École de gestion TELFER School of Management

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ADM 2304
APPLIED STATISTICAL METHODS IN BUSINESS

8 February 2014

9:00 – 11:00 AM

NAME (please print): ____________________________________

Student Number: __________________________ Section: _____________

Instructions

Length of Exam: 5 pages, plus 1 page of Minitab output (please return).

Please show all your work and explain your answers briefly when asked. All tests must include hypotheses, test statistics and rejection regions, decisions, and conclusions for full marks.

You are encouraged to use the Minitab output as much as possible.

You are permitted to have a non-programmable calculator and a sheet (8.5 x 11 inch) of notes.

Three pages of statistical tables (normal and t) are provided separately (please keep).

Marks:	________	+	________	+	_______	+	________	=	___________
	9		10		10		11		40

Question 1. [9 marks]

The unemployment rate in Canada in December 2013 was reported to be 7.2%. Over the previous three months, it was a steady 6.9%. Although the unemployment rate is estimated from the Labour Force Survey, using a multi-stage sampling design, assume that it is based on a simple random sample of 48,000 respondents.

(a) Does the estimated unemployment rate for December 2013 provide sufficient evidence to show that the true rate is higher than 7.0%? Use the critical value approach and the 0.05 level of significance.

[4]

```
Test of p = 0.07 vs p > 0.07
                                        95%
                                      Lower
Sample      X      N  Sample p        Bound  Z-Value  P-Value
1        3456  48000  0.072000     0.070059     1.72    0.043

-Ho: p = .070; Ha: p > .07
-Z = .002 / sqrt(.07*.93 / 48000) = 1.72
-rejection region:  z > 1.645
-Reject Ho (since z = 1.72 > 1.645) and conclude rate exceeds 7.0%
```

(b) Calculate an appropriate 95% 1-sided confidence interval to estimate the unemployment rate (calculate this to five decimal places). Explain how this interval confirms your decision and conclusion in part (a).

[3]

Lower Bound of 0.072 – 1.645 * sqrt(072*.928/48000) = 072 – 1.645 * 0.0011798
= 0.072 - .001941 = 0.07006 or CI is (.07006, infinity)

Since this interval does not cover the hypothesized value of 0.070, we reject Ho.

-0.5 mark for standard error calculation, 0.5 for z-value, 1 for final calculation of lower bound and noting it is a lower bound (1 mark out of 2 for 2-sided CI)
-1 mark for noting confirmation (2-sided interval would not confirm (a)).

(c) What sample size would be required to estimate the unemployment rate with a margin of error of plus-or-minus 0.001 (or ± 0.1%) using a 95% confidence level?

[2]

$n = (1.96 / .001)^2 * .072 * .928 = 256{,}681$

If p = 0.70 used and n = 250,089, then only 1 mark.

Question 2. [10 marks]

An operating room (OR) manager is an attempting to run her business better. As a result she wants to ensure that she has an accurate assessment of the length of surgeries for each surgeon. For surgeon A, she collects a sample of 26 surgical times and determines a sample mean of 167.038 minutes and a sample standard deviation of 158.902 minutes.

(a) Having taken 2nd year statistics, she knows that she needs to provide a confidence interval around this estimate but has forgotten how. Provide her with a 95% confidence interval for the population mean.

[2]

$167 \pm 2.06 * 31.2 = 167 \pm 64.3 = (102.7, 231.3)$

(b) Surgeon A has taken statistics himself and demands to see the boxplot of the sample – provided in Appendix A. Based on the boxplot, is the calculation of the 95% confidence interval in part (a) valid? Why or why not?

[2]

boxplot shows some extreme outliers which combined with the small sample size suggests the normal assumption is not warranted so CI is invalid.

(c) Appendix A also provides a boxplot with four unusual values taken out. The mean of this "censored" data set is 102.955 with a standard error of 31.0276. Conduct the appropriate parametric hypothesis test to determine if the true mean surgical time is less than 120 minutes.

[4]

Ho: mu = 120; Ha: mu < 120

t = (102.955 – 120) / (31.0276/sqrt(22) = - 17.045 / 6.615 = - 2.58

Rejection region is t < - 1.72 (based on 21 d.f.)

Reject Ho (since t = -2.58 < -1.72) and conclude mean time is less than 120 minutes.

```
                       Test of mu = 120 vs < 120
                                                  95% Upper
Variable         N    Mean  StDev  SE Mean      Bound      T      P
Censored Data   22  102.95  31.03     6.62     114.34  -2.58  0.009
```

Many solutions gave the rejection region as | t | > 1.72. However, this must be accompanied by the condition that t < 0; otherwise, a positive t would lead to a rejection .

(d) Explain whether this test is valid given the data distribution.

[2]

boxplot stills show a bit of a right skewness but much reduced. I would accept both the argument that the skewness makes the t-test suspect and the argument that the skew is not bad so the t-test is fine (it lies in the gray zone in my opinion) as long as they describe the box-plot correctly as still having some skewness.

Question 3. [10 marks]

In 2009 and 2013, two surveys were conducted to determine the percentage of smokers among young women in the 20 to 24 age group. In 2009, a random sample of size 350 was taken and it was found that 80 were smokers. In 2013 another random sample of size of 200 was taken and it was found that 36 were smokers.

(a) Test the hypothesis that there has been a change in the percentage of female smokers in the age groups under consideration. Use the critical value approach and a 0.05 level of significance.

[5]

```
Sample    X     N  Sample p
1        80   350  0.228571
2        36   200  0.180000

Difference = p (1) - p (2)
Estimate for difference:  0.0485714
95% CI for difference:  (-0.0204958, 0.117639)
Test for difference = 0 (vs not = 0):  Z = 1.34  P-Value = 0.179
```

-Ho: p(2009) – p(2013) = 0 ; Ha: $\neq 0$

-Pooled p-bar = 116 / 550 = 0.211

-Z = 0.0486 / sqrt(0.211*0.789*(1/350 + 1/200) = .0486/.036 = 1.35

-rejection region is |z| > 1.96

-Do not reject Ho (since |z| = 1.35 is not > 1.96) ; conclude there is insufficient evidence to show there has been change in percentage of female smokers in 20-24 group.

(b) Find the p-value for the result in part (a).

[1] p-value = P(|Z| > 1.35) = 2 P(Z < -1.35) = 2 (.0885) = 0.177

0.5 mark for .0885 only

A common incorrect answer was p-value = P(Z < 1.35) = 0.9115.

(c) Calculate the 95% two-sided (symmetric) confidence interval for the true difference using the data provided.

[2] -SE = sqrt(.228571*.771429/350 + .18*.82/200) = 0.03524

-CI is 0.0486 ±1.96 * 0.03524 = 0.0486 ± 0.0691= (.-0.0205, 0.1177)

(d) Explain how the p-value and the confidence interval lead you to the same decision and conclusion as part (a).

[2] p-value not < .05 and the CI covers zero, therefore do not reject Ho.

Many wrote incorrectly, "Since the p-value is covered by the CI, we do not reject the null hypothesis." The CI estimates the difference in proportions whereas the p-value estimates the rarity of the test statistic assuming the null hypothesis is true.

Question 4. [11 marks]

The Programme for International Student Assessment (PISA) is a worldwide study by the Organization for Economic Co-operation and Development (OECD). It tests the academic skills of 15-year-old students in dozens of countries including Canada. In December 2013, they published a country-by-country comparison of how these countries performed on various academic tests. One of the most publicized results had to do with the average mathematics score. Canada's ranking appears to be slipping. For each country, there is a comparison of how boys and girls perform. In Canada, the mean math score for boys was listed as 523 with a standard error of 2.1; for girls, the mean was 513 with the same standard error.

a) The math test was designed to have an average of 500 with a standard deviation of 100. Assuming that the Canadian results have a sample standard deviation also equal to 100, how many boys or girls took the PISA math test in Canada?

[1]

SE = 2.1 = 100 / sqrt(n) = 2.1 therefore n = $(100/2.1)^2$ = 2267 boys and 2267 girls

or n = $(z_{\alpha/2} * s / z_{\alpha/2} * SE)^2 = (s/SE)^2 = (100/2.1)^2$ = 2267 where M = $z_{\alpha/2}$*SE

b) Test at the 0.01 level of significance whether the data are sufficient to show there is a difference between the mean math scores of boys and girls.

[5]

-Ho: mu1 = mu2; Ha: mu1 ≠ mu2

-SE = sqrt($2.1^2 + 2.1^2$) = sqrt(100^2/2267 + 100^2/ 2267) = 2.97 *(sample sizes not req'd)*

-t = (523-513)/ 2.97 = 3.37 *(t = (523 – 513)/2.1 only for a 1-sample test)*

-Reject Ho if |t| > 2.576.

-We reject Ho (since 3.37 > 2.576) and conclude the mean math scores are different.

c) Suppose you had no idea whether boys or girls do better on math tests. However, after you see that the sample mean score for boys is higher than the mean score for girls, you decide to perform a one-sided test. Is this fair or ethical? Explain briefly.

[1]

A 1-sided test must be based on an a priori belief that boys do better than girls. Deciding on a 1-sided test after seeing the direction of the data makes it easier to reject the null hypothesis than a 2-sided test. This is unethical.

d) What distributional assumption(s) should you check to satisfy yourself that the test in (b) is valid? Explain why this is a reasonable assumption.

[2]

Assumption: Since the samples are very large, we need only the assumption that the data are not extremely skewed. Assumption of normality is not acceptable.

Could accept the answer that no distributional assumptions are necessary given the extremely large sample; however, population cannot have infinite variance.

Reason: This is reasonable since the scores are constrained to have a minimum of zero and a maximum score equal to perfect. ?No reason needed for 2nd answer above?

e) Suppose you wanted to estimate the average boys' score using a 99% symmetric (two-sided) confidence interval with a plus-or-minus margin of error of 1.5. What sample size would be required?

[2] $n = (z * \sigma / M)^2 = (2.576 * 100 / 1.5)^2 = 29492$