NAME:____________________ STUDENT ID# :______________

LAKEHEAD UNIVERSITY

FACULTY OF ENGINEERING

Midterm Exam

Solutions Key

Engineering	**3021**	**AA**	**Summer Transition Program**
SUBJECT	COURSE NO.	SECTION	

Engineering Analysis A	**Dr. H. Naser**
COURSE TITLE	INSTRUCTOR

Wed, July 18, 2012	**9:00 a.m. - 11 a.m.**	**2 hours**	**UC2011**
EXAM DATE	EXAM TIME	DURATION	ROOM

- **NO CALCULATOR OR ELECTRONIC GADGETS ALLOWED**
- **SHOW YOUR STEPS – NO STEPS NO MARKS**
- **CLOSED BOOK EXAM**

This exam **MAY NOT** be removed from the room

STUDENTS PLEASE NOTE

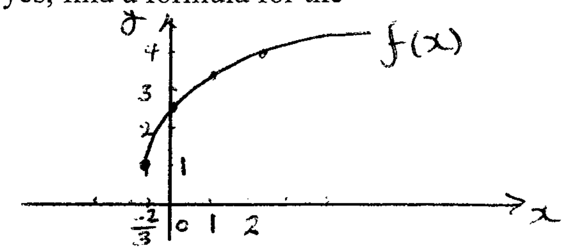
YOU MUST count the number of pages in this examination question paper **BEFORE** beginning to write, and report any discrepancy immediately to a proctor.

This is page 1 of 12.

This section is for faculty use only

Q	1	2	3	4	5	6	7	8.a	8.b	8.c	8.d	**Total**
M												

1. Determine whether the following function is one-to-one. If yes, find a formula for the inverse of the function.



$$f(x) = 1 + \sqrt{2+3x}$$

solve the equation for x:

$$y = 1 + \sqrt{2+3x} \Rightarrow y - 1 = \sqrt{2+3x}$$

$$\Rightarrow (y-1)^2 = 2 + 3x \Rightarrow x = \frac{(y-1)^2 - 2}{3}$$

for every value of y, there is clearly one and only one value for x.

$\therefore$ The horizontal line test passed $\Rightarrow$ the function is one-to-ene

The inverse: $x = \dfrac{(y-1)^2 - 2}{3}$

- 5 marks for Hto1
- 5 marks for the formula of inverse func.

or after interchanging x with y

$$y = \frac{(x-1)^2 - 2}{3}$$

2. Find the functions $f \circ g$ and $g \circ f$ and their domains.

$$f(x) = 2x + 1 \qquad g(x) = \cos(x)$$

$D_f = \mathbb{R}$ $\qquad$ $D_g = \mathbb{R}$

$f \circ g(x) = f(g(x)) = f(\cos x) = 2\cos x + 1$ 3

$g \circ f(x) = g(f(x)) = g(2x+1) = \cos(2x+1)$ 3

$D_{f\circ g} = \{\forall x \in D_g \text{ such that } g(x) \in D_f\}$

$= \{\forall x \in \mathbb{R} \text{ such that } \cos x \in \mathbb{R}\}$ 2

$\Rightarrow D_{f\circ g} = \mathbb{R} = (-\infty, \infty)$

$D_{g\circ f} = \{\forall x \in D_f \text{ s.t. } f(x) \in D_g\}$

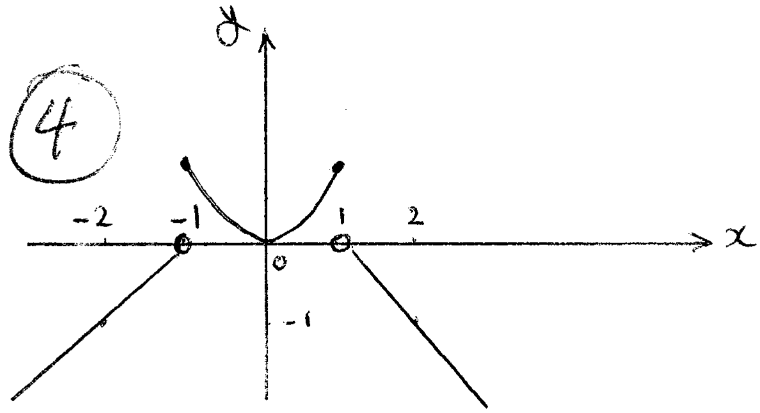
$= \{\forall x \in \mathbb{R} \text{ s.t. } (2x+1) \in \mathbb{R}\}$ 2

$\Rightarrow D_{g\circ f} = \mathbb{R} = (-\infty, \infty)$

3. Sketch the graph of the function and determine at which points the function is continous. Explain why by using the definition of continuty.

$$f(x)=\begin{cases}1+x & if \quad x<-1\\ x^2 & if \quad -1\le x<1\\ 1-x & if \quad x\ge 1\end{cases}$$

4



2 The function is continuous everywhere except at -1 and 1.

$\lim_{x\to -1^-} f(x) = \lim_{x\to -1} 1+x = 0$ $\quad \lim_{x\to -1^+} f(x) = \lim_{x\to -1} x^2 = 1$

2 Since $\lim_{x\to -1^-} f(x) \neq \lim_{x\to -1^+} f(x)$, the $\lim_{x\to -1} f(x)$ does not exist

Hence $f(x)$ is not continuous at -1.

Similarly :

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1} x^2 = 1$ $\quad \lim_{x\to 1^+} f(x) = \lim_{x\to 1} 1-x = 0$

2 Since $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$, then $\lim_{x\to 1} f(x)$ does not exist.

Hence $f(x)$ is not continuous at $x=1$.

4. Solve the inequality $|x+5| \geq 2$ in terms of intervals and illustrate the solution set on the real number line.

$$|x+5| = \begin{cases} x+5 & \text{if} \quad x \geq -5 \\ -(x+5) & \text{if} \quad x < -5 \end{cases}$$ 2

4

if $x \geq -5$ $\quad |x+5| \geq 2 \Rightarrow x+5 \geq 2 \Rightarrow x \geq -3$

$\Rightarrow \boxed{x \geq -3}$ Solution 1

4

if $x < -5$ $\quad |x+5| \geq 2 \Rightarrow -(x+5) \geq 2 \Rightarrow -x-5 \geq 2$

$\Rightarrow -x \geq 7 \Rightarrow x \leq -7$

$\therefore \boxed{x \leq -7}$ Solution 2

Final answer: $x \geq -3$ or $x \leq -7$

5. Find the limit if it exists. If the limit does not exist explain why.

$$\lim_{x \to 2} \frac{x^2 + x - 6}{x - 2}$$

$$\lim_{x \to 2} \frac{x^2+x-6}{x-2} = \lim_{x \to 2} \frac{(x-2)(x+3)}{(x-2)} \quad 5$$

$$= \lim_{x \to 2} (x+3) = 5$$

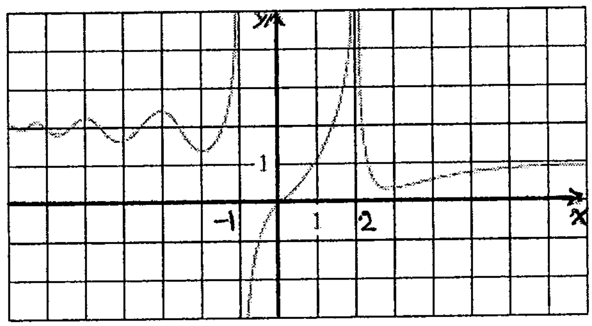
5

6. For the function f whose graph is given, state the following.

a. $\lim_{x \to 2} f(x)$

b. $\lim_{x \to \infty} f(x)$

c. $\lim_{x \to -1^+} f(x)$



4 a) $\lim_{x \to 2^+} f(x) = \infty$ $\quad \lim_{x \to 2^-} f(x) = \infty \Rightarrow \lim_{x \to 2} f(x) = \infty$ (the limit does not exist).

3 b) $\lim_{x \to \infty} f(x) = 1$

3 c) $\lim_{x \to -1^+} f(x) = -\infty$

7. Use the formula $m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ to find an equation of the tangent line to the graph of $y = \sqrt{x}$ at point P(1, 1).

The slope of the tangent line at $a = 1$:

$$m = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} \frac{\sqrt{x} - \sqrt{1}}{x - 1}$$

$$= \lim_{x \to 1} \frac{\sqrt{x} - 1}{x - 1} \quad 2 \quad = \lim_{x \to 1} \frac{\sqrt{x} - 1}{(\sqrt{x} - 1)(\sqrt{x} + 1)} \quad 3$$

or $= \lim_{x \to 1} \frac{\sqrt{x} - 1}{x - 1} \cdot \frac{\sqrt{x} + 1}{\sqrt{x} + 1} = \frac{x - 1}{(x-1)(\sqrt{x} + 1)} = \frac{1}{\sqrt{x} + 1}$

$$= \lim_{x \to 1} \frac{1}{\sqrt{x} + 1} = \frac{1}{1 + 1} = \frac{1}{2}$$

2

$$\therefore \boxed{m = \frac{1}{2}}$$

The equation of the tangent line at $(1, 1)$:

$$y - f(1) = m(x - 1)$$

$$\boxed{y - 1 = \frac{1}{2}(x - 1)} \quad 3$$

8. Differentiate the following functions

(a) $f(x) = \cos(1+x^3)$ (Use chain rule) 2

$y = \cos(1+x^3)$ Let $u \triangleq 1+x^3 \Rightarrow \frac{du}{dx} = 3x^2$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{d}{du}(\cos u) \cdot \frac{du}{dx}$$ 3

2

3

$$= (-\sin u) \cdot (3x^2) = -\sin(1+x^3) \cdot (3x^2)$$

$$= -3x^2 \sin(1+x^3)$$

(b) $y = e^{\left(\frac{x+1}{\sqrt{x-2}}\right)}$ (Use logarithmic differentiation)

2 $\ln y = \frac{x+1}{\sqrt{x-2}} \Rightarrow \frac{d}{dx} \ln y = \frac{d}{dx}\left(\frac{x+1}{\sqrt{x-2}}\right)$

$$\frac{y'}{y} = \frac{(x+1)'\sqrt{x-2} - (x+1)\left(\sqrt{x-2}\right)'}{x-2} \Rightarrow$$

6 $$\frac{y'}{y} = \frac{1 \cdot \sqrt{x-2} - (x+1)\left[\frac{1}{2}(x-2)^{-1/2}\right]}{x-2}$$

$$\frac{y'}{y} = \frac{\sqrt{x-2} - \frac{x+1}{2\sqrt{x-2}}}{x-2} = \frac{x-5}{2\sqrt{(x-2)^3}}$$

2

$$\Rightarrow y' = e^{\left(\frac{x+1}{\sqrt{x-2}}\right)} \cdot \frac{x-5}{2\sqrt{(x-2)^3}}$$

(c) $x^3 + x^2y + 4y^2 = 6$ (Use implicit differentiation)

8 $3x^2 + [2xy + x^2y'] + 8yy' = 0 \Rightarrow$

$3x^2 + 2xy + (x^2 + 8y)y' = 0 \Rightarrow$

2 $y' = -\dfrac{3x^2 + 2xy}{x^2 + 8y}$

(d) $y = \dfrac{x}{1 - \ln(x-1)}$

Use the Law of Quotient:

$y' = \dfrac{1 \cdot (1 - \ln(x-1)) - x\left[0 - \frac{1}{x-1}\right]}{[1 - \ln(x-1)]^2} \Rightarrow$ 10

$y' = \dfrac{1 - \ln(x-1) + \frac{x}{x-1}}{[1 - \ln(x-1)]^2}$