update - an update function compatible with value (function)
    Postconditions:
        returns
        updated - True if value is in bst and is updated, False if
        value is not in bst, but adds value to bst in that case.
        (Recursive algorithm.)
    -----
    """
    # Find the node containing the key _data.
    updated = self._update_r_aux(self._root, key, update)
    return updated

def _update_r_aux(self, node, key, update):
    """
    -----
    Updates a _data in a bst.
    -----
    Preconditions:
        node - a BST node
        key - a comparable part of a data element.
        update - an update function compatible with key.
    Postconditions:
        returns
        Attempts to key in node. Returns the full node _data if found,
        otherwise returns None. Updates the _data with update if found.
        (Recursive algorithm.)
    -----
    """
    # Find the node containing the key _data.

```

```

if node is not None:
    result = self._cf(key, node._data)

    if result == self.LESSER:
        value = self._update_r_aux(node._left, key, update)
    elif result == self.GREATER:
        value = self._update_r_aux(node._right, key, update)
    else:
        # Update the node data.
        update(node._data)
        value = deepcopy(node._data)
else:
    value = None
return value

"""
-----
"""

def join(self, bst2):
    self._root = self._join_aux(self._root, bst2._root)
    return

def _join_aux(self, node1, node2):
    return

def apply(self):
    self._apply_aux(self._root)
    return

def _apply_aux(self, node):
    if node is not None:
        self._apply_aux(node._left)
        node._data.apply()
        self._apply_aux(node._right)
    return

def inorder(self):
    """
    -----
    Generates a list of the contents of the tree in inorder order.
    Use: bst.inorder()
    -----
    Postconditions:
        returns
        a - copy of the contents of the tree in inorder (list of ?)
    -----
    """
    a = []
    self._inorder_aux(self._root, a)
    return a

def _inorder_aux(self, node, a):
    """
    -----
    Traverses node subtree in inorder.
    Private recursive operation called only by inorder.
    Use: self._inorder_aux(node, a)
    -----
    Preconditions:
        node - an BST node (_BSTNode)
        a - target list of data (list of ?)
    Postconditions:
        a contains the contents of node and its children in inorder.
    """

```

```

-----
"""
if node is not None:
    self._inorder_aux(node._left, a)
    a.append(deepcopy(node._data))
    self._inorder_aux(node._right, a)
return

def preorder(self):
    """
    -----
    Generates a list of the contents of the tree in preorder order.
    Use: bst.preorder()
    -----
    Postconditions:
        returns
        a - copy of the contents of the tree in preorder (list of ?)
    -----
    """
    a = []
    self._preorder_aux(self._root, a)
    return a

def _preorder_aux(self, node, a):
    """
    -----
    Traverses node subtree in preorder.
    Private recursive operation called only by preorder.
    Use: self._preorder_aux(node, a)
    -----
    Preconditions:
        node - an BST node (_BSTNode)
        a - target of data (list of ?)
    Postconditions:
        a contains the contents of node and its children in preorder.
    -----
    """
    if node is not None:
        a.append(deepcopy(node._data))
        self._preorder_aux(node._left, a)
        self._preorder_aux(node._right, a)
    return

def postorder(self):
    """
    -----
    Generates a list of the contents of the tree in postorder order.
    Use: bst.postorder()
    -----
    Postconditions:
        returns
        a - copy of the contents of the tree in postorder (list of ?)
    -----
    """
    a = []
    self._postorder_aux(self._root, a)
    return a

def _postorder_aux(self, node, a):
    """
    -----
    Traverses node subtree in postorder.
    Private recursive operation called only by postorder.
    Use: self._postorder_aux(node, a)

```

```

-----
Preconditions:
    node - an BST node (_BSTNode)
    a - target of data (list of ?)
Postconditions:
    a contains the contents of node and its children in postorder.
-----
"""
if node is not None:
    self._postorder_aux(node._left, a)
    self._postorder_aux(node._right, a)
    a.append(deepcopy(node._data))
return

def levelorder(self):
    """
    -----
    Copies the contents of the tree in levelorder order to a list.
    Use: values = bst.levelorder()
    -----
    Postconditions:
        returns
        values - a list containing the values of bst in levelorder.
        (list of ?)
    -----
    """
    values = []

    if self._root is not None:
        # Put the nodes for one level into a queue.
        queue = []
        queue.append(self._root)

        while len(queue) > 0:
            # Add a copy of the data to the sublist
            node = queue.pop(0)
            values.append(deepcopy(node._data))

            if node._left is not None:
                queue.append(node._left)
            if node._right is not None:
                queue.append(node._right)
    return values

"""
-----
"""

def valid_avl(self):
    """
    -----
    Determines if an AVL is valid.
    Use: b = avl.valid()
    -----
    Postconditions:
        returns:
        is_valid - True if the tree is an AVL, False otherwise.
    -----
    """
    is_valid = self.valid_avl_aux(self._root)
    return is_valid

def valid_avl_aux(self, node):
    """

```

```

-----
Helper function to determine the AVL validity of node.
Private operation called only by is_valid.
Use: b = self._is_valid_aux(node)
-----
Preconditions:
    node - the node to check the validity of (_AVLNode)
Postconditions:
    returns
    is_valid - True if node is an AVL, False otherwise (boolean)
-----
"""
if node is None or (node._left is None and node._right is None):
    # Base case: node is empty or a leaf, so tree must be an AVL.
    is_valid = True
elif abs(self._node_height(node._left) -
         self._node_height(node._right)) > 1:
    # Base case: left or right subtree is too deep.
    # print("Height Violation at value: {}".format(node._data))
    is_valid = False
elif (node._left is not None and node._left._data > node._data) \
     or (node._right is not None and node._right._data < node._data):
    # Base case: does not follow the BST property.
    # print("Binary Tree Violation")
    is_valid = False
else:
    # General case: check the nodes children for validity.
    is_valid = self.valid_avl_aux(node._left) and \
               self.valid_avl_aux(node._right)
return is_valid

def preorder_i(self):
    """
    -----
    Generates a list of the contents of the tree in preorder order.
    (Iterative algorithm)
    Use: bst.preorder_i()
    -----
    Postconditions:
        returns
        a - copy of the contents of the tree in preorder (list of ?)
    -----
    """
    a = []
    stack = Stack()
    stack.push(self._root)

    while not stack.empty():
        node = stack.pop()

        if node is not None:
            a.append(deepcopy(node._data))
            stack.push(node._right)
            stack.push(node._left)
    return a

def count(self):
    """
    -----
    Returns the number of nodes in a BST.
    Use: number = bst.count()
    -----
    Postconditions:
        returns

```

```

        number - count of nodes in tree (int)
    """
    number = self._count_aux(self._root)
    return number

def _count_aux(self, node):
    """
    -----
    Returns the number of nodes in a BST subtree.
    -----
    Preconditions:
        node - a BST node (_BSTNode)
    Postconditions:
        returns
            number - count of nodes in the current subtree (int)
    """
    if node is None:
        # Base case: node does not exist
        number = 0
    else:
        # General case: node exists.
        number = 1 + self._count_aux(node._left) + \
            self._count_aux(node._right)
    return number

def count_apply(self, func):
    """
    -----
    Returns the number of values in a BST where func(value) is True.
    Use: number = bst.count_apply(func)
    -----
    Preconditions:
        func - a function that given a value in the bst returns
            True for some condition, otherwise returns False.
    Postconditions:
        returns
            number - count of nodes in tree where func(value) is True (int)
    """
    number = self._count_apply_aux(func, self._root)
    return number

def _count_apply_aux(self, func, node):
    """
    -----
    Returns the number of nodes in a BST subtree
    where the result of calling func(value) is True.
    Use: number = self._count_apply_aux(func, node)
    -----
    Preconditions:
        node - a BST node (_BSTNode)
    Postconditions:
        returns
            number - count of nodes in the current subtree (int)
    """
    if node is None:
        # Base case: node does not exist
        number = 0
    else:
        # General case: node exists.
        if func(node._data):

```

```
        number = 1
    else:
        number = 0
    number = number + self._count_apply_aux(func, node._left) + \
        self._count_apply_aux(func, node._right)
return number

def __iter__(self):
    """
    -----
    Generates a Python iterator. Iterates through a BST node
    in level order.
    Use: for v in bst:
    -----
    Postconditions:
        yields
        value - the values in the BST node and its children (?)
    -----
    """
    if self._root is not None:
        # Put the nodes for one level into a queue.
        queue = []
        queue.append(self._root)

        while len(queue) > 0:
            # Add a copy of the data to the sublist
            node = queue.pop(0)
            yield node._data

            if node._left is not None:
                queue.append(node._left)
            if node._right is not None:
                queue.append(node._right)
```

```
"""
-----
list_linked.py
Linked version of the list ADT.
-----
Author:  David Brown
ID:      999999999
Email:   dbrown@wlu.ca
__updated__ = "2018-03-21"
-----
"""
from copy import deepcopy

class _ListNode:

    def __init__(self, value, next_):
        """
        -----
        Initializes a list node.
        Use: node = _ListNode(value, _next)
        -----
        Preconditions:
            _data - data value for node (?)
            _next - another list node (_ListNode)
        Postconditions:
            Initializes a list node that contains a copy of value
            and a link to the next node in the list.
        -----
        """
        self._data = deepcopy(value)
        self._next = next_
        return

class List:

    def __init__(self):
        """
        -----
        Initializes an empty list.
        Use: l = List()
        -----
        Postconditions:
            Initializes an empty list.
        -----
        """
        self._front = None
        self._count = 0
        return

    def empty(self):
        """
        -----
        Determines if the list is empty.
        Use: b = l.empty()
        -----
        Postconditions:
            returns
            True if the list is empty, False otherwise.
        -----
        """
        return self._front is None
```

```

def is_empty(self):
    """
    -----
    Determines if the list is empty.
    Use: b = l.is_empty()
    -----
    Postconditions:
        returns
        True if the list is empty, False otherwise.
    -----
    """
    return self._front is None

def __len__(self):
    """
    -----
    Returns the size of the list.
    Use: n = len(l)
    -----
    Postconditions:
        returns
        the number of values in the list.
    -----
    """
    return self._count

def insert(self, i, value):
    """
    -----
    Inserts a copy of value into the list at index i.
    Use: l.insert(i, value)
    -----
    Preconditions:
        i - index value (int)
        value - a data element (?)
    Postconditions:
        a copy of value is added to index i, all other values are pushed
        right
        If i outside of range of length of list, appended to end
    -----
    """
    if i < 0:
        # negative index
        i = self._count + i

    n = 0
    previous = None
    current = self._front

    while n < i and current is not None:
        # find the proper location in the list
        previous = current
        current = current._next
        n += 1

    if previous is None:
        # Insert a new node into the front of the list.
        self._front = _ListNode(value, self._front)
    else:
        # Insert a new node elsewhere in the list
        previous._next = _ListNode(value, current)
    self._count += 1
    return

```

```

def insert_front(self, value):
    """
    -----
    Inserts a copy of value into the front of the list.
    Use: l.insert_front(value)
    -----
    Preconditions:
        value - a data element. (?)
    Postconditions:
        value is added to the front of the list.
    -----
    """
    self.insert(0, value)
    return

def _linear_search(self, key):
    """
    -----
    Searches for the first occurrence of key in the list.
    Private helper methods - used only by other ADT methods.
    Use: p, c, i = self._linear_search(key)
    -----
    Preconditions:
        key - a partial data element (?)
    Postconditions:
        returns
        previous - pointer to the node previous to the node containing key
        current - pointer to the node containing key (_ListNode)
        index - index of the node containing key, -1 if key not found (int)
    -----
    """
    previous = None
    current = self._front
    index = 0

    while current is not None and current._data != key:
        previous = current
        current = current._next
        index += 1

    if current is None:
        index = -1

    return previous, current, index

def remove(self, key):
    """
    -----
    Finds, removes, and returns the first value in list that matches key.
    Use: value = l.remove(key)
    -----
    Preconditions:
        key - a partial data element (?)
    Postconditions:
        returns
        value - the full value matching key, otherwise None (?)
    -----
    """
    # search list for key.
    previous, current, _ = self._linear_search(key)

    if current is None:
        # Key is not found.

```

```

        value = None
    else:
        value = current._data
        self._count -= 1

        if previous is None:
            # Remove the first element.
            self._front = current._next
        else:
            # Remove any other element.
            previous._next = current._next
    return value

def remove_front(self):
    """
    -----
    Removes the first node in the list.
    Use: value = l.remove_front()
    -----
    Postconditions:
        returns
        value - the first value in the list (?)
    -----
    """
    assert self._front is not None, "Cannot remove from an empty list"

    value = self._front._data
    self._front = self._front._next
    self._count -= 1
    return value

def remove_many(self, key):
    """
    -----
    Finds and removes all values in the list that match key.
    Use: l.remove_many(key)
    -----
    Preconditions:
        key - a data element (?)
    Postconditions:
        Removes all values matching key.
    -----
    """
    previous = None
    current = self._front

    while current is not None:

        if current._data == key:
            # Do not update previous
            self._count -= 1

            if previous is None:
                # Remove the first element.
                self._front = current._next
            else:
                # Remove any other element.
                previous._next = current._next
        else:
            previous = current
            current = current._next
    return

def find(self, key):

```

```

"""
-----
Finds and returns a copy of the first value in list that matches key.
Use: value = l.find(key)
-----
Preconditions:
    key - a partial data element (?)
Postconditions:
    returns
    value - a copy of the full value matching key, otherwise None (?)
-----
"""
_, current, _ = self._linear_search(key)

if current is not None:
    value = deepcopy(current._data)
else:
    value = None
return value

def peek(self):
    """
    -----
    Returns a copy of the first value in list.
    Use: value = l.peek()
    -----
    Postconditions:
        returns
        value - a copy of the first value in the list (?)
    -----
    """
    assert self._front is not None, "Cannot peek at an empty list"

    value = deepcopy(self._front._data)
    return value

def index(self, key):
    """
    -----
    Finds location of a value by key in list.
    Use: n = l.index(key)
    -----
    Preconditions:
        key - a partial data element (?)
    Postconditions:
        returns
        i - the index of the location of key in the list, -1 if
            key is not in the list.
    -----
    """
    _, _, i = self._linear_search(key)
    return i

def _valid_index(self, i):
    """
    -----
    Private helper method to validate an index value.
    Python index values can be positive or negative and range from
    -len(list) to len(list) - 1
    Use: assert self._valid_index(i)
    -----
    Preconditions:
        i - an index value (int)
    Postconditions:
    """

```

```

        returns
        True if i is a valid index, False otherwise.
    """
    -----
    n = self._count
    return -n <= i < n

def __getitem__(self, i):
    """
    -----
    Returns a copy of the nth element of the list.
    Use: value = l[i]
    -----
    Preconditions:
        i - index of the element to access (int)
    Postconditions:
        returns
        value - the i-th element of list (?)
    -----
    """
    assert self._valid_index(i), "Invalid index value"

    current = self._front

    if i < 0:
        # negative index - convert to positive
        i = self._count + i
    j = 0

    while j < i:
        current = current._next
        j += 1

    value = deepcopy(current._data)
    return value

def __setitem__(self, i, value):
    """
    -----
    Places a copy of value into the list at position n.
    Use: l[i] = value
    -----
    Preconditions:
        i - index of the element to access (int)
        value - a data value (?)
    Postconditions:
        The i-th element of list contains a copy of value. The
        existing value at i is overwritten.
    -----
    """
    assert self._valid_index(i), "Invalid index value"

    current = self._front

    if i < 0:
        # negative index - convert to positive
        i = self._count + i
    j = 0

    while j < i:
        current = current._next
        j += 1

    current._data = deepcopy(value)

```

```

    return

def __contains__(self, key):
    """
    -----
    Determines if the list contains key.
    Use: b = key in l
    -----
    Preconditions:
        key - a partial data element (?)
    Postconditions:
        returns
        True if the list contains key, False otherwise.
    -----
    """
    _, current, _ = self._linear_search(key)
    return current is not None

def max(self):
    """
    -----
    Finds the maximum value in list.
    Use: value = l.max()
    -----
    Postconditions:
        returns
        max_data - a copy of the maximum value in the list (?)
    -----
    """
    assert self._front is not None, "Cannot find maximum of an empty list"

    max_data = self._front._data
    current = self._front._next

    while current is not None:
        if max_data < current._data:
            max_data = current._data
            current = current._next
    max_data = deepcopy(max_data)
    return max_data

def min(self):
    """
    -----
    Finds the minimum value in list.
    Use: value = l.min()
    -----
    Postconditions:
        returns
        min_data - a copy of the minimum value in the list (?)
    -----
    """
    assert self._front is not None, "Cannot find maximum of an empty list"

    min_data = self._front._data
    current = self._front._next

    while current is not None:
        if min_data > current._data:
            min_data = current._data
            current = current._next
    min_data = deepcopy(min_data)
    return min_data

```

```

def count(self, key):
    """
    -----
    Finds the number of times key appears in list.
    Use: n = l.count(key)
    -----
    Preconditions:
        key - a partial data element (?)
    Postconditions:
        returns
        number - number of times key appears in list (int)
    -----
    """
    number = 0
    current = self._front

    while current is not None:
        if key == current._data:
            number += 1
        current = current._next
    return number

def reverse(self):
    """
    -----
    Reverses the order of the elements in list.
    Use: l.reverse()
    -----
    Postconditions:
        The contents of list are reversed in order with respect
        to their order before the method was called.
    -----
    """
    new_front = None

    while self._front is not None:
        temp = self._front._next
        self._front._next = new_front
        new_front = self._front
        self._front = temp

    self._front = new_front
    return

def reverse_r(self):
    """
    -----
    Reverses the order of the elements in list.
    Use: l.reverse_r()
    -----
    Postconditions:
        The contents of list are reversed in order with respect
        to their order before the method was called.
    -----
    """
    self._reverse_r_aux(None)
    return

def _reverse_r_aux(self, new_front):
    """
    -----
    Auxiliary recursive function for reverse_r (fruitful).
    Use: l.reverse_r_aux(new_front)
    -----

```

```

Preconditions:
    new_front - the new front node to process (_ListNode)
Postconditions:
    The list is in reverse order.
-----
"""
if self._front is None:
    self._front = new_front
else:
    temp = self._front._next
    self._front._next = new_front
    new_front = self._front
    self._front = temp
    self._reverse_r_aux(new_front)
return

def reverse_r2(self):
    """
    -----
    Reverses the order of the elements in list.
    Use: l.reverse_r()
    -----
    Postconditions:
        The contents of list are reversed in order with respect
        to their order before the method was called.
    -----
    """
    self._front = self._reverse_r2_aux(self._front, None)
    return

def _reverse_r2_aux(self, current, previous):
    """
    -----
    Auxiliary recursive function for reverse_r (fruitful).
    -----
    Preconditions:
        current - the current node to process (_ListNode)
        previous - the node previous to current (_ListNode)
    Postconditions:
        returns
        new_front - the new front of the list so far (_ListNode)
    -----
    """
    if current is None:
        new_front = previous
    else:
        new_front = self._reverse_r2_aux(current._next, current)
        current._next = previous
    return new_front

def append(self, value):
    """
    -----
    Appends a copy of value to the end of the List.
    Use: l.append(value)
    -----
    Preconditions:
        value - a data element (?)
    Postconditions:
        a copy of value is added to the end of the List.
    -----
    """
    previous = None
    current = self._front

```

```

while current is not None:
    # Find the last node in the list
    previous = current
    current = current._next

if previous is None:
    # Insert into the front of the list.
    self._front = _ListNode(deepcopy(value), self._front)
else:
    # Insert elsewhere in the list
    previous._next = _ListNode(deepcopy(value), current)
self._count += 1
return

def clean(self):
    """
    -----
    Removes duplicates from the list. (iterative algorithm)
    Use: l.clean()
    -----
    Postconditions:
        The list contains one and only one of each value formerly present
        in the list. The first occurrence of each value is preserved.
    -----
    """
    key_node = self._front

    while key_node is not None:
        # Loop through every node - compare each node with the rest
        previous = key_node
        current = key_node._next

        while current is not None:
            # Always search to the end of the list (may have > 1 duplicate)
            if current._data == key_node._data:
                # Remove the current node by connecting the node before it
                # to the node after it.
                self._count -= 1
                previous._next = current._next
            else:
                previous = current
                # Move to the _next node.
                current = current._next
            # Check for duplicates of the _next remaining node in the list
            key_node = key_node._next
        return

def pop(self, *i):
    """
    -----
    Finds, removes, and returns the value in list whose index matches i.
    Use: value = l.pop(i)
    -----
    Preconditions:
        i - an array of arguments (?)
            i[0], if it exists, is the index
    Postconditions:
        returns
            value - if i exists, the value at position i, otherwise the last
                    value in the list, value is removed from the list (?)
    -----
    """
    assert self._front is not None, "Cannot pop from an empty list"

```

```

assert len(i) <= 1, "No more than 1 argument allowed"

previous = None
current = self._front

if len(i) == 1:
    if i[0] < 0:
        # index is negative
        i[0] = self._count + i[0]
        j = 0

        while j < i[0]:
            previous = current
            current = current._next
            j += 1
    else:
        # find and pop the last element
        j = 0

        while j < (self._count - 1):
            previous = current
            current = current._next
            j += 1

value = current._data

if previous is None:
    # Update the front
    self._front = current._next
else:
    # Update any other node
    previous._next = current._next
self._count -= 1
return value

def _swap(self, pln, prn):
    """
    -----
    Swaps the position of two nodes.
    Use: self._swap(pln, prn)
    -----
    Preconditions:
        pln - node before list node to swap (_ListNode)
        prn - node before list node to swap (_ListNode)
    Postconditions:
        The nodes in pln.next and prn.next have been swapped,
        and all links to them updated.
    -----
    """
    if pln is not prn:
        # Swap only if two nodes are not the same node

        if pln is None:
            # Make r the new front
            l = self._front
            self._front = prn._next
        else:
            l = pln._next
            pln._next = prn._next

        if prn is None:
            # Make l the new front
            r = self._front

```

```

        self._front = l
    else:
        r = prn._next
        prn._next = l

    # Swap next pointers
    # l._next, r._next = r._next, l._next
    temp = l._next
    l._next = r._next
    r._next = temp
return

def identical(self, rs):
    """
    -----
    Determines whether two lists are identical. (iterative version)
    Use: b = l.identical(rs)
    -----
    Preconditions:
        rs - another list (List)
    Postconditions:
        returns
        is_identical - True if this list contains the same values as
            rs in the same order, otherwise False.
    -----
    """
    if self._count != rs._count:
        is_identical = False
    else:
        current1 = self._front
        current2 = rs._front

        while current1 is not None and current1._data == current2._data:
            current1 = current1._next
            current2 = current2._next

        is_identical = current1 is None
    return is_identical

def identical_r(self, rs):
    """
    -----
    Determines whether two lists are identical. (recursive version)
    Use: b = l.identical_r(rs)
    -----
    Preconditions:
        rs - another list (List)
    Postconditions:
        returns
        is_identical - True if this list contains the same values as rs
            in the same order, otherwise False.
    -----
    """
    if self._count != rs._count:
        is_identical = False
    else:
        is_identical = self._identical_r_aux(self._front, rs._front)
    return is_identical

def _identical_r_aux(self, node1, node2):
    """
    -----
    An auxiliary function for identical_r
    -----
    """

```

```

Preconditions:
    node1 - a List node (_ListNode)
    node2 - a List node (_ListNode)
Postconditions:
    returns
        identical - True if node1 contains the same values as node2
                    in the same order, otherwise False.
-----
"""
if node1 is None:
    is_identical = True
elif node1._data != node2._data:
    is_identical = False
else:
    is_identical = self._identical_r_aux(node1._next,
                                         node2._next)

return is_identical

def split_alt(self):
    """
    -----
    Split a List into two parts. even contains the even indexed
    elements, odd contains the odd indexed elements.
    Order of even and odd is not significant. (iterative version)
    Use: even, odd = l.split_alt()
    -----
    Postconditions:
        returns
            even - the even indexed elements of the list (List)
            odd - the odd indexed elements of the list (List)
            The List is empty.
    -----
    """
    even = List()
    odd = List()
    i = 0

    while self._front is not None:

        if i % 2 == 0:
            even._move_front(self)
        else:
            odd._move_front(self)
        i += 1

    even.reverse()
    odd.reverse()
    return even, odd

def split_alt_r(self):
    """
    -----
    Split a list into two parts. even contains the even indexed
    elements, odd contains the odd indexed elements.
    Order of even and odd is not significant. (recursive version)
    Use: even, odd = l.split_alt()
    -----
    Postconditions:
        returns
            even - the even numbered elements of the list (List)
            odd - the odd numbered elements of the list (List)
            The List is empty.
    -----
    """

```

```

    even = List()
    odd = List()
    self._split_alt_r_aux(even, odd)
    return even, odd

def _split_alt_r_aux(self, even, odd):
    """
    -----
    Split a list into two parts. even contains the even indexed
    elements, odd contains the odd numbered elements.
    Order of even and odd is not significant.
    -----
    Postconditions:
        returns
        even - the even numbered elements of the list (List)
        odd - the odd numbered elements of the list (List)
        The List is empty.
    -----
    """
    if self._front is not None:
        even._move_front(self)
        # Reverse the order of the arguments.
        self._split_alt_r_aux(odd, even)
    return

def _linear_search_r(self, key):
    """
    -----
    Searches for the first occurrence of key in the list.
    Private helper methods - used only by other ADT methods.
    (recursive version)
    Use: p, c, i = self._linear_search(key)
    -----
    Preconditions:
        key - a partial data element (?)
    Postconditions:
        returns
        previous - pointer to the node previous to the node containing key
    (_ListNode)
        current - pointer to the node containing key (_ListNode)
        index - index of the node containing key, -1 if key not found (int)
    -----
    """
    previous, current, index = self._linear_search_r_aux(
        key, None, self._front, 0)
    return previous, current, index

def _linear_search_r_aux(self, key, previous, current, index):
    """
    -----
    Auxiliary method for _linear_search.
    Use: p, c, i = self._linear_search(key, previous, current, index)
    -----
    Preconditions:
        key - a partial data element (?)
        previous - pointer to the node previous to the node containing key
    (_ListNode)
        current - pointer to the node containing key (_ListNode)
        index - index of the node containing key, -1 if key not found (int)
    Postconditions:
        returns
        previous - pointer to the node previous to the node containing key
    (_ListNode)
        current - pointer to the node containing key (_ListNode)
    """

```

```

        index - index of the node containing key, -1 if key not found (int)
-----
"""
# Note: can be done with three parameters, if these cases added:
#     if previous is None:
#         current = self._front
#     else:
#         current = previous._next

if current is None:
    index = -1
elif current._data != key:
    previous, current, index = self._linear_search_r_aux(
        key, current, current._next, index + 1)

return previous, current, index

def intersection(self, rs):
    """
    -----
    Returns a list that contains only values that appear in both
    the current List and rs. (iterative version)
    Use: l2 = l1.intersection(rs)
    -----
    Preconditions:
        rs - another List (List)
    Postconditions:
        returns
        new_list - A List that contains only the values found in both
        the current List and rs. Values do not repeat. (List)
    -----
    """
    new_list = List()
    temp = rs._front

    while temp is not None:
        _, current, _ = self._linear_search(temp._data)

        if current is not None:
            # Value exists in both lists.
            _, current, _ = new_list._linear_search(temp._data)

            if current is None:
                # Value does not appear in new list.
                new_list._front = _ListNode(temp._data, new_list._front)
                new_list._count += 1

            temp = temp._next
    return new_list

def intersection_r(self, rs):
    """
    -----
    Returns a list that contains only values that appear in both
    the current List and rs. (recursive version)
    Use: l2 = l1.intersection_r(rs)
    -----
    Preconditions:
        rs - another List (List)
    Postconditions:
        returns
        new_list - A List that contains only the values found in both
        the current List and rs. Values do not repeat. (List)
    -----
    """

```

```

"""
new_list = List()
self._intersection_r_aux(rs._front, new_list)
new_list.reverse()
return new_list

def _intersection_r_aux(self, rs_node, new_list):
    """
    -----
    Auxiliary function for intersection_r.
    Use: self._intersection_r_aux(rs_node, new_list)
    -----
    Preconditions:
        rs_node - right side list node (_ListNode)
        new_list - the list to contain the intersection (List)
    Postconditions:
        new_list contains one copy of values common to current list
        and rs
    -----
    """
    if rs_node is not None:
        _, current, _ = self._linear_search(rs_node._data)

        if current is not None:
            # Value exists in both lists.
            _, current, _ = new_list._linear_search(rs_node._data)

            if current is None:
                # Value does not appear in new list.
                new_list.insert(0, rs_node._data)
            self._intersection_r_aux(rs_node._next, new_list)
    return

def union(self, rs):
    """
    -----
    Returns a list that contains all values in both
    the current List and rs. (iterative algorithm)
    Use: nl = l.union(rs)
    -----
    Preconditions:
        rs - another list (List)
    Postconditions:
        returns
        new_list - contains all values found in both the current
        List and rs. Values do not repeat. (List)
    -----
    """
    new_list = List()
    temp = self._front

    while temp is not None:
        _, current, _ = new_list._linear_search(temp._data)

        if current is None:
            # Value does not exist in new list.
            new_list.insert(0, temp._data)
            temp = temp._next

    temp = rs._front

    while temp is not None:
        _, current, _ = new_list._linear_search(temp._data)

```

```

        if current is None:
            # Value does not exist in new list.
            new_list.insert(0, temp._data)

        temp = temp._next
    new_list.reverse()
    return new_list

def union_r(self, rs):
    """
    -----
    Returns a list that contains all values in both
    the current List and rs. (recursive algorithm)
    Use: nl = l.union(rs)
    -----
    Preconditions:
        rs - another list (List)
    Postconditions:
        returns
        new_list - contains all values found in both the current
        List and rs. Values do not repeat. (List)
    -----
    """
    new_list = List()
    new_list._union_r_aux(self._front)
    new_list._union_r_aux(rs._front)
    new_list.reverse()
    return new_list

def _union_r_aux(self, rs_node):
    """
    -----
    Auxiliary function for union_r.
    Use: self._union_r_aux(rs_node)
    -----
    Preconditions:
        rs_node - right side list rs_node (_ListNode)
    Postconditions:
        self contains one copy of all values in rs_node
    -----
    """
    if rs_node is not None:
        _, current, _ = self._linear_search(rs_node._data)

        if current is None:
            # Value does not exist in current list.
            self.insert(0, rs_node._data)
            self._union_r_aux(rs_node._next)
    return

def clean_r(self):
    """
    -----
    Removes duplicates from the list. (recursive algorithm)
    Use: l.clean_r()
    -----
    Postconditions:
        The list contains one and only one of each value formerly present
        in the list. The first occurrence of each value is preserved.
    -----
    """
    self._clean_r_aux(self._front)
    return

```

```

def _clean_r_aux(self, key_node):
    """
    -----
    Auxiliary function to remove duplicates from a list.
    Use: self._clean_r_aux(key_node)
    -----
    Preconditions:
        key_node - node containing the current key value to be searched
                for (_ListNode)
    Postconditions:
        All nodes containing the same value as key_node are removed
        from the rest of the list.
    -----
    """
    # Compares key_node against the rest of the list.
    if key_node is not None:
        # Update the _next node in the list.
        self._clean_r_aux_2(key_node._data, key_node)
        # Move to the _next key node in the list for comparison.
        self._clean_r_aux(key_node._next)
    return

def _clean_r_aux_2(self, data, previous):
    """
    -----
    Auxiliary function to remove duplicates from a list.
    Use: self._clean_r_aux_2(data, node)
    -----
    Preconditions:
        data - the key value to be searched for (?)
        previous - the node pointing to the rest of the list to
                be searched for data (_ListNode)
    Postconditions:
        All nodes containing the same value as data are removed from
        the rest of the list.
    -----
    """
    # Removes all duplicates to the key node from the rest of the list.
    if previous._next is not None:
        current = previous._next

        if current._data == data:
            # Remove the current node.
            self._count -= 1
            previous._next = current._next
            self._clean_r_aux_2(data, previous)
        else:
            # Move to the next node.
            self._clean_r_aux_2(data, previous._next)
    return

def split_th(self):
    """
    -----
    Splits list into two parts. ls contains the first half,
    rs the second half. Uses Tortoise / Hare algorithm.
    Current list is empty.
    Use: ls, rs = l.split_th()
    -----
    Postconditions:
        returns
        ls - a new List with >= 50% of the original List (List)
        rs - a new List with <= 50% of the original List (List)
    -----
    """

```

```

"""
ls = List()
rs = List()
# Initialize both temporary pointers to beginning of the list.
hare = self._front
tortoise = self._front

while hare is not None and hare._next is not None:
    # Move hare down two nodes.
    hare = hare._next._next

    if hare is not None:
        # Update tortoise only if hare is not None.
        tortoise = tortoise._next

# Split the list.
if tortoise is not None:
    ls._front = self._front
    rs._front = tortoise._next
    tortoise._next = None
    rs._count = self._count // 2
    ls._count = self._count - rs._count
    self._count = 0
return ls, rs

def split(self):
    """
    -----
    Splits list into two parts. ls contains the first half,
    rs the second half. Uses counting algorithm.
    Current list is empty.
    Use: ls, rs = l.split_th()
    -----
    Postconditions:
        returns
        ls - a new List with >= 50% of the original List (List)
        rs - a new List with <= 50% of the original List (List)
    -----
    """
    ls = List()
    rs = List()
    # Find the middle of the list.
    middle = self._count // 2 + self._count % 2 - 1
    current = self._front
    i = 0

    while i < middle:
        current = current._next
        i += 1

    if current is not None:
        ls._front = self._front
        rs._front = current._next
        current._next = None
        rs._count = self._count // 2
        ls._count = self._count - rs._count
        self._count = 0
    return ls, rs

def split_apply(self, func):
    """
    -----
    Splits list into two parts. ls contains all the values
    where the result of calling func(value) is True,

```

```

rs contains the remaining values.
Use: ls, rs = l.split_apply(func)
-----
Postconditions:
    returns
    ls - a new List with values where func(value) is True (List)
    rs - a new List with values where func(value) is False (List)
    self is empty. Order of values in new lists is maintained.
-----
"""
ls = List()
rs = List()
# Split
while self._front is not None:

    if func(self._front._data):
        ls._move_front(self)
    else:
        rs._move_front(self)
ls.reverse()
rs.reverse()
return ls, rs

def copy(self):
    """
    -----
    Duplicates the current list to a new list in the same order.
    (iterative version)
    Use: new_list = l.copy()
    -----
    Postconditions:
        returns
        new_list - a copy of self (List)
    -----
    """
    new_list = List()

    if self._front is not None:
        # Set up the new list front.
        new_list._front = _ListNode(self._front._data, None)
        previous = new_list._front
        current = self._front._next

        while current is not None:
            # Add a node in the new list.
            new_node = _ListNode(current._data, None)
            previous._next = new_node
            previous = new_node
            # Move to the next node in the current list.
            current = current._next
        new_list._count = self._count
    return new_list

def copy_r(self):
    """
    -----
    Duplicates the current list to a new list in the same order.
    (recursive version)
    Use: new_list = l.copy()
    -----
    Postconditions:
        returns
        new_list - a copy of self (List)
    -----
    """

```

```

"""
new_list = List()
new_list._front = self._copy_r_aux(self._front)
new_list._count = self._count
return new_list

def _copy_r_aux(self, current):
"""
-----
Auxiliary function for copy_r. Private helper function
Use: new_node = self._copy_r_aux(current)
-----
Preconditions:
    current - current list node to be copied (_ListNode)
Postconditions:
    returns
    new_node - new node of new list (_ListNode)
-----
"""
if current is None:
    new_node = None
else:
    next_node = self._copy_r_aux(current._next)
    new_node = _ListNode(current._data, next_node)
return new_node

def reverse_pc(self):
"""
-----
Reverses a list through partitioning and concatenation.
Use: l.reverse_pc()
-----
Postconditions:
    The contents of the current list are reversed.
-----
"""
self._front = self._reverse_pc_aux(self._front)

def _reverse_pc_aux(self, current):
"""
-----
Auxiliary function for reverse_pc.
Use: node = self._reverse_pc_aux(node)
-----
Preconditions:
    current - the current node to process (_ListNode)
Postconditions:
    returns
    result -
    Recursively split and concatenate the list until the end
    of the list is reached.
-----
"""
if current is None:
    result = None
else:
    head, tail = self._partition(current)
    tail = self._reverse_pc_aux(tail)
    result = self._concatenate(tail, head)
return result

def _partition(self, current):
"""
-----

```

```

Partitions a list at the current node.
-----
Parameters:
    current - the current node to process
Preconditions:
    current - a valid node dictionary
Postconditions:
    Partition a list by removing the first node as 'head' and
    naming the remainder of the list 'tail'. 'node' cannot be None.
-----
"""
tail = current._next
head = current
head._next = None
return head, tail

def _concatenate(self, tail, head):
    """
    -----
    [function description]
    -----
Parameters:
    head - the head node of a list
    tail - the tail node of a list
Preconditions:
    head - a valid node dictionary
    tail - a valid node dictionary
Postconditions:
    Appends the 'head' node to the end of 'tail'.
    -----
    """
    previous = None
    current = tail

    # Find the end of 'tail'.
    while current is not None:
        previous = current
        current = current._next

    # Append 'head' to the end of 'tail'.
    if previous is None:
        tail = head
    else:
        previous._next = head
    # 'tail' is the new head of the list.
    return tail

def _move_front(self, rs):
    """
    -----
    Moves the front node from the rs List to the front
    of the current List. Private helper method.
    Use: self._move_front(rs)
    -----
Preconditions:
    rs - a non-empty linked List (List)
Postconditions:
    The current List contains the old front of the rs List and
    its count is updated. The rs List front and count are updated.
    -----
    """
    assert rs._front is not None, \
        "Cannot move the front of an empty List"

```

```

# Move the front node from rs to self
temp = rs._front
rs._front = rs._front._next
temp._next = self._front
self._front = temp
# Update the list counts
self._count += 1
rs._count -= 1
return

def combine(self, s2):
    """
    -----
    Combines contents of two lists into a third.
    Use: new_list = l1.combine(s2)
    -----
    Preconditions:
        s2- an linked-based List (List)
    Postconditions:
        returns
        new_list - the contents of the current List and s2
        are interlaced into new_list - current List and s2
        are empty (List)
    -----
    """
    new_list = List()

    while self._front is not None and s2._front is not None:
        new_list._move_front(self)
        new_list._move_front(s2)

    while self._front is not None:
        new_list._move_front(self)

    while s2._front is not None:
        new_list._move_front(s2)

    new_list.reverse()
    return new_list

def combine_r(self, rs):
    """
    -----
    Combines contents of two lists into a third.
    Use: new_list = l1.combine(rs)
    -----
    Preconditions:
        rs- an linked-based List (List)
    Postconditions:
        returns
        new_list - the contents of the current List and rs
        are interlaced into new_list - current List and rs
        are empty (List)
    -----
    """
    new_list = List()
    new_list._combine_r_aux(self, rs)
    new_list.reverse()
    return new_list

def _combine_r_aux(self, l1, l2):
    """
    -----
    Auxiliary recursive function for combine_r.
    
```

```
Use: self._combine_aux(l1, l2)
-----
Preconditions:
    l1 - (List)
    l2 - (List)
Postconditions:
    Contents of l1 and l2 are interlaced into the current List.
    l1 and l2 are empty.
-----
"""
if l1._front is not None and l2._front is not None:
    self._move_front(l1)
    self._combine_r_aux(l2, l1)
elif l1._front is not None:
    self._move_front(l1)
    self._combine_r_aux(l1, l2)
elif l2._front is not None:
    self._move_front(l2)
    self._combine_r_aux(l2, l1)
return

def __iter__(self):
    """
    USE FOR TESTING ONLY
    -----
    Generates a Python iterator. Iterates through the list
    from front to rear.
    Use: for v in s:
    -----
    Postconditions:
        yields
        value - the next value in the list (?)
    -----
    """
    current = self._front

    while current is not None:
        yield current._data
        current = current._next
```

```
"""
-----
queue_circular.py
Array version of the CircularQueue ADT.
-----
Author:  David Brown
ID:     999999999
Email:  dbrown@wlu.ca
__updated__ = "2018-02-03"
-----
"""

from copy import deepcopy

class CircularQueue:

    def __init__(self, max_size):
        """
        -----
        Initializes an empty queue. Data is stored in a list.
        Use: cq = CircularQueue(max_size)
        -----
        Preconditions:
            max_size - maximum size of the queue (int > 0)
        Postconditions:
            Initializes an empty queue.
        -----
        """
        assert max_size > 0, "CircularQueue size must be > 0"

        self._max_size = max_size
        self._values = [None] * self._max_size
        self._front = 0
        self._rear = 0
        self._count = 0
        return

    def empty(self):
        """
        -----
        Determines if the queue is empty.
        Use: b = cq.empty()
        -----
        Postconditions:
            Returns True if the queue is empty, False otherwise.
        -----
        """
        return self._count == 0

    def is_empty(self):
        """
        -----
        Determines if the queue is empty.
        Use: b = cq.is_empty()
        -----
        Postconditions:
            Returns True if the queue is empty, False otherwise.
        -----
        """
        return self._count == 0

    def full(self):
        """
        -----

```

```

Determines if the queue is full.
Use: b = cq.full()
-----
Postconditions:
    Returns True if the queue is full, False otherwise.
-----
"""
return self._count == self._max_size

def is_full(self):
    """
    -----
    Determines if the queue is full.
    Use: b = cq.is_full()
    -----
    Postconditions:
        Returns True if the queue is full, False otherwise.
    -----
    """
    return self._count == self._max_size

def __len__(self):
    """
    -----
    Returns the length of the queue.
    Use: n = len(cq)
    -----
    Postconditions:
        Returns the number of values in the queue.
    -----
    """
    return self._count

def insert(self, value):
    """
    -----
    Inserts a copy of value into the queue.
    Use: cq.insert( value )
    -----
    Preconditions:
        value - a data element (?)
    Postconditions:
        a copy of value is added to the rear of the queue.
    -----
    """
    assert self._count < self._max_size, "queue is full"

    self._values[self._rear] = deepcopy(value)
    self._rear = (self._rear + 1) % self._max_size
    self._count += 1
    return

def remove(self):
    """
    -----
    Removes and returns value from the queue.
    Use: v = cq.remove()
    -----
    Postconditions:
        returns
        value - the value at the front of the queue - the value is
        removed from the queue (?)
    -----
    """

```

```
    assert self._count > 0, "Cannot remove from an empty queue"

    value = self._values[self._front]
    self._values[self._front] = None
    self._front = (self._front + 1) % self._max_size
    self._count -= 1
    return value

def peek(self):
    """
    -----
    Peeks at the front of queue.
    Use: v = cq.peek()
    -----
    Postconditions:
        returns
        value - a copy of the value at the front of the queue -
        the value is not removed from the queue (?)
    -----
    """
    assert self._count > 0, "Cannot peek at an empty queue"

    value = deepcopy(self._values[self._front])
    return value

def __iter__(self):
    """
    USE FOR TESTING ONLY
    -----
    Generates a Python iterator. Iterates through the queue
    from front to rear.
    Use: for v in cq:
    -----
    Postconditions:
        returns
        value - the next value in the queue (?)
    -----
    """
    j = self._front
    i = 0

    while i < self._count:
        yield self._values[j]
        i += 1
        j = (j + 1) % self._max_size
```