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EECS 2001 E Quiz 1 (Version A)

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Guidelines and Instructions:

1. This is a 60-minute, closed-book test.
2. This exam paper has 4 pages (including this page) and 7 questions.
3. Print your name and write down your student number clearly.
4. If you need more space to write your answer use the back of the page but clearly indicate which question you are answering.
5. You may not leave within the first and last 10 minutes of the exam. Remain seated until all exams are collected.

Question 1	/3
Question 2	/4
Question 3	/4
Question 4	/4
Question 5	/4
Question 6	/5
Question 7	/6
Total	/33

Question 1. [3 marks] Let $A = \{n \mid 1 \leq n \leq 15, and\ n = 5k\ for\ some\ k \in N\}$ and $B = \{a, b\}$. Give an explicit listing of

a) $A \cup B$
A = {5, 10, 15}, B = {a, b} => $A \cup B$ = {5, 10, 15, a, b} (1 mark for the correct answer)

b) $A \times B$ = {(5, a), (5, b), (10, a), (10, b), (15, a), (15, b)} (1 mark for the correct answer)

c) Power set of B = { ∅, {a}, {b}, {a, b}} (1 mark for the correct answer)

Question 2. [4 marks] Are the following statements true or false? Assume the domain of x and y is the set of integers. Briefly explain your answer.

a) $\forall x \exists y\ x + y = 5$

True! For any x, there is y = 5 -x which satisfies the equation. (1 mark for true and 1 for the correct explanation)

b) $\exists y \forall x\ x \cdot y = x$

True! Let y =1, then for any x we have x.y = x (1 mark for true and 1 for the correct explanation)

Question 3. [4 marks] show that ¬(¬p ↔ (r ∨ p)) is logically equivalent to $r \to p$

One can use a truth table (1 mark for each correct row)

r	p	¬(¬p ↔ (r ∨ p))	$r \to p$
T	T	¬(¬T ↔ (T ∨ T)) = ¬(F ↔ T) = T	T
F	F	¬(¬F ↔ (F ∨ F)) = ¬(T ↔ F) = T	T
F	T	¬(¬T ↔ (F ∨ T)) = ¬(F ↔ T) = T	T
T	F	¬(¬F ↔ (T ∨ F)) = ¬(T ↔ T) = F	F

Question 4. [4 marks] A palindrome is a string that reads the same forward as backward, like "madam". Give a **recursive definition** of the set of all palindromes P over an alphabet Σ.

Set of all palindromes P over an alphabet Σ can be defined as

1) $\varepsilon \in P$; $\forall a \in \Sigma$, $a \in P$
2) $\forall a \in \Sigma\ \forall x \in P$, $axa \in P$
3) No other string is in P

(1 mark for the correct base case: line 1, 2 marks for recursion: line 2, and 1 for line 3).

Question 5. [4 marks] Prove that for any simple undirected graph G the sum of the degrees of all nodes in G is even.

See Theorem 0.21 of the Text: Every edge in G is connected to two nodes. Each edge contributes 1 to the degree of each node at its end points. Therefore each node contributes 2 to the total sum. Hence if G contains e edges, then the sum of all degrees is 2e implying it is even.

(The key is to notice that each edge contributes 2 to the sum. Deduct marks for any incorrect reasoning or hole in the proof.)

Question 6. [5 marks] Prove that $\log_2 3$ is irrational.

Assume $\log_2 3$ is rational. This implies $\log_2 3 = p/q$ for some positive integers p and q. But $\log_2 3 = p/q$ implies q $\log_2 3 = p$ which implies $\log_2 3^q = p$. By definition of log we have $2^p = 3^q$. But this is impossible since left side is even and right side is odd. Therefore $\log_2 3$ cannot be rational.

(This is done by contradiction. Give 0 for any attempt by induction. Give partial mark (<= 3) if they attempt proof by contradiction, but cannot make it work. Full mark only if they can arrive correctly at the contradiction).

Question 7. [6 marks] Use induction to prove that any natural number n, $n \geq 12$, can be written as $n = 4a + 5b$ where a and b are from the set of non-negative integers (i.e. natural numbers with 0 included).

Let P(n) be the statement that a natural number $n \geq 12$, can be written as $n = 4a + 5b$.
Base case: P(12) is true since $12 = 4 \times 3 + 5 \times 0$.

Induction hypothesis: Assume P(n) is true for some natural number $n \geq 12$. That means we can write $n = 4a + 5b$ for non-negative integers a and b.

Induction step: we show $P(n) \rightarrow P(n+1)$. By induction hypothesis we can write $n + 1 = 4a + 5b + 1$.
Now there are two cases: a) either a > 0, or b) a = 0.
Case a) a >0: then we can write $n + 1 = 4a + 5b + 1 = 4(a-1) + 5(b+1)$. Note that both (a-1) and (b+1) are non-negative integers.
Case b) a = 0. In this case since $n \geq 12$ we must have $b \geq 3$. Then we can write $n + 1 = 4a + 5b + 1 = 4a + 5(b-3) + 15 + 1 = 4a + 5(b-3) + 4 \times 4 = 4(a+4) + 5(b-3)$. Here also both (b-3) and (a+4) are non-negative integers and we are done.

(2 marks for the base case and 2 marks each for the induction step cases. Give partial marks depending on the correctness of the arguments)