Name (please print):
Signature:
Student number:

FACULTY OF SCIENCE
YORK UNIVERSITY
DEPARTMENT OF PHYSICS AND ASTRONOMY

PHYS 1800 03 – Engineering Mechanics
Fall 2016

Final examination

Instructions:

1. Print your name and student number on this page in the upper left corner.
2. Check that your exam contains 7 questions.
3. The total number of marks is 82.
4. Simple scientific calculators are allowed as aid.
5. Answer all questions in the space provided on this exam paper.
6. Formulae are provided on the last page.

1.	
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Total	

CLO1	
CLO2	
CLO3	
CLO4	
CLO5	
CLO6	
CLO7	
CLO8	

1. (10 marks)

An object (mass m = 0.20 kg) is moving in the xy plane. The position of the object is given by the following vector: $\mathbf{r} = (2.0t^3 - 4.0t)\mathbf{i} + (2.0t^2 - 1.0t)\mathbf{j}$, where r is in meters and t is in seconds.

a. Determine the instantaneous velocity of the particle at t = 1.0 s. Express velocity using unit vectors **i** and **j**.

b. Determine the average velocity of the particle for the time interval from t = 1.0 s to t = 2.0 s. Express the average velocity using unit vectors **i** and **j**.

c. What is the force (magnitude and direction) acting on object at the t = 1.0 s?
Hint: F = ma

a)

$$v_x = \frac{dx}{dt} = 6.0t^2 - 4.0 \qquad v_y = \frac{dy}{dt} = 4.0t - 1.0$$

$$v_x(t=1.0\,s) = (6.0)(1.0) - 4.0 = 2.0\ m/s \qquad v_y(t=1.0\,s) = (4.0)(1.0) - 1.0 = 3.0\ m/s$$

$$\vec{v} = 2.0\,\vec{i} + 3.0\,\vec{j}\ (m/s)$$

b)

$$\bar{v} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{[(2.0)(2.0)^3 - (4.0)(2.0)]\vec{i} + [2.0(2.0)^2 - (1.0)(2.0)]\vec{j} - \{[(2.0)(1.0) - (4.0)(1.0)]\vec{i} + [(2.0)(1.0) - 1.0(1.0)]\vec{j}\}}{2.0 - 1.0}$$

$$\bar{v} = \frac{(8.0\,\vec{i} + 6.0\,\vec{j}) - (-2.0\,\vec{i} + 1.0\,\vec{j})}{1.0} = 10\,\vec{i} + 5.0\,\vec{j}\ (m/s)$$

c)

$$\vec{F} = ma$$

$$a_x = \frac{dv_x}{dt} = 12t \qquad a_y = \frac{dv_y}{dt} = 4.0\ m/s^2$$

$$a_x(t=1.0) = 12\ m/s^2$$

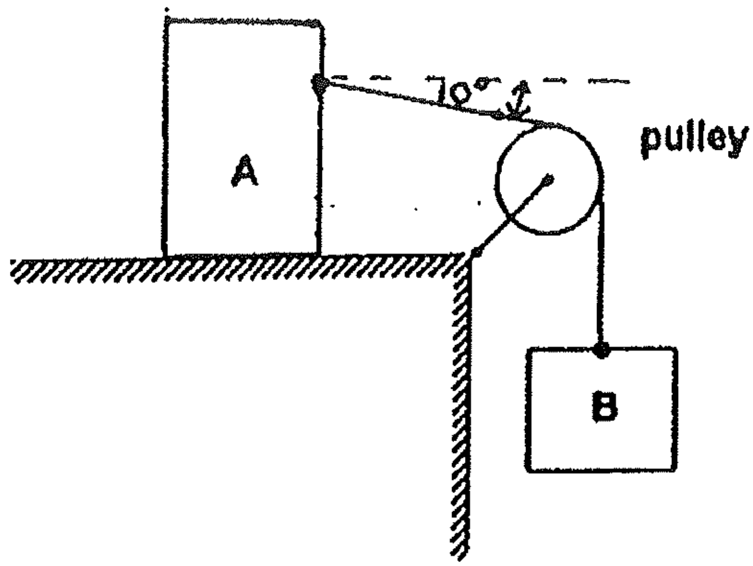
$$\vec{F} = m\vec{a} = (0.20)(12\,\vec{i} + 4.0\,\vec{j})\ (m/s^2) = 2.4\,\vec{i} + 0.80\,\vec{j}\ (N)$$

$$F = \sqrt{(2.4)^2 + (0.80)^2} = 2.5\ N.$$

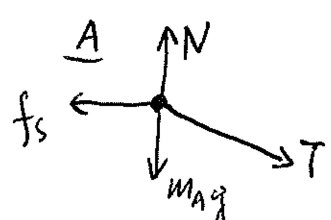
$$\tan\theta = \frac{0.80}{2.4} \Rightarrow \theta = 18^\circ.$$

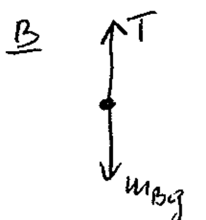
2. (8 marks)

Two blocks are connected over a massless and frictionless pulley as shown below. The mass of block A is 30.0 kg and the mass of block B is 13.0 kg. The coefficient of static friction between block A and the surface is 0.35. The angle of the rope and the horizontal is 10.0^0. Assume that the pulley is frictionless.



a. Draw a free-body force diagram for blocks A and B.





b. Verify if blocks start to move when released from rest. Show all necessary calculations.

BLOCKS REMAIN STATIONARY IF

$$f_{s,MAX} > T_x$$

BLOCK A $\quad f_{s,MAX} = \mu_s N$ $\qquad$ $T_x = T\cos 10.0^\circ$

$\Sigma F_y = 0$

$N - T\sin 10^\circ - m_A g = 0$

$N = m_A g + T\sin 10^\circ$

$N = (30.0)(9.81) + (127.4)\sin 10 = 316\,N$

BLOCK B

$\Sigma F_y = 0$

$T - m_B g = 0$

$T = m_B g = (13.0)(9.81) = 127.4\,N.$

$f_{s,MAX} = \mu_s N = (0.35)(316) = 111\,N.$

$T_x = (127.4)\cos 10^\circ = 125\,N.$

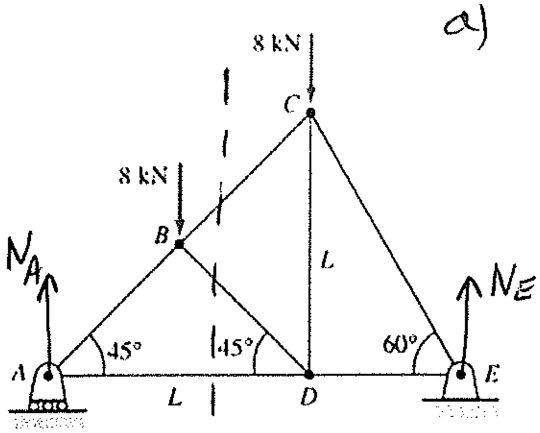
$f_{s,MAX} < T_x$ BLOCKS START TO MOVE

3. (10 marks)

Consider the truss shown below. Two loads, each of 8000.0 N, are applied at joints B and C. The length of truss elements AD and DC is L = 4.0 m. Assume that the roller is frictionless.

a. Determine the support reaction at A and E.

b. Using the <u>method by sections</u>, determine the forces in the following truss members: AD, BC and BD.



a)

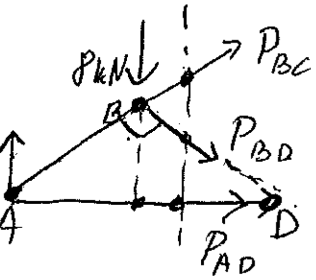
$$\Sigma M_A = -(8000.0)\left(\frac{4.0}{2}\right) - (8000.0)(4.0) + N_E\left(4.0 + \frac{4.0}{\tan 60°}\right) = 0$$

$$N_E = 7608\,N \approx 7600\,N$$

$$\Sigma M_E = (8000.0)\left(\frac{4.0}{\tan 60}\right) + (8000.0)\left(\frac{4.0}{2} + \frac{4.0}{\tan 60}\right) - N_A\left(4.0 + \frac{4.0}{\tan 60}\right) = 0$$

$$N_A = 8392\,N \approx 8400\,N.$$

b)



b)

$$\Sigma M_A = -(8000.0)\left(\frac{4.0}{2}\right) - P_{BD}(4.0 \cos 45°) = 0$$

$$P_{BD} = -5660\,N \quad \text{(COMPRESSION)}$$

$$\Sigma M_B = P_{AD}\left(\frac{4.0}{2}\tan 45°\right) - (8392)\left(\frac{4.0}{2}\right) = 0$$

$$P_{AD} = 8392\,N \approx 8400\,N$$

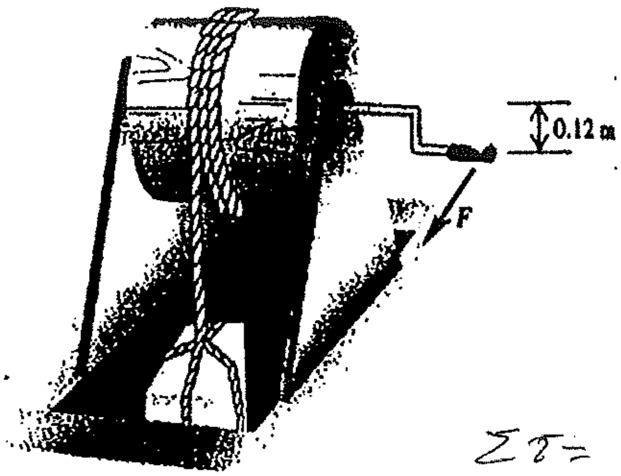
$$\Sigma M_D = -P_{BC}(4.0 \cos 45°) + (8000.0)\left(\frac{4.0}{2}\right) - (8392)(4.0) = 0$$

$$P_{BC} = -6193\,N \approx -6200\,N$$

(COMPRESSION)

4. (8 marks)

A crate of mass m = 50.0 kg is raised as shown below. A rope is wrapped around a wooden cylinder that turns on a metal axle. The cylinder has a radius R = 0.25 m and rotational inertia I = 2.9 kg m^2 about the axle. A crank handle is attached to a wooden cylinder. When the crank is turned, the handle rotates about the axle in a vertical circle of radius r = 0.12 m.
What magnitude of the force F applied tangentially to the rotating crank is required to raise the crate with an acceleration of a = 0.80 m/s^2? The mass of the rope and the crank can be neglected.



$$T - mg = ma$$
$$T = mg + ma$$
$$T = (50.0)(9.81) + (50.0)(0.80) = 530N.$$

$$\Sigma \tau = I\alpha \quad , \alpha = \frac{a}{R}$$
$$Fr - TR = I\left(\frac{a}{R}\right)$$

$$F = \frac{TR + I\left(\frac{a}{R}\right)}{r}$$

$$F = \frac{(530)(0.25) + (2.9)\left(\frac{0.80}{0.25}\right)}{0.12} = 1181 N \simeq 1200 N$$

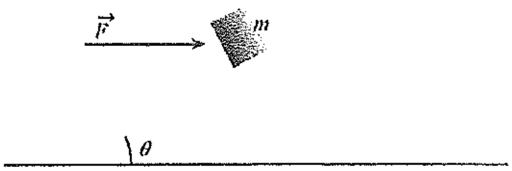
5. (6 marks)

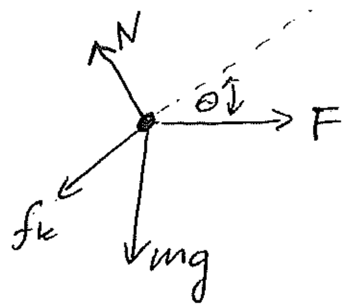
A horizontal force F = 510.0 N is used to push a block m = 25.0 kg a distance of 5.0 m along an inclined plane, as shown below. The angle $\theta = 30.0^0$ and the force of kinetic friction acting on the block is $f_k = 92$ N.

a. Draw a free-body force diagram for the block.

b. Calculate the work done by each force acting on the block.

c. Use the work-kinetic energy theorem to calculate the speed of the block at the end of the 5.0 distance. Assume that the block was initially at rest.





$$W_F = \vec{F} \cdot \vec{\Delta x} = (510)(5.0)\cos 30.^\circ = 2208\text{ J} \approx 2200\text{ J}.$$

$$W_{f_k} = \vec{f_k} \cdot \vec{\Delta x} = (92)(5.0)\cos 180^\circ = -460\text{ J}.$$

$$W_{mg} = (m\vec{g}) \cdot \vec{\Delta x} = (25.0)(9.8)(5.0)\cos(90+30^\circ) = -613\text{ J}.$$
$$\approx 610\text{ J}$$

$$W_N = \vec{N} \cdot \vec{\Delta x} = 0$$

TOTAL WORK

$$W_{TOTAL} = 2208 - 460 - 613 = 1135\text{ J}.$$

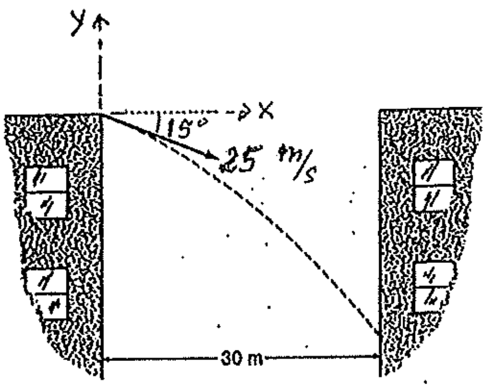
$$W_{TOTAL} = \Delta K = \tfrac{1}{2}mv^2 - 0$$

$$v = \sqrt{\frac{2W_{TOTAL}}{m}} = \sqrt{\frac{2(1135)}{25.0}} = 9.5\text{ m/s}.$$

6. (20 marks, 5 marks each).

Answer the following four questions.

A. Two tall buildings of equal height are 30.0 m apart. A stone is thrown from the top of one building at an angle 15.0^0 below the horizontal and directly at the other building with speed 25.0 m/s.
What is the magnitude of velocity of the stone just before it strikes the building?



$$v = \sqrt{v_x^2 + v_y^2}$$

$$v_x = v_{0x} = (25.0)\cos 15.0^\circ = 24.1 \text{ m/s}$$

$$v_y = v_{0y} + a_y t = -(25.0)\sin 15.0^\circ - (9.8)(t)$$

TO FIND t:

$$30.0 = (25.0)(\cos 15.0^\circ)\, t$$

$$t = 1.24 \text{ s}.$$

$$v_y = (-25.0)\sin 15.0^\circ - (9.8)(1.24 \text{ s}) = -18.2 \text{ m/s}$$

$$v = \sqrt{(24.1)^2 + (-18.2)^2} = 30.2 \text{ m/s}.$$

B. You are standing at rest in Toronto (at a latitude angle $\beta = 42.0^0$) on the spinning Earth of radius $R_E = 6370$ km , as shown below. Determine your centripetal acceleration.

$$a_c = \frac{v^2}{r}$$

$$r = R_E \cos\beta$$

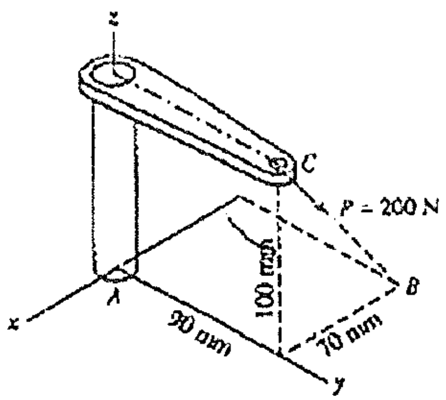
$$v = \frac{2\pi r}{T}.$$

$$a_c = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2 R_E \cos\beta}{T^2}$$

$$a_c = \frac{4\pi^2 (6370\times10^3 \text{ m}) \cos 42^\circ}{[(24)(60)(60)\text{ s}]^2} = 0.025 \text{ m/s}^2.$$

C. Compute the moment of the force P = 200.0 N about point A.

Hint: Write the force $\vec{P} = P_x\vec{i} + P_y\vec{j} + P_z\vec{k}$ and use determinant to calculate the moment of P.



$$\vec{P} = 200.0\left[\frac{-70\vec{i} + 90\vec{j} - 100\vec{k}}{\sqrt{(-70)^2 + (90)^2 + (-100)^2}}\right]$$

$$\vec{P} = (200.0)(-0.462\vec{i} + 0.593\vec{j} - 0.659\vec{k})\ (N)$$

$$\vec{P} = -92.4\vec{i} + 118.6\vec{j} - 131.8\vec{k}\ (N)$$

$$\vec{r}_{AC} = 0\vec{i} + 0.09\vec{j} + 0.1\vec{k}$$

$$\vec{M} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0.09 & 0.1 \\ -92.4 & 111.6 & -131.8 \end{vmatrix} = -23\vec{i} - 9.2\vec{j} + 8.3\vec{j}\ (Nm)$$

D. A mass, m = 0.80 kg, is attached to one end of spring and the system is set into simple harmonic motion. The position x of the mass is given by the following equation: x = 0.15cos(2.30t), where x is in meters and t is in seconds. Determine the spring constant, the kinetic energy of the mass and the spring potential energy at t = 2.0 s.

$$\omega = \sqrt{\frac{k}{m}}$$

$$k = \omega^2 m = (2.30\ Hz)(0.80\ kg) = 4.23\ N/m$$

$$x(t=2.0s) = 0.15\cos[(2.30\tfrac{1}{s})(2.0s)] = -0.0168m$$

$$U_s = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(4.23\ N/m)(-0.0168m)^2 = 5.97\times10^{-4}\ J.$$

$$K = \tfrac{1}{2}mv^2$$

$$v = \frac{dx}{dt} = -(0.15)(2.30)\sin[(2.30)(2.0)] = -0.34\ m/s$$

$$K = \tfrac{1}{2}(0.80)(0.34)^2 = 0.047\ J.$$

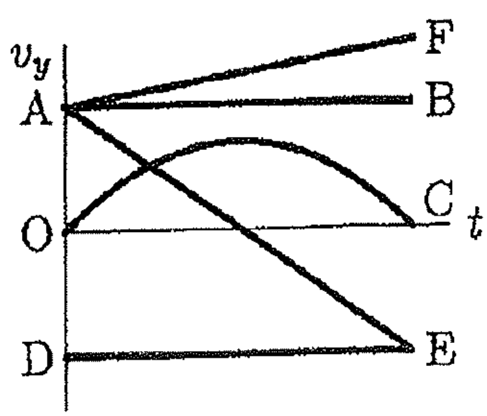
7. (20 marks, 2 mark each)

Answer the following multiple-choice questions. Circle the correct answer in the answer table provided below.

a.	1	~~2~~	3	4	5
b.	~~1~~	2	3	4	5
c.	1	2	~~3~~	4	5
d.	1	2	3	4	~~5~~
e.	1	2	3	4	~~5~~
f.	~~1~~	2	3	4	5
g.	1	2	~~3~~	4	5
h.	1	2	3	~~4~~	5
i.	~~1~~	2	3	4	5
j.	1	2	3	~~4~~	5

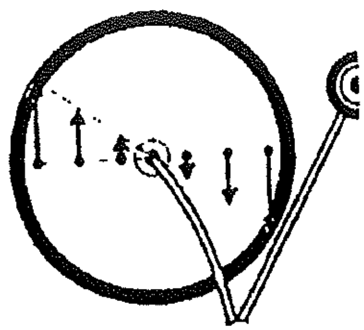
a. Which of the curves on the graph below best represents the vertical component v_y of the velocity versus time t for a projectile fired at an angle of 45^0 above the horizontal?

1. OC
2. AE
3. AB
4. DE
5. AF



b. A bicycle is turned upside-down, the front wheel is spinning. Six points on the wheel are shown with arrows associated with them. The arrows represent:

1. tangential acceleration.
2. angular velocity.
3. centripetal acceleration.
4. angular acceleration.
5. angular velocity and angular acceleration.



c. A wheel initially has an angular velocity of 18 rad/s but it is slowing at a rate of 2.0 rad/s^2. By the time it stops it will have turned through;

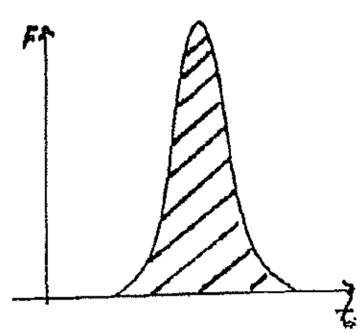
1. 245 rad.
2. 160 rad
3. 81 rad
4. 330 rad
5. 410 rad

$$\omega^2 = \omega_o^2 + 2\alpha \Delta\theta$$

$$\Delta\theta = \frac{\omega^2 - \omega_o^2}{2\alpha} = \frac{0^2 - 18^2}{2(-2)} = 81 \text{ RAD}$$

d. The shaded area on the graph of the force F versus time t represents:

1. impulse
2. work
3. change of momentum
4. momentum
5. impulse and change of momentum



e. A car of mass 2250 kg and travelling at 20.0 m/s smashes into a tree. The car is stopped in 0.25 s. The average force on the car during the collision is

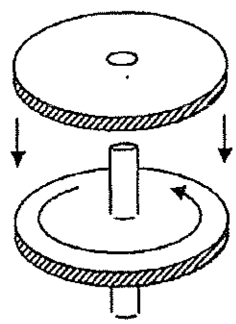
1. $2.2\text{x}10^3$ N
2. $8.0\text{x}10^2$ N
3. $1.8\text{x}10^3$ N
4. $2.2\text{x}10^4$ N
5. $1.8\text{x}10^5$ N

$$F = \frac{\Delta p}{\Delta t} = \frac{(2250)(20.0)}{0.25} = 1.8\times10^5\ N.$$

f. A wheel, mounted on a vertical shaft of negligible moment of inertia, is rotating at 500.0 rpm (revolutions per minute). Another identical (but not rotating) wheel is suddenly dropped onto the same shaft, as shown below. The resultant combination of the two wheels and the shaft will rotate at:

1. 250 rpm
2. 354 rpm
3. 500 rpm
4. 707 rpm
5. 1000 rpm

$$L_i = L_f$$
$$I\omega_i = 2I\omega_f$$
$$\omega_f = \frac{\omega_i}{2} = \frac{500.0}{2} = 250\ \text{rpm}$$

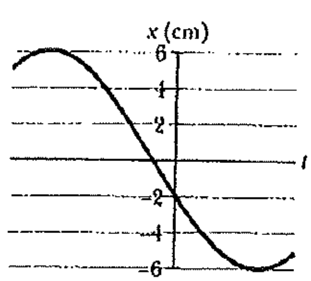


g. What is the phase constant ϕ for the simple harmonic motion (spring-mass system) with the position x versus time t shown below? Assume that the position function has the form $x = A\cos(\omega t + \phi)$

1. 70^0
2. 3.0 rad
3. 1.9 rad
4. $\frac{\pi}{3}$ rad
5.0.33 rad

AT $t=0$, $x=-2$ cm

$$-2 = 6\cos\phi$$
$$\phi = 1.9\ \text{RAD.}$$



h. A travelling wave is represented by the following equation: $D(x,t) = 0.40 \sin(3.0\pi x - 12.0t)$. The speed of the wave is:

1. 4.0 m/s
2. 0.25 m/s
3. 4.0π m/s
4. 1.3 m/s
5. 36 m/s

$v = \frac{\omega}{k}$

$v = \frac{12.0}{3.0\pi} = \frac{4.0}{\pi} = 1.3$ m/s

i. A wire 2.5 m long has cross-sectional area of 2.5 mm^2. It is hung vertically and a 4.50 kg block is hang from it. By how much does the wire stretch if Young's modulus for that material is $2.00\times10^{11}\ \frac{N}{m^2}$?

1. 0.22 mm
2. 2.2 mm
3. 0.022 mm
4. 0.72 mm
5. 0.072 mm

$\frac{F}{A} = Y\frac{\Delta L}{L}$

$\Delta L = \frac{(F)(L)}{(A)(Y)} = \frac{(mg)(L)}{(A)(Y)} = \frac{(4.5)(9.81)(2.5)}{2.5\times(10^{-3}m)^2(2.0\times10^{11})}$

$\Delta L = 2.2\times10^{-5}m = 0.22\ mm$

j. The yield point of a given material is

1. its ultimate tensile stress.
2. the maximum stress that the material will take such that its strain is directly proportional to the applied stress.
3. slightly lower than the proportional point.
4. the maximum stress for which the deformation of the material is reversible.
5. the maximum point on the stress-strain curve.

THE END

$x = x_0 + v_0t + \frac{1}{2}at^2$ $\quad v = v_0 + at$ $\quad v^2 = v_0^2 + 2a(x - x_0)$ $\quad v_{avg} = \frac{1}{2}(v_0 + v)$ $\quad (x - x_0) = \frac{1}{2}(v_0 + v)t$

$\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$ $\quad \mathbf{v}_{avg} = \frac{\Delta r}{\Delta t}$ $\quad \mathbf{v} = \frac{dr}{dt} = \frac{dx}{dt}\boldsymbol{i} + \frac{dy}{dt}\boldsymbol{j} + \frac{dz}{dt}\boldsymbol{k}$

$\mathbf{a}_{avg} = \frac{\Delta v}{\Delta t}$ $\quad \mathbf{a} = \frac{dv}{dt} = \frac{dv_x}{dt}\boldsymbol{i} + \frac{dv_y}{dt}\boldsymbol{j} + \frac{dv_z}{dt}\boldsymbol{k}$

$\mathbf{F} = m\mathbf{a}$ $\quad f = \mu N$ $\quad a_c = \frac{v^2}{r}$ $\quad F_g = mg$ $\quad F_g = \frac{GMm}{r^2}$ $\quad F = -kx$

$\theta = \frac{s}{r}$ $\quad \omega = \frac{d\theta}{dt}$ $\quad \alpha = \frac{d\omega}{dt}$ $\quad \Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ $\quad \omega = \omega_0 + \alpha t$ $\quad \Delta\theta = \left(\frac{\omega_i + \omega_f}{2}\right)t$

$v = r\omega$ $\quad \mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$ $\quad a_c = \frac{v^2}{r} = r\omega^2$ $\quad \mathbf{a}_c = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$ $\quad a_t = \frac{dv}{dt} = r\alpha$ $\quad \mathbf{a}_t = \boldsymbol{\alpha} \times \mathbf{r}$

$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$ $\quad \boldsymbol{\tau} = I\,\boldsymbol{\alpha}$ $\quad I_{CM}^{rod} = (1/12)\,ML^2$ $\quad I_{CM}^{disk} = (1/2)\,ML^2$ $\quad I_{CM}^{sphere} = (2/5)\,ML^2$ $\quad I = I_{cm} + Md^2$

$\mathbf{p} = m\mathbf{v}$ $\quad \mathbf{F} = \frac{dp}{dt}$ $\quad \Delta p = \mathbf{F}_{av}\Delta t$ $\quad \mathbf{L} = I\boldsymbol{\omega}$ $\quad \boldsymbol{\tau} = \frac{dL}{dt}$ $\quad \Delta\mathbf{L} = \boldsymbol{\tau}_{av}\Delta t$

$\sigma = Y\,\varepsilon$ $\quad \frac{F}{A} = Y\frac{\Delta L}{L}$ $\quad \frac{F}{A} = G\frac{\Delta L}{L}$ $\quad \Delta p = -B\frac{\Delta V}{V}$ $\quad Y = 2G(1 + \nu) = 3B(1 - 2\nu)$

$F = -kx$ $\quad \tau = -D\,\theta$ $\quad x = A\cos(\omega t + \varphi)$ $\quad x = (A_o e^{-t/\tau})\cos(\omega t + \varphi)$

$\omega = \frac{2\pi}{T} = 2\pi f t$ $\quad T = 2\pi\sqrt{\frac{m}{k}}$ $\quad T = 2\pi\sqrt{\frac{I}{D}}$ $\quad T = 2\pi\sqrt{\frac{I}{MgL}}$ $\quad T = 2\pi\sqrt{\frac{L}{g}}$

$D(x,t) = A\sin(kx - \omega t + \varphi)$ $\quad D(x,t) = A\sin(kx)\cos(\omega t)$ $\quad k = \frac{2\pi}{\lambda}$ $\quad \omega = \frac{2\pi}{T} = 2\pi f t$ $\quad v = \lambda f = \frac{\omega}{k}$

$v = \sqrt{\frac{T}{\mu}}$ $\quad v = \sqrt{\frac{kL}{\mu}}$ $\quad v = \sqrt{\frac{Y}{\rho}}$

$W = \mathbf{F} \bullet \Delta\mathbf{x}$ $\quad W = \int_{x_1}^{x_2} F(x)\,dx$ $\quad \Delta K + \Delta U = W_{nc}$ $\quad \Delta K + \Delta U = 0$ $\quad K = \frac{1}{2}mv^2$ $\quad K = \frac{1}{2}I\omega^2$

$\Delta U_g = mgh$ $\quad \Delta U_g = -\frac{GMm}{R}$ $\quad \Delta U_s = \frac{1}{2}kx^2$

$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$ $\quad F_x = F\cos\alpha,$ $\quad F_y = F\cos\beta,$ $\quad F_z = F\cos\gamma$ $\quad \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$g = 9.81\ m/s^2$ $\quad G = 6.67\times10^{-11}\ N\,m^2/kg^2$ $\quad \bar{x} = \frac{\sum_i x_i}{N}$ $\quad \sigma = \sqrt{\frac{\sum_i (x_i - \bar{x})^2}{N - 1}}$ $\quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}} = \sqrt{\frac{\sum_i (x_i - \bar{x})^2}{N(N - 1)}}$

$\frac{d(x^n)}{dx} = nx^{n-1}$ $\quad \frac{d(\cos x)}{dx} = -\sin x$ $\quad \frac{d(\sin x)}{dx} = \cos x$