Lecture 16

Truel

November 7, 2019

Outline

Auction Results

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Penny Jar.

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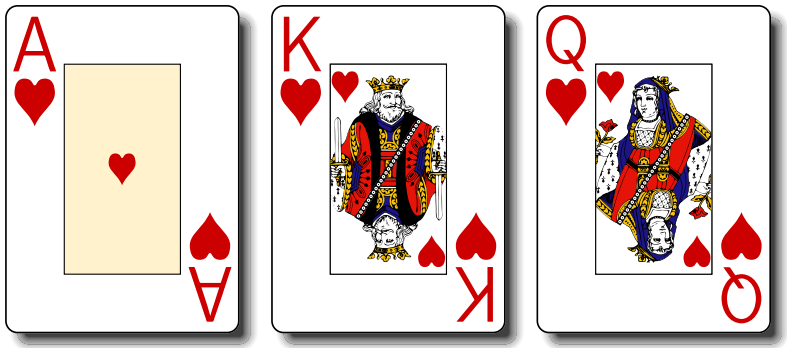
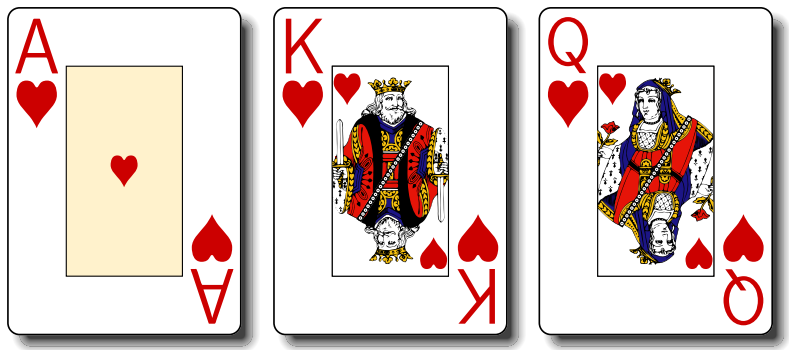
Winners will be contacted!

AKQ Game



Format

AKQ Game



Format

- Two players, each get one card from the *three-card* deck.
- Three cards: Ace, King, Queen (with their usual ranking A>K>Q)
- Both players *ante* one chip.
- One round of betting: P1 can check or bet one chip; P2 can call or fold. (P2 cannot bet when checked to.)

AKQ Game

So far:

Player 1	Action	Player 2	Action
A	Bet	A	Call
K	Check	K	??
Q	??	Q	Fold

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Thus the questions are: should Player 1 bluff with a Queen? And should Player 2 call with a King?

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Solution. Fold.

Best responses and Oscillating strategies

Let's try to use the method of best responses.

You're Player 2. You're holding the King. Suppose you know that I never bluff with a Queen. What is your best play when I bet?

Solution. Fold.

Note: when Player 1 never bluffs and Player 2 never calls, both break even (EV=0).

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You're Player 1. You're holding the Queen. Suppose you know Player 2 always folds a King. What is your best play?

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Solution. Bet!

Best responses and Oscillating strategies

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Solution. Bet!

Note: When Player 2 always folds a King and Player 1 always bluffs, Player 1 makes a profit: the EV is $\frac{1}{2} \cdot 2 + \frac{1}{2} \cdot (-1) = \frac{1}{2}$.

Best responses and Oscillating strategies

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Note: When Player 1 always bluffs a Queen and Player 2 always calls, both break even.

Adapting and Exploiting

Player 1 never bluffs with a Q

$\Longrightarrow$

Player 2 never calls with a K

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Each time one player settles on a strategy, the other player adapts to it and exploits it (by playing a best response to it). This process never converges, and there is no NE.

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Big Question: How do we decide on these frequencies??

Rock-Paper-Scissors

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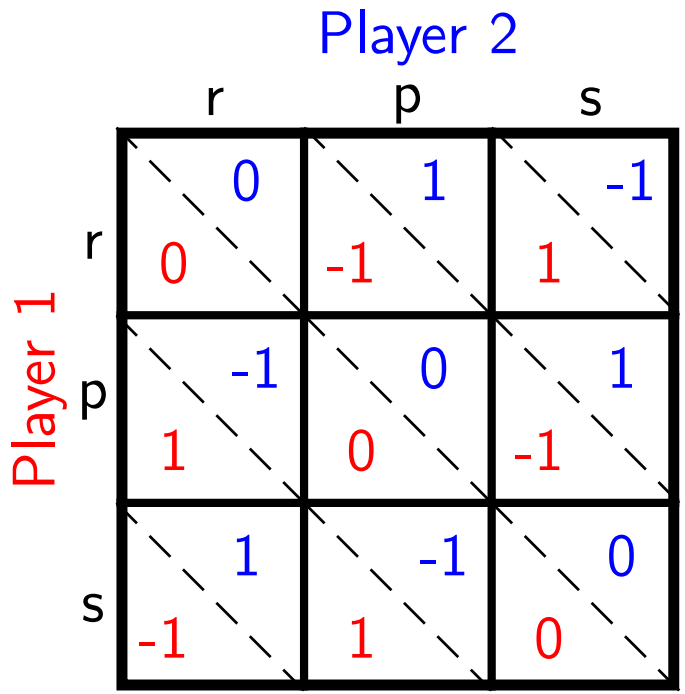
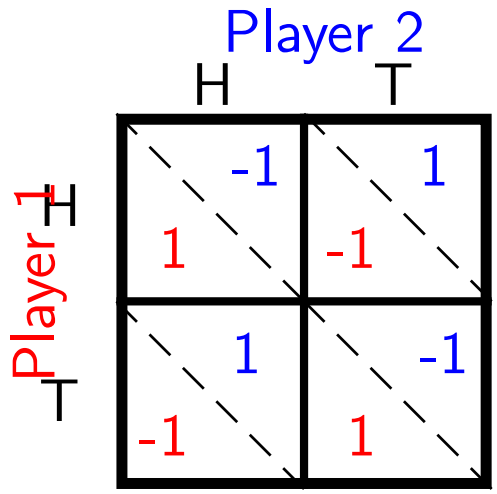


Rock-Paper-Scissors

Player 1 \ Player 2	r	p	s
r	0, 0	1, -1	-1, 1
p	-1, 1	0, 0	1, -1
s	1, -1	-1, 1	0, 0

- No NE in pure strategies!
- Every pure strategy is exploitable.

Matching pennies



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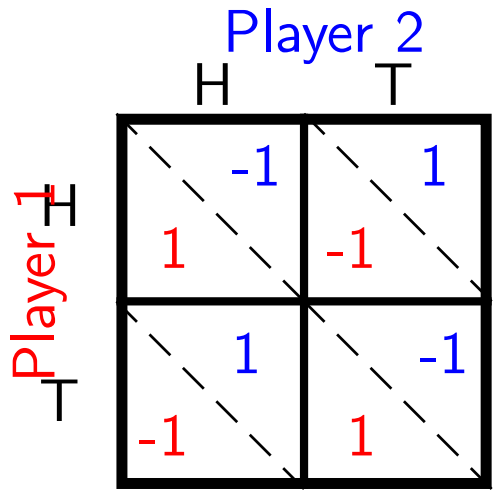

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In games like Poker, Rock-Paper-Scissors, "matching pennies", sticking to one pure strategy doesn't work: your opponent will *exploit* your predictability by choosing their best response against your pure strategy.

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In games like Poker, Rock-Paper-Scissors, "matching pennies", sticking to one pure strategy doesn't work: your opponent will *exploit* your predictability by choosing their best response against your pure strategy. Instead, you must be *unpredictable*:

> A mixed strategy is a probabilistic combination of the available pure strategies.

Concretely, this means that you *randomize* your choice: you play each of your pure strategies a certain percentage of the time, letting chance decide.

Example

In the “matching pennies" game, an example of a mixed strategy would be

Play H 1/3 of the time, and play T 2/3 of the time.

Since it is difficult to act truly randomly, it is best to have a random experiment available.

Truel

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Preferences (for each of the participants):

- Highest priority: maximize chances of survival
- Next priority: kill others
- If someone is the only one with a bullet, flip a coin to decide whom to aim at.

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Question 4: What do you do?

Solution

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  - Case 2a (probability 2/3) Ugly kills Bad. You survive.

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- Case 2 (probability 2/3) You miss. Then Ugly will try to shoot Bad.
  - Case 2a (probability 2/3) Ugly kills Bad. You survive.
  - Case 2b (probability 1/3) Ugly misses. Then Bad will flip a coin. Half of the time he will aim at you, and you have 1/6 chance of surviving.

Solution, Continued

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$$P = \frac{1}{3}\cdot\frac{1}{3} + \frac{2}{3}\cdot\left(\frac{2}{3}\cdot 1 + \frac{1}{3}\cdot\left(\frac{1}{2}\cdot\frac{1}{6}+\frac{1}{2}\cdot 1\right)\right) \approx 69\%$$

Solution, continued

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- Case 1 (probability 2/3) Bad is killed, and you survive.
- Case 2 (probability 1/3) Ugly misses, and now Bad flips a coin; half of the time he aims at you, which you survive with probability 1/6.

Overall probability of survival:

$$P = \frac{2}{3} \cdot 1 + \left(\frac{1}{3} \cdot \left(\frac{1}{2} \cdot \frac{1}{6} + \frac{1}{2} \cdot 1\right)\right) \approx 86\%.$$

Solution, continued

What are the chances of survival for The Bad and The Ugly if you choose the “deliberate miss"-strategy?

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The Bad survives 1/3 of the time (because The Ugly has 2/3 accuracy).

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Moral: It is not always a bad thing to be the weakest player.

Real Estate

Campeau Corporation

Robert Campeau (1923-2017)

Through hostile takeovers, took control of *Allied Stores* and *Federated Department Stores* (now *Macy's*).

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(When they don't succeed in obtaining a majority, the offer is off the table.)

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- The price is calculated on a *prorated basis*:

$$\text{price per share} = \$90 + \$15 \cdot \frac{50}{X}$$

where $X \geq 50\%$ is the percentage of shares tendered.

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- If a majority is obtained, then the non-tendered shares are bought for \$90 each.

Which offer is best?

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1. Tender to Campeau
2. Tender to Macy's
3. Don't tender

Analysis

Three cases:

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3. **You're the tie breaker.** In this case, tendering pays slightly less than \$105 per share. (Assuming you don't have a lot of shares.)

Thus, tendering to Campeau is a dominant strategy. As a result, everyone tenders, and everyone ends up getting \$97.50 for their shares.