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University of Toronto
Faculty of Arts and Sciences

April 2019 Examinations

MAT223 Practice Final Solutions

Duration: 3 hours
Aids Allowed: None

Exam Reminders:

- Fill out your name, student number, and email address at the top of this page.
- Do not begin writing the actual exam until the announcements have ended and the Exam Facilitator has started the exam.
- As a student, you help create a fair and inclusive writing environment. If you possess an unauthorized aid during an exam, you may be charged with an academic offence.
- Turn off and place all cell phones, smart watches, electronic devices, and unauthorized study materials in your bag under your desk. If it is left in your pocket, it may be an academic offence.
- When you are done with your exam, raise your hand for someone to come and collect your exam. Do not collect your bag and jacket before your exam is handed in.
- If you are feeling ill and unable to finish your exam, please bring it to the attention of an Exam Facilitator so it can be recorded before leaving the exam hall.
- In the event of a fire alarm, do not check your cell phone when escorted outside.

Special Instructions:

- Write legibly and darkly.
- Cross out any work you do not wish to have scored, and clearly indicate if there is work on another page you want scored.
- Show all of your work. Unsupported answers may not earn credit.

Exam Format and Grading Scheme:

Answers must be written on the examination paper.

Question:	1	2	3	4	5	6	7	8	9	10	11	12	Total
Points:	28	8	8	16	8	5	6	13	14	6	10	15	137

Students must hand in all examination materials at the end

1. Complete the following sentences with a mathematically correct definition. No marks will be awarded for a "close" but incorrect definition.

   (a) (2 points) $M \subseteq \mathbb{R}^n$ is a *subspace* if
   See notes

   (b) (2 points) The vectors $\{\vec{v}_1, \ldots, \vec{v}_k\}$ are *linearly independent* if
   See notes

   (c) (2 points) The vectors $\vec{x}$ and $\vec{y}$ are *orthogonal* if
   See notes

   (d) (2 points) The *rank* of a linear transformation $\mathcal{T} : \mathbb{R}^n \to \mathbb{R}^m$ is
   See notes

   (e) (2 points) The *range* of a linear transformation $\mathcal{T} : \mathbb{R}^n \to \mathbb{R}^m$ is
   See notes

   (f) (2 points) A linear transformation $\mathcal{T} : \mathbb{R}^n \to \mathbb{R}^m$ is *one-to-one* if
   See textbook

   (g) (2 points) Let $\mathcal{B}$ be a basis for $\mathbb{R}^n$ and let $\mathcal{T} : \mathbb{R}^n \to \mathbb{R}^n$. The matrix $[\mathcal{T}]_{\mathcal{B}}$ is
   See notes

(h) (2 points) E is a 3×3 *elementary matrix* if
See notes

(i) (2 points) A matrix A is *diagonalizable* if
See notes

(j) (2 points) A matrix A is invertible if
See notes

(k) (2 points) The matrices A and B are *similar* if
See notes

(l) (2 points) The *eigenspace*, E, of a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^n$ with corresponding eigenvalue λ is
See notes

(m) (2 points) The *characteristic polynomial* of the matrix A is
See notes

(n) (2 points) The *determinant* of a linear transformation $\mathcal{T} : \mathbb{R}^n \to \mathbb{R}^n$ is
See notes

2. Let S be the set of all solutions to the system $\begin{cases} x_1 + 6x_2 + 2x_3 - 5x_4 - 2x_5 = -4 \\ 2x_3 - 8x_4 - 1x_5 = 3 \\ x_5 = 7. \end{cases}$

(a) (3 points) Express S in vector form.

$$\vec{x} = t \begin{bmatrix} -6 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -3 \\ 0 \\ 4 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 5 \\ 0 \\ 7 \end{bmatrix}$$

(b) (3 points) Is S a subspace? Prove your answer.

No. Every subspace must contain $\vec{0}$, but $\vec{0} \notin S$.

(c) (2 points) Is S a *translated* subspace? If so, find a subspace V and a vector $\vec{w}$ so that $S = V + \{\vec{w}\}$.

Yes. Let $\vec{w} = -\begin{bmatrix} 0 \\ 0 \\ 5 \\ 0 \\ 7 \end{bmatrix}$. Then $V = S + \{\vec{w}\} = \text{span}\left\{ \begin{bmatrix} -6 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 4 \\ 1 \\ 0 \end{bmatrix} \right\}$ is a subspace.

3. Let $\mathcal{E}$ be the standard basis for $\mathbb{R}^4$. You know the following about the linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^4$.

$$T \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix}_{\mathcal{E}} = \begin{bmatrix} w - x \\ x - y \\ y - z \\ z - w \end{bmatrix}_{\mathcal{E}}$$

(a) (2 points) Find a matrix for T in the standard basis (i.e., find $[T]_{\mathcal{E}}$).

$$[T]_{\mathcal{E}} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 \end{bmatrix}$$

(b) (2 points) Give a basis for the null space of T.

A basis is $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$

(c) (2 points) Give a basis for the range of T

A basis is $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix} \right\}$

(d) (2 points) What is the volume of the image of the unit 4-cube under the transformation T?

Zero

4. For each of the following, give an example if possible. Otherwise, explain why it is impossible.

(a) (2 points) A *non-diagonal* 2×2 matrix A with eigenvalues 7 and 101.

$$\begin{bmatrix} 7 & 1 \\ 0 & 101 \end{bmatrix}$$

(b) (2 points) A 6×4 matrix B whose column space is three dimensional.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(c) (2 points) A linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ whose null space is the y-axis.

Let T be projection onto the xz-plane.

(d) (2 points) A subspace of dimension 3 that is the span of 3 linearly independent vectors in $\mathbb{R}^5$.

$V = \text{span}\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$

(e) (2 points) A subspace of dimension 2 that is the span of 3 linearly independent vectors in $\mathbb{R}^3$.

Impossible. The span of three linearly independent vectors produces a subspace of dimension three.

(f) (2 points) A linear transformation $X : \mathbb{R}^5 \to \mathbb{R}$ whose nullity is 5.

$X(\vec{x}) = \vec{0}$

(g) (2 points) A 3×2 matrix A with positive determinant.

Impossible. Only square matrices have determinants.

(h) (2 points) A 3×3 matrix A with 5 distinct eigenvectors (not necessarily linearly independent).

$I_{3\times3}$

5. Let $P \subset \mathbb{R}^4$ be the subspace with equation $2x + 3y - z + w = 0$, let $Q \subset \mathbb{R}^4$ be the hyper-plane with equation $x + y - z = 0$, let $R \subset \mathbb{R}^4$ be the hyper-plane with equation $x - y + w = 0$, and let $l = P \cap Q \cap R$

(a) (2 points) Is l a subspace? If so, what is the dimension? Explain.

Yes. ℓ is the intersection of subspaces and so it is a subspace. Solving, we see

$$\ell = \text{span}\left\{ \begin{bmatrix} -1 \\ 0 \\ -1 \\ 1 \end{bmatrix} \right\},$$

and so ℓ is one dimensional.

(b) (2 points) Give an example of a linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^3$ so that $l = null(T)$.

T is the transformation induced by the matrix $M = \begin{bmatrix} 2 & 3 & -1 & 1 \\ 1 & 1 & -1 & 0 \\ 1 & -1 & 0 & 1 \end{bmatrix}$.

(c) (2 points) Give an example of a linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^4$ so that $l = range(T)$.

$$T \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} -x \\ 0 \\ -x \\ x \end{bmatrix}$$

(d) (2 points) Find an eigenvalue and a corresponding eigenvector of the linear transformation above.

T above has an eigenvalue of 0 and a corresponding eigenvector of $\begin{bmatrix} 0 \\ 1 \\ 1 \\ 99.7345 \end{bmatrix}$.

6. Suppose we have a 10×10 matrix A such that the matrix-vector equation $A\vec{x} = \vec{b}$ is always consistent.

(a) (1 point) Does $A\vec{x} = \vec{b}$ always have a unique solution? Justify your answer.

Yes. Since $A\vec{x} = \vec{b}$ is always consistent, the column space of A is 10 dimensional. By the rank-nullity theorem we know the nullity of A is zero, and therefore $A\vec{x} = \vec{b}$ always has a unique solution.

(b) (2 points) What are the possible dimensions of the row space of matrix A? Justify your answer.

Only 10. The dimension of the row space of A is always equal to the dimension of the column space.

(c) (2 points) What are the possible dimensions of the null space of matrix A? Justify your answer.

Only 0. As explained in part (a).

7. Let M be a 3×3 matrix with three distinct eigenvalues $\lambda_1, \lambda_2, \lambda_3$ and corresponding eigenvectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$.

(a) (3 points) Prove that $\{\vec{v}_1, \vec{v}_2\}$ is linearly independent.

Since λ_1 and λ_2 are distinct, without loss of generality, assume $\lambda_1 \neq 0$. Now, suppose

$$\alpha\vec{v}_1 + \beta\vec{v}_2 = \vec{0} \qquad \Longrightarrow \qquad \alpha\vec{v}_1 = -\beta\vec{v}_2.$$

Applying M to both sides we have

$$\alpha\lambda_1\vec{v}_1 = M(\alpha\vec{v}_2) = M(-\beta\vec{v}_2) = -\beta\lambda_2\vec{v}_2 \qquad \Longrightarrow \qquad \alpha\vec{v}_1 = -\beta\tfrac{\lambda_2}{\lambda_1}\vec{v}_2.$$

This means $-\beta\vec{v}_2 = -\beta\frac{\lambda_2}{\lambda_1}\vec{v}_2$. Since $\lambda_1 \neq \lambda_2$, we know $\frac{\lambda_2}{\lambda_1} \neq 1$. Since $\vec{v}_2 \neq \vec{0}$, we know $\beta = 0$. Further, since $\vec{v}_1 \neq \vec{0}$, we know $\alpha = 0$.

Since the only way for $\alpha\vec{v}_1 + \beta\vec{v}_2 = \vec{0}$ to be satisfied is with $\alpha = \beta = 0$, by definition, $\{\vec{v}_1, \vec{v}_2\}$ is linearly independent.

(b) (3 points) Is M diagonalizable? Justify your answer.

Yes. The geometric multiplicity of each eigenvector is 1 and so the sum of the geometric multiplicities is 3. Since M is a 3×3 matrix, it must be diagonalizable.

8. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation . Let $\mathcal{E} = \{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$ be the standard basis for $\mathbb{R}^3$. Let $\mathcal{C} = \{\vec{c}_1, \vec{c}_2, \vec{c}_3\}$ where $c_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}_{\mathcal{E}}, c_2 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}_{\mathcal{E}}, c_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}_{\mathcal{E}}$, and suppose $[T]_{\mathcal{E}} = \begin{bmatrix} 2 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$.

(a) (3 points) What are the eigenvalues of $[T]_{\mathcal{E}}$?

$2, 2, 1$ are the eigenvalues.

(b) (2 points) Find the matrix Q which changes vectors from the $\mathcal{C}$ basis to the $\mathcal{E}$ basis.

$$Q = \begin{bmatrix} 0 & 2 & 1 \\ 0 & -1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

(c) (2 points) Find a matrix that changes vectors from the $\mathcal{E}$ basis to the $\mathcal{C}$ basis.

$Q^{-1} = \begin{bmatrix} 0 & 1 & 1 \\ 0 & -1 & 0 \\ 1 & 2 & 0 \end{bmatrix}$. (Solve many equations to get here.)

(d) (2 points) Find a matrix D such that $[T]_{\mathcal{E}} = QDQ^{-1}$.

$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

(e) (2 points) Calculate D^{10} (where D is the matrix you found in the previous part).

$$D^{10} = \begin{bmatrix} 2^{10} & 0 & 0 \\ 0 & 1^{10} & 0 \\ 0 & 0 & 2^{10} \end{bmatrix}$$

(f) (2 points) Is $[T]_{\mathcal{E}}^{10}$ diagonalizable? If so, find a matrix P and a diagonal matrix A so that $[T]_{\mathcal{E}}^{10} = PAP^{-1}$.

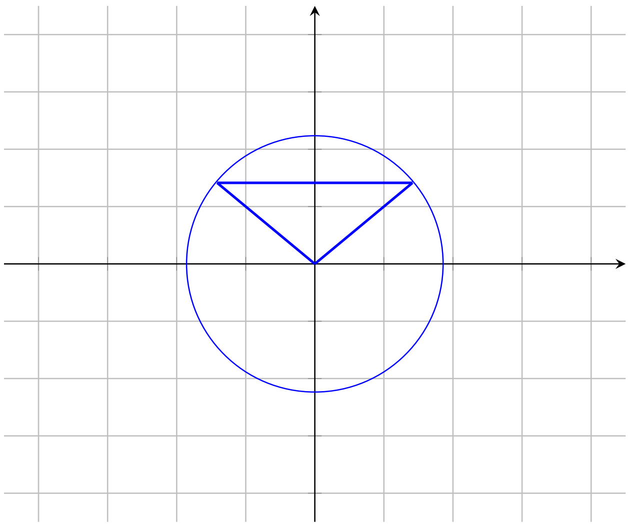
Yes. $P = Q$, where Q is from the previous parts.

9. Let $S \subseteq \mathbb{R}^2$ be the sides of the triangle with corners $\{\vec{0}, \vec{e}_1, \vec{e}_2\}$ and C be the unit circle centered at the origin, let T be the linear transformation with standard matrix

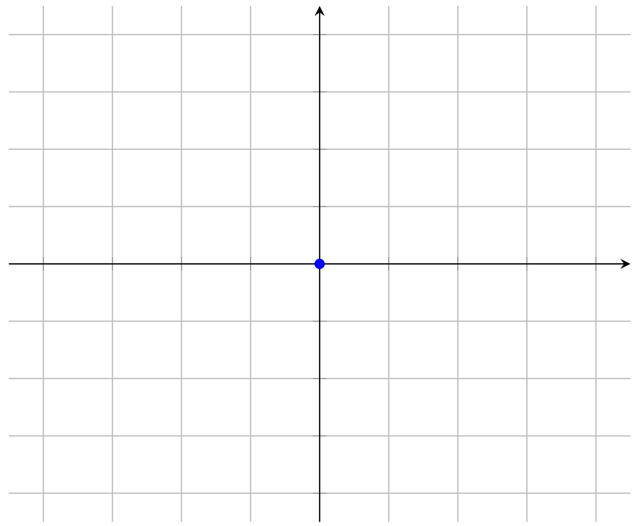
$$M = \begin{bmatrix} \sqrt{2} & -\sqrt{2} \\ \sqrt{2} & \sqrt{2} \end{bmatrix}$$

Draw the following subsets of $\mathbb{R}^2$.

(a) (2 points) $T(S) \cup T(C)$



(b) (2 points) $null(T)$



(c) (2 points) Find the area of $T(S)$ and $T(C)$

The area of $T(S)$ is 2 and the area of $T(C)$ is 4π.

(d) (4 points) Describe, in complete sentences, what T does geometrically.

T rotates counter clockwise by 45° and then stretches in all directions by a factor of 2.

(e) (4 points) If possible, express T as a composition of two, simpler linear transformations.

Let R be rotation counter clockwise by 45° and let $D : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $D\vec{x} = 2\vec{x}$. Then $T = D \circ R$.

10. (6 points) Ron and Jake are arguing about the following claim by a student they met at a food truck:

> *Any square matrix can be written as a product $E_k \cdots E_1 P$ where each E_i is an elementary matrix and P is the matrix for a projection.*

Ron thinks the claim is true because elementary matrices can be used as a replacement for row reduction. Jake is not yet convinced.

Explain to Ron and Jake, using complete English sentences, whether or not the claim is true by either filling out the details of Ron's argument, or showing that the claim is false and explaining where Ron's argument went wrong.

Answer 1: The statement is *true*, but Ron's argument misses the bulk of the proof. What is clearly true is for a matrix A, we can always write $\text{rref}(A) = E_1 \cdots E_k A$ for elementary matrices E_1, ..., E_k (because elementary matrices perform row operations).

Fact (i): If matrices A and B have the same shape and the same row space, then $\text{rref}(A) = \text{rref}(B)$. This follows because the reduced row echelon form of a matrix is unique. Since row reduction doesn't change the row space, $\text{rref}(A)$ and $\text{rref}(B)$ are both row-reduced matrices with identical row spaces, and so they must be equal.

Fact (ii): If P is a matrix for a projection, then the row space and column space of P are the same. Proof: Let V be a subspace, let $\mathcal{P}$ be projection onto V and let $P = [\mathcal{P}]_{\mathcal{E}}$ be the matrix for P. We know that $\text{row}(P)$ consist of all vectors orthogonal to $\text{null}(P)$. Geometrically, we know $P\vec{v} = \vec{0}$ if and only if $\vec{v} \perp V$ (because $\mathcal{P}$ is a projection). Since $\text{col}(P) = V$, we have both the column space and the row space of P consist of all vectors orthogonal to $\text{null}(P)$, and so $\text{row}(P) = \text{col}(P)$.

Fix a square matrix A and let P be the matrix that projects onto the row space of A. By fact (ii), we know $\text{row}(P) = \text{col}(P) = \text{row}(A)$. Therefore, by fact (i), $\text{rref}(P) = \text{rref}(A)$ and so there exists elementary matrices $E_1, \ldots, E_k$ and $E'_1, \ldots, E'_{k'}$ so that

$$E_1 \cdots E_k A = E'_1 \cdots E'_{k'} P.$$

Since the inverse of an elementary matrix is an elementary matrix, we have

$$A = E_K^{-1} \cdots E_1^{-1} E'_1 \cdots E'_{k'} P,$$

which completes the proof.

Answer 2: The statement is *true*, but Ron's argument is misleading. Every *invertible* matrix can be written as a product of elementary matrices. One way to see this is because the algorithm for computing the inverse of a matrix could be written out in terms of elementary matrices replacing the row-reduction steps.

We now see the claim is equivalent to the claim "Every square matrix A can be written as $A = BP$ where B is invertible and P is the matrix for a projection." We will prove this claim.

Fix an $n \times n$ matrix A, and define the following transformations:

$$P = \text{ projection onto row}(A) \qquad Q = \text{ projection onto null}(A).$$

Let $\vec{n}_1, \dots, \vec{n}_k$ be a basis for null(A) and let $\vec{w}_1, \dots, \vec{w}_k$ be a basis for null(A^T) (notice there are the same number of $\vec{n}_i$'s as $\vec{w}_i$'s).

Extend $\vec{n}_1, \dots, \vec{n}_k$ and $\vec{w}_1, \dots, \vec{w}_k$ to both be bases for $\mathbb{R}^n$. Then define the $n \times n$ matrix W by

$$W\vec{n}_i = \vec{w}_i.$$

Because $\{\vec{n}_i\}_{i=1}^n$ is a basis, this completely defines W. Because $\{\vec{w}_i\}_{i=1}^n$ is a bases, W is invertible. Further, by construction, if $\vec{x} \in$ null(A), then $W\vec{x} \in$ null(A^T).

Fact (i) $PP = P$. Since P does projection, applying it twice does the same thing as applying it once.

Fact (ii): $I = P + Q$. We can see this because the row space and null space of A are orthogonal, and so $Q = I - P$. Therefore $P + Q = P + I - P = I$.

Fact (iii): $AP = A$. By fact (ii), $A = A(P + Q)$. Distributing, $A = AP + AQ = AP$, since $AQ = 0$ (because Q outputs vectors in null(A)).

Fact (iv): $QP = 0$. P outputs vectors in the row space of A. Q projects onto the null space of A. Since the row space and null space of A are orthogonal, we must have $QP\vec{x} = \vec{0}$ for all $\vec{x}$. Alternatively, $Q = (I - P)$ and so $QP = (I - P)P = P - PP = 0$ since $PP = P$ (which follows from fact(i)).

Now, define B as

$$B = AP + WQ$$

and note that by applying facts (i)–(iv), we see

$$BP = (AP + WQ)P = APP + WQP = AP = A.$$

Claim: B is invertible. (Showing this will complete the proof.)

Suppose $B\vec{x} = (AP + WQ)\vec{x} = \vec{0}$. Then $AP\vec{x} = A(P\vec{x}) = W(-Q\vec{x}) = -WQ\vec{x}$. By construction the column space of A is orthogonal to the column space of W. We conclude

$$AP\vec{x} = -WQ\vec{x} = \vec{0}.$$

But W is invertible and $AP = A$, so $\vec{x} \in$ null$(Q) =$ row(A) and $\vec{x} \in$ null(A). Because the row space of A and the null space of A are orthogonal, we conclude $\vec{x} = \vec{0}$. Therefore, $B\vec{x} = \vec{0}$ if and only if $\vec{x} = \vec{0}$. Since B is a square matrix, B is invertible.

11. In this question you will be working with a new definition.
A square matrix is *positive definite* if $(A\vec{x}) \cdot \vec{x} > 0$ for all $\vec{x} \neq \vec{0}$.

(a) (2 points) Given an example of such a matrix.

$I_{2\times 2}$

(b) (3 points) What is the null space of a positive definite matrix? Justify your answer.

If A is positive definite, $\text{null}(A) = \{\vec{0}\}$. Suppose $A\vec{x} = \vec{0}$. Then $(A\vec{x}) \cdot \vec{x} = \vec{0} \cdot \vec{x} = 0$, and so $\vec{x} = \vec{0}$.

(c) (5 points) If a 2×2 matrix has two distinct positive eigenvalues, could the matrix positive definite? Justify your answer.

Yes. Let $A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$. A has eigenvalues of 2 and 3. Further,

$$A \begin{bmatrix} x \\ y \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = 2x^2 + 2y^2 > 0$$

if $x, y \neq= 0$ and equal to zero otherwise.

12. Suppose a matrix A is such that

(i) the matrix-vector equation $A\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ has a unique solution; and

(ii) $A \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

(a) (1 point) What is the size of the matrix A.

3×2

(b) (2 points) What is the dimension of the row space of matrix A? Justify your answer.

2. Since $A\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ has a unique solution, then nullity of A is 0. By the rank-nullity theorem, the rank of A is 2 and so the dimension of the row space of A is 2.

(c) (3 points) Is it possible that both $\vec{e}_1$ and $\vec{e}_2$ are in the range of T_A (the transformation induced by A)?

No. $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is in the range of T_A. If $\vec{e}_1, \vec{e}_2$ were also in the range of T_A then the range of T_A would contain $\left\{ \vec{e}_1, \vec{e}_2, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$, which is a set of three linearly independent vectors, making the range 3 dimensional. But, we already know the range must be two dimensional.

Consider property (iii)

(iii) $A \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$

(d) (3 points) How many matrices exist satisfying properties (i), (ii), and (iii)? Justify your answer.

None. Since $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^2$, we can explicitly solve for the entries in A, finding

$$A = \begin{bmatrix} 0 & 1/2 \\ 0 & 1/2 \\ 0 & 1/2 \end{bmatrix}.$$

However, this matrix has a non-trivial null space and so won't satisfy property (i).

(e) (3 points) How many matrices exist satisfying properties (i) and (ii)? Justify your answer.

Infinitely many. If the columns of A are linearly independent, then $A\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ will have a unique solution. For example,

$$A = t \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1/2 \end{bmatrix} + (1-t) \begin{bmatrix} 0 & 1/2 \\ 1 & 0 \\ 0 & 1/2 \end{bmatrix}$$

will satisfy properties (i) and (ii) for any $t \in [0, 1]$.

(f) (3 points) How many matrices exist satisfying properties (ii) and (iii)? Justify your answer.

One. As outlined in part (d),

$$A = \begin{bmatrix} 0 & 1/2 \\ 0 & 1/2 \\ 0 & 1/2 \end{bmatrix}$$

is the only one.

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