**EECS2001**
**ASSIGNMENT 1**
**DEADLINE: 3 October, 2019**

NOTES:

- Your assignment must be deposited in the EECS 2001 drop box by 6pm.
- Your assignments must have **a separate** cover page containing:
  - your full name and student number,
  - the course name and number,
  - this assignment's number (nr 1),
  - clearly written the number of a problem that you would like to be marked (**optional**).

## Part I: Alphabets, Strings, and Languages

<u>PROBLEM 1</u>
Consider the string $(011)^R((01)^2(01)^0)^R$

(a) What is the length of the string over the alphabet $\{0, 1\}$?

(b) What is the length of the string over the alphabet $\{01, 1\}$?

(c) What is the length of the string over the alphabet $\{11, 01\}$?

Justify your answer for cases (b) and (c).

<u>PROBLEM 2</u>
Consider two alphabets:

$A1 = \{if, then, else, =, x, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, (, )\}$
$A2 = \{i, f, t, h, e, n, l, s, = 0, = 1, = 2, = 3, = 4, = 5, = 6, = 7, = 8, = 9,$
$(0), (, ), (), (1), (2), (3), (4), (5), (6), (7), (8), (9)\}$.

(a) What is the intersection of $A1^*$ and $A2^*$?

(b) Give an example of a finite language over $A1$ and of an infinite language over $A2$.

(c) Is $A2$ a language over $A1$?

(d) Is the concatenation of three 1-element strings (, 5, and ) a string over $A2$?

PROBLEM 3
Let A be an alphabet. Which of the following properties of strings and languages over A are true and which are false? Justify your answers.

(a) if x, y, z are in $A^*$, then $(xyz)^R = z^R y^R x^R$;

(b) if L1 and L2 are languages over A, then so is the Cartesian product L1 ×L2;

(c) let A be an alphabet and x one of its elements; if L is a language over A, then so is $\{x\}L^R$, where $L^R$ denotes the set of all reversed strings from L, i.e. $L^R = \{w^R : w \in L\}$.

PROBLEM 4 Let A be an alphabet. By $A^{*e}$ let us denote the set of all strings from $A^*$ of even length. Show that

$$(AA)^* = A^{*e}$$

## Part II: FAs and Regular Languages

PROBLEM 5
Let $M =< Q, \Sigma, \delta, q0, F >$ be the DFA defined as follows:
$\Sigma = \{a, b\}, Q = \{q0, q1, q2, q3\}, F = \{q3\}$. The transition function $\delta$ is given by this table:

| d | a | b |
|---|---|---|
| q0 | q1 | q0 |
| q1 | q2 | q0 |
| q2 | q2 | q3 |
| q3 | q2 | q3 |

where 'd' in the table is $\delta$.

(a) Draw the transition diagram for M.

(b) Trace the computations of M for the strings w defined below by computing $\delta^*(q0, w)$, when $w$ is:

  (1) abaa
  (2) aaaabb

(c) Which of the strings from (b) are accepted by M?

(d) Describe formally $L(M)$ as

$$L(M) = \{w : \quad ????\},$$

where ??? is some necessary and sufficient condition that w must satisfy in order to be in $L(M)$.

PROBLEM 6 Design FAs to accept the following languages over the alphabet $\{a, b, c, d, e\}$:

(a) The set of all strings that end with a letter that has appeared before.

(b) The set of all strings that end with a letter that has not appeared before.

PROBLEM 7

Let $M$ be a DFA with 100 states and suppose that w is a string in $L(M)$ of length 101. Which of the following statements are true and which are false?

(a) $L(M)$ must be finite;

(b) there is a string $v \in L(M)$ shorter than w.

PROBLEM 8

Let L be the union of infinitely many regular languages L1, L2, L3, ... over the same alphabet A. Answer the following questions (with justifications):

(a) can L be regular?

(b) is L always regular?

PROBLEM 9

Suppose that a language L over some alphabet A is the union of two languages L1 and L2. We know that if L1 and L2 are regular, then so is L. Is the converse of this fact also true, i.e.,

if L is regular, then L1 and L2 must be regular?