54. The sum of two positive numbers is 16. What is the smallest possible value of the sum of their squares?

55. Evaluate the integral by interpreting it in terms of areas.

$$\int_{-2}^{2} \sqrt{4-x^2}\, dx$$

56. Use the properties of integrals to verify that

$$2 \le \int_{-1}^{1} \sqrt{1+x^2}\, dx \le 2\sqrt{2}$$

57. Evaluate the integral $\int_{1}^{8} \sqrt[3]{x}\, dx$

58. Evaluate the integral $\int_{1}^{2} \frac{(x-1)^3}{x^2}\, dx$

59. Find the general indefinite integral $\int \frac{\sin 2x}{\sin x}\, dx$

60. Use Part 1 of the Fundamental theorem of calculus to fo find the derivative of the function.

(a) $g(x) = \int_{0}^{x} \sqrt{1+2t}\, dt$ (b) $G(x) = \int_{x}^{1} \cos\sqrt{t}\, dt$

61. Evaluate the indefinite integral.

(a) $\int x \sin(x^2)\, dx$ (b) $\int \frac{x}{(x^2+1)^2}\, dx$ (c) $\int \frac{x}{1+x^4}\, dx$

62. Evaluate the integral.

(a) $\int \ln(2x+1)\, dx$ (b) $\int \sin^{-1} x\, dx$ (c) $\int_0^1 \frac{y}{e^{2y}}\, dy$

63. First make a substitution and then use integration by parts to evaluate the integral

$$\int \cos \sqrt{x}\, dx$$

64. Evaluate the integral $\int \frac{dx}{x^2 \sqrt{4-x^2}}$

65 Use long division to evaluate the integral

$$\int_0^1 \frac{x^3 - 4x - 10}{x^2 - x - 6}\, dx$$

66 Evaluate the integral $\int_2^3 \frac{1}{x^2-1}\, dx$

67. Sketch the region enclosed by the given curves. Decide whether to integrate with respect to x or y. Find the area of the region.

$$x = 1 - y^2, \qquad x = y^2 - 1$$

68. Sketch the region enclosed by the given curves and find its area

$$y = 12 - x^2, \qquad y = x^2 - 6$$

69. Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

(a) $y = \frac{1}{x}$, $x = 1$, $x = 2$, $y = 0$; about the x-axis

(b) $y = \ln x$, $y = 1$, $y = 2$, $x = 0$; about the y-axis.

70. The region enclosed by the given curves is rotated about the specified line. Find the volume of the resulting solid.

$x - y = 1$, $y = x^2 - 4x + 3$; about $y = 3$

71. Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the y-axis.

$y = \frac{1}{x}$, $y = 0$, $x = 1$, $x = 2$

72. Find the exact length of the curve

$x = 1 + 3t^2$, $y = 4 + 2t^3$, $0 \leq t \leq 1$

73. Find the exact length of the curve.

$y = \frac{1}{4}x^2 - \frac{1}{2}\ln x$, $1 \leq x \leq 2$

74. Find the average value of the function on the given interval. $g(x) = \sqrt{x}$, $[1, 4]$

75. Find the average value of f on the given interval.

$f(x) = (x-3)^2$, $[2, 5]$

76. Determine whether the sequence converges or diverges. If it converges, find the limit.

$$a_n = \frac{n3}{n+1}$$

77. Does the sequence $a_n = \frac{3^{n+2}}{5^n}$ converges?

If so, find the limit,

78. Determine whether the sequence is increasing, decreasing, or not monotonic. Is this sequence bounded?

$$a_n = \frac{1}{2n+3}$$

79. Determine whether the geometric series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \frac{(-6)^{n-1}}{5^{n-1}}$$

80. Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \frac{1+2^n}{3^n}$$

81. Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \left(\frac{3}{5^n} + \frac{2}{n}\right)$$

82. Use the comparison test to determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{n}{2^{n^3}+1}$$

83. Test the series for convergence or divergence.

$$\sum_{n=1}^{\infty} (-1)^n \frac{3n-1}{2n+1}$$

84. For what values of p is the following series convergent?

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^p}$$

85. Find the radius of convergence and interval of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n+1}$$

86. Find the radius of convergence and interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{x^n}{n\,3^n}$$

87. Find the radius of convergence and interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt{n}} (x+3)^n$$

88. Find a power series representation for the function and determine the interval of convergence.

$$f(x) = \frac{x}{1-x}$$

89. Find the Maclaurin series for $f(x)$ using the definition of a Maclaurin series. Also find the radius of convergence.

$$f(x) = (1-x)^{-2}$$

90. Find the Taylor series for $f(x)$ centered at the given value of a.

$$f(x) = 1 + x + x^2, \quad a = 2$$