Physics 1007/BIT 1203
Fall 2019

Lecture 10

Work and Energy
Potential Energy
Conservation of Energy

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Work-Energy Theorem

- The work-energy theorem is

$$W = \Delta K$$

  - This assumes any work done by a force goes ONLY into a change of kinetic energy
  - Works for motion in the horizontal direction where there is no change of height of the moving object.

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Potential Energy

- Sometimes when we do work on a system we do not change the kinetic energy.
- The energy added to the system is stored in the system as potential energy
- When potential energy is released, it may be converted to other forms of energy, often kinetic energy

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Moving the Furniture Up the Ramp Example 6.1

- How much work is done by the movers, exerting a force F_m if we move it 4.0 m up the ramp, raising it by a height of 1.0 metres?
  - Assume no friction, and moving at constant speed
  - No change in kinetic energy

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$$W_m = +1400\ N.m\ or +\ 1400\ J$$

- The work done by the movers represents a transfer of energy from the movers to the chest
- But the chest is moving at constant speed, so the kinetic energy is constant
- Where has the energy transferred from the movers to the chest gone?
- It has been transferred as potential energy (stored energy)

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- Examples of potential energy storage:
- Raising an object in gravity
  - Increases gravitational potential energy
- Compressing or stretching a spring
  - Increases elastic potential energy

https://www.youtube.com/watch?v=YtqmpJegrno&feature=youtu.be

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Conservative Forces

- Forces which have potential energies associated with them are called Conservative forces
  - Gravity, elastic force, electric force
- A force is conservative if the work done by a force is independent of the path taken,

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Non-Conservative Forces

- Friction forces and air resistance are not conservative
- They cannot have a potential energy associated with them
- They always remove energy from the system (by doing negative work)
- The length of the path length determines how much work is done.

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Work Done and Potential Energy

- If a conservative force does work, then there is a change in potential energy
- The work done is the negative of the change in potential energy

$$W = -\Delta U$$

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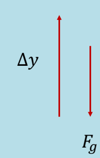
Gravitational Potential Energy

- Suppose we raise an object in the Earth's gravity field



$$W_g = |F_g||\Delta y| \cos 180°$$

$$W_g = -|F_g||\Delta y|$$

$$W = -\Delta U$$

- Gravity did negative work, so the gravitational potential energy change is positive:
- We increase GPE by raising an object

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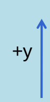
- If the gravity field is constant over the displacement (a good approximation near to the surface of the earth)

$$\Delta U_{grav} = mg\Delta y$$

$$\Delta U_{grav} = mg(y_f - y_i)$$

$$\Delta U_{grav} = mgh$$



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- We cannot measure an absolute value for gravitational potential at a point in space
- We can calculate the change in gravitational potential when an object is displaced in a gravitational field
  - When solving a problem, we choose one height as an arbitrary zero, to make calculations easy

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If we raise a mass of 1.1 kg, by a height of 1.5 m near the surface of the Earth, then the change in gravitational potential energy is positive.

$$\Delta U = mg\Delta y$$

$$\Delta U = (1.1\ kg)(9.8\ m/s^2) \times 1.5\ m$$

$$\Delta U = +16\ J\ to\ 2\ sf$$

- Raising an object in a gravity field always increases the GPE. Lowering the object reduces GPE

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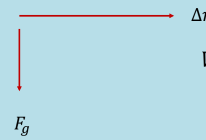
- If we move the object perpendicularly to the force of gravity, then no work is done by the gravitational force, and there is no change in gravitational potential energy



$$W = F_g \Delta r \cos 90° = 0$$

- When an object moves, we only have to calculate the vertical height change to calculate the change in GPE

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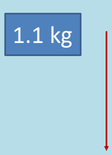
Lowering Potential Energy

- If an object has potential energy, and is free to move, it will always move in such a way as to reduce the potential energy.



If our object has positive GPE, then it will reduce it by falling

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Mechanical Energy

- The sum of the kinetic energy and the gravitational potential energy is known as the Mechanical Energy

$$E_{mech} = K + U$$

- For systems which do not involve electrical or magnetic interactions, this is a good approximation for the energy available to change

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Work-Energy Theorem and Potential Energy

- The work-energy theorem is

$$W = \Delta K$$

  - This assumes any work goes ONLY into a change of kinetic energy
- If a change in potential energy is allowed too

$$W = \Delta K + \Delta U$$

  - This allows for changes in potential energy as well as kinetic energy

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Conservative Forces Do Not Change the Mechanical Energy

- If the only forces acting on an object are conservative, then the mechanical energy of the object is constant (conserved)
  - i.e. we have to neglect air resistance, which is non-conservative when we consider the motion of objects moving in air

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Falling Coconuts Are Dangerous

- Assume a coconut, mass 2.0 kg, drops from rest from a tree 35 metres high
- We can write this problem in terms of conservation of mechanical energy, if we neglect air resistance.

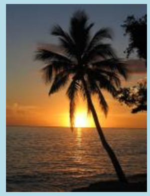

Image: Wikimedia Commons

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- The mechanical energy of the system is constant

$$E_i = E_f$$

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f$$

Stationary coconut, so $K_i = 0$

At h = 35 m

At ground level, h = 0

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$$0 + mgh_i = \frac{1}{2}mv_f^2 + 0$$

$$gh_i = \frac{1}{2}v_f^2$$

We can cancel the mass

$$2gh_i = v_f^2$$

$$v_f = \sqrt{2gh_i}$$

$$v_f = \sqrt{2(9.8\, m.s^{-2})(35\, m)}$$

$$v_f = 26\, m.s^{-1} to\ 2\ s.f.$$

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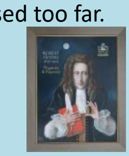
Ideal Springs

- The English scientist Robert Hooke (1635-1703) noticed that for many objects, the deformation was proportional to the force causing the deformation
- Many springs obey Hooke's Law, as long as they are not stretched or compressed too far.



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Hooke's Law

- In 1676 Hooke published his findings on the force required to stretch springs
- cediinnoopsssttuu
- This is an anagram in Latin of the phrase
- *Ut Pondus sic Tensia*
- *"As the weight, so the tension"*

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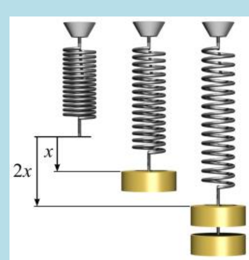
- The change in length from the equilibrium position is the displacement
- It applies for both elongation and compression



The change in length is proportional to the applied force.

In this example the applied force is the weight added under the spring

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Force Required to Distort a Spring

- Can be either stretching or compression

$$\vec{F}_{app} = k\Delta\vec{x}$$

Applied Force

k is the spring constant

Displacement of the spring from its unstrained length (the change in length)

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The Restoring Force/Spring Force

- When a spring is distorted, then it exerts an internal restoring force which tries to move it back to its unstrained length
- Sometimes called "The Spring Force"
- The restoring force always points in a direction opposite to the displacement of the spring.

$$F = -k\Delta x$$

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The Spring Constant

- The spring constant has the SI unit of newtons per metre (N/m)

$$k = \frac{F}{\Delta x}$$

- The stiffer the spring, the higher the value of k
- A large value of k means that a large force is required to create a small change in length

https://www.youtube.com/watch?v=H_CJqb9dacw

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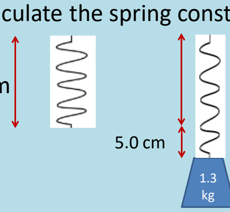
Stretching a String

- A weight is attached to the end of a vertical spring of length 25.0 cm. The extension of the spring is 5.0 cm, and the mass is 1.3 kg. Calculate the spring constant in N/m.



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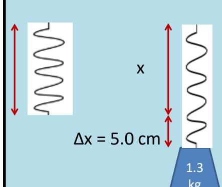
- The original length of the spring is not important, it is the extension of the spring that matters



The applied force here is the weight

$$k = \frac{F_{app}}{\Delta x}$$

$$k = \frac{mg}{\Delta x}$$

$$k = \frac{(1.3\ kg)(9.8\ m.s^{-2})}{0.050\ \text{m}}$$

$$k = 254.8\ N.m^{-1}$$

$k = 250\ N.m^{-1}$ to 2 f

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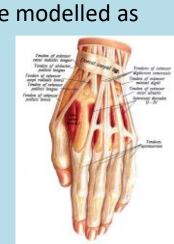
Elastic Properties of Materials

- Many solid materials behave just like springs when compressed or stretched
- Chemical bonds can also be modelled as springs
- Tendons behave as springs



Hooke's Law for springs applies to these systems too.

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Work Done by a Variable Force

- This equation for work

$$W = F\Delta r \cos\theta$$

- Is only valid for constant forces of magnitude F
- If the force is not constant, but varies with position, then we can add up all the contributions to the total work done for small increments of Δr (this is integration!)

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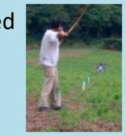
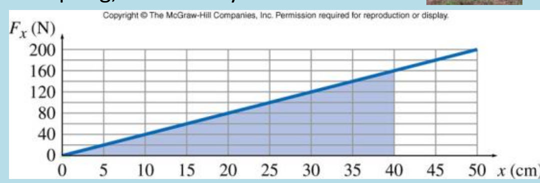
Work Done Pulling a Longbow

- For a simple bow, the force required is linear with distance
- The bow is acting like an ideal spring, which obeys Hooke's Law





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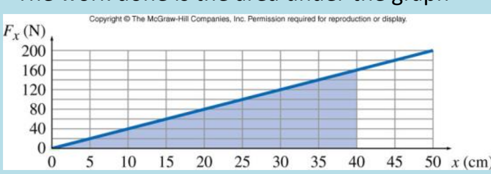
- Plot the graph of force (as y) versus displacement (as x)
- The work done is the area under the graph



Calculus alert: The area underneath the graph of force versus position is the integral

$$W = \int_{x_i}^{x_f} \vec{F}(\vec{x}).d\vec{x}$$

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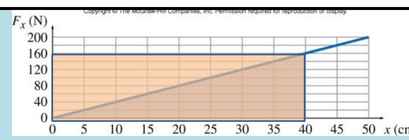

- For a linear graph through the origin, determine the area of the rectangle, and divide by 2 to get the area under the graph

$$Area = \Delta F \times \Delta x = 160\ N \times 0.40\ m$$

$$W = \frac{1}{2}(\Delta x)F(x)$$

$$W = \frac{1}{2}(0.40\ \text{m})(160\ \text{N}) = +32\ \text{J}$$

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Elastic Potential Energy

- This is the energy stored up when a spring is either compressed or stretched.
- If the spring is released, it will lose this stored potential energy, and convert it into kinetic energy (the spring moves back to the original length)
- Most materials, and even chemical bonds act like springs unless stretched too much.

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Elastic Potential Energy Stored in an Ideal Spring

$$U_{el} = \frac{1}{2}k(\Delta x)^2$$ Equation 6.24

– We usually define U = 0 for a displacement Δx = 0 for convenience (but remember we can't actually calculate absolute values for elastic potential energy)

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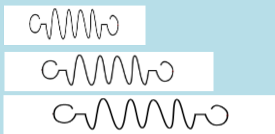
$$U_{el} = \frac{1}{2}k(\Delta x)^2$$

- Elastic potential energy is always positive, because the displacement term is squared.
- The spring stores elastic potential energy if is compressed ($\Delta x < 0$) OR if it is stretched($\Delta x > 0$)

Compressed $\Delta x < 0$

Equilibrium length $\Delta x = 0$

Stretched $\Delta x > 0$



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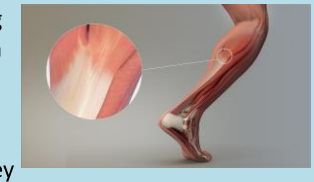
- Tendons are strong elastic fibres which attach muscle to bone. To a reasonable approximation, they obey Hooke's law.



- A 125 kg mass was hung from a tendon, it stretched by 0.660 mm.
- Find the spring constant of this tendon in N/m

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- A 125 kg mass was hung from a tendon, it stretched by 0.660 mm.

$$F_{app} = kx$$

$$k = \frac{mg}{x}$$

$$k = \frac{(125\ kg)(9.81\ m.s^{-2})}{0.660 \times 10^{-3} m}$$

$$k = 1.86 \times 10^{6} N.m^{-1}$$

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- How much elastic potential energy is there in the stretched tendon?

$$U_{el} = \frac{1}{2}k(\Delta x)^2$$

$$U_{el} = \frac{1}{2}(1.86 \times 10^{6} N.m^{-1})(0.660 \times 10^{-3} m)^2$$

$$U_{el} = 0.405\ J$$

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