AK/ADMS3530 3.0
Fall 2008
Assignment #1 Solutions

Instructions:

(1) This assignment is to be done individually. **You must sign and submit the standard cover page supplied as the last page of this assignment.**
(2) Before you start, please read the note "**Writing Style Required for ADMS3530 Assignments**" posted in the "Assignments" folder on the course web site. Please stick to the writing guidelines suggested in the note.
(3) This assignment is **due in class**, the **week of October 14**, 2008.
(4) This assignment can be typed or handwritten. Work that is too difficult to read due to messiness and poor handwriting will receive zero credit. You must show your work to receive full credit.
(5) This assignment carries a total mark of **100 points**.
(6) For **Internet section students**, the assignment must be uploaded to the Centre for Distance Education: **http://www.atkinson.yorku.ca/cde/assignmentupload** and identified precisely in accordance with the course outline by Tuesday, **October 14, midnight**.
(7) Late assignments will not be accepted whether for technical or any other reason.
(8) Decimal places: please keep at least 4 in your calculations and 2 in your final answers.

Notations

We may denote the PV and FV annuity factors respectively by PVAF(r,n) and FVAF(r,n), i.e.:

$$PVAF(r,n) \equiv \frac{1-(1+r)^{-n}}{r} \quad ; \quad FVAF(r,n) \equiv \frac{(1+r)^{n}-1}{r}$$

Question 1 (TVM) (16 marks)

This question has the following two independent parts, (a) and (b).

(a) April is saving $250 in *nominal* dollars at the end of each year for the next 12 years. The expected annual nominal interest rate is 3% for the first 4 years, 3.6% for the next 6 years and 4% for the last 2 years. The expected annual rate of inflation is 2% for the first 4 years, 2.2% for the next 6 years and 2.5% for the last 2 years. At the end of 12 years, how much will April have in *real* dollars? What is the present value today of the savings account future value that you have just calculated? (8 marks)

(b) You plan to retire in 45 years and want to have $2 million in your account by then. You deposit an equal amount into your account at the end of each month consecutively for the next 30 years and then stop the contributions. The monthly interest rate is 1.06%. What will this equal amount be? (8 marks)

Solution

(a) First, in 12 years April will have in nominal dollars the amount of:

FV = $250 x FVAF (4, 3%) × $(1.036)^6$ × $(1.04)^2$
+ $250 x FVAF (6, 3.6%) × $(1.04)^2$
+ $250 x FVAF (2, 4%)
FV = $3,684.30

Then, in terms of real dollars, ($3,684.30) / [$(1.02)^4(1.022)^6(1.025)^2$], which is equal to $2,843.16 in real dollars.

There are two equivalent ways to compute the present value of the savings account future value:

(1) In terms of nominal quantities:

PV = $3,684.30 / [$(1.03)^4(1.036)^6(1.04)^2$] = $2,447.83

(2) In terms of real quantities:

The expected annual real interest rate is:

$$\frac{(1+\text{nominal rate})}{(1+\text{inflation rate})}-1=\frac{(1+3\%)}{(1+2\%)}-1=0.9804\%$$ for the first 4 years,

1.3699% for the next 6 years, and 1.4634% for the last 2 years. It follows that the PV of the $2,843.16 real dollars is:

$2,843.16 / [$(1.009804)^4$ $(1.013699)^6$ $(1.014634)^2$] = $2,447.83.

(b) This is a 30-year (or 360-month) annuity followed by a single lump sum for another 15 years. First, we compute the EAR:

$$EAR=(1+1.06\%)^{12}-1=13.4884\%.$$

Next, consider the single lump sum. At the beginning of Year 31, the PV of this single sum is calculated as follows:

$$PV = FV \times (1 + \text{EAR})^{-n}$$
$$PV = \$2{,}000{,}000 \times (1 + 13.4884\%)^{-15}$$
$$PV = \$299{,}748.55$$

The above PV is the amount to which the 30-year annuity will have accrued over the next 30 years. Therefore, we have:

$$FV = PMT \times \left[\frac{(1 + r_{\text{month}})^{n} - 1}{r_{\text{month}}} \right]$$
$$\$299{,}748.55 = PMT \times \left[\frac{(1 + 1.06\%)^{360} - 1}{1.06\%} \right]$$
$$PMT = \$73.01$$

The monthly deposit is \$73.01.

Question 2 (TVM) (12 marks)

This question has three parts, with (c) being independent from parts, (a) and (b).

(a) You've inherited \$15,000,000 and have decided to use this money to open a shelter for abused women in Aurora one year from today. The annual operating costs are estimated to be \$350,000 and are expected to grow at a constant rate of 2% (inflation) starting in year 2. If the interest rate is 4.5%, would you be able to fund this shelter? (4 marks)

(b) Given the above scenario, what would be the maximum amount available for annual operating expenses? (4 marks)

(c) What is the future value of a 20-year growing annuity if the growth rate is 2%, the annual interest rate is 6% and the payment at the end of year 1 is \$5,000? (4 marks)

Solution

(a) This is a growing perpetuity:

PV = C/ (r - g)
PV = \$350,000/(4.5% –2%) = \$14,000,000

Since the present value of this growing perpetuity is \$14,000,000, you will have enough money to fund this shelter with your \$15,000,000 inheritance.

(b) The initial operating cost (in the growing perpetuity) is given by:

$$15,000,000 = C/(0.045-0.02)$$
$$\underline{C = \$375,000}$$

Therefore, $<u>375,000 is the maximum amount</u> available for annual operating expenses.

(c) The FV formula for a growing annuity gives us:

$$FV = \frac{C_1}{r-g} \times \left[(1+r)^n - (1+g)^n\right]$$
$$= \frac{5,000}{0.06-0.02} \times \left[(1+6\%)^{20} - (1+2\%)^{20}\right]$$
$$= \underline{\$215,148.51}$$

Question 3 (TVM) (16 marks)

Consider the following monthly cash flows (see the diagram below):

- Cash flows of an amount X occur in months 1, 3, 5, …, 17 and 19 (the 10 odd-numbered months)

- Cash flows of an amount Z occur in months 2, 4, 6, …, 18 and 20 (the 10 even-numbered months)

The APR is 6% and is compounded on a monthly basis.

(a) How much money will be accumulated 20 months from now if X = $1,000 and Z = $0? (4 marks)

(b) What is the present value of these cash flows today if X = $1,000 and Z = - $1,000? (6 marks)

(c) If the present value of these cash flows today is $12,000, and the future value of the Z's cash flows 20 months from now is $5,000, what is the value of X? (6 marks)

Solution

(a) The monthly interest rate is 0.5% but since the X's cash flows are made every two months, we need to calculate the 2-months equivalent interest rate:

$$r = (1+0.5\%)^2 - 1 = 1.0025\%$$

The future value 19 months from now is simply given by the annuity formula:

$$FV_{19} = \$1{,}000 \times \left[\frac{(1+1.0025\%)^{10} - 1}{1.0025\%} \right] = \$10{,}463.40$$

And the future value 20 months from now is given by:

$$FV_{20} = \$10{,}463.40 \times (1+0.5\%) = \$10{,}515.72$$

(b) The present value of the Z's cash flows is given by:

$$PV_Z = -\$1{,}000 \times \left[\frac{1}{1.0025\%} - \frac{1}{1.0025\% \times (1+1.0025\%)^{10}} \right]$$

$$PV_Z = -\$9470.03$$

And the present value of the X's is given by:

$$PV_X = \$1{,}000 \times \left[\frac{1}{1.0025\%} - \frac{1}{1.0025\% \times (1+1.0025\%)^{10}} \right] \times (1+0.5\%)$$

$$PV_X = \$9{,}517.38$$

Note: We first compute the value one period before today as an annuity with bi-monthly payments of $1,000 and a bi-monthly rate of 1.0025%; and then we multiply by (1+0.5%) – compound over one month, to bring the value to today.

The total present value is equal to:

$$PV = \$9{,}517.38 - \$9470.03 = \$47.35$$

(c) We know the future value of the Z's 20 months from today, this allows us to find the value of Z:

$$\$5{,}000 = Z \times \left[\frac{(1+1.0025\%)^{10} - 1}{1.0025\%} \right]$$
$$Z = \$477.86$$

and also their present value today:

$$PV_Z = \frac{\$5{,}000}{(1+1.0025\%)^{10}} = \$4{,}525.31$$

The present value of the X's is then equal to:

$12,000 - $4,525.31 = $7,474.69

The value of X can be derived from the following equation:

$$\$7{,}474.69 = X \times \left[\frac{1}{1.0025\%} - \frac{1}{1.0025\% \times (1+1.0025\%)^{10}} \right] \times (1+0.5\%)$$
$$X = \$785.37$$

Question 4 (Mortgage) **(16 marks)**

One contributing factor to the recent real estate and credit crisis in the United Sates was the surge in Adjustable Rate Mortgages (ARM's) offered to customers who had little or no understanding of finance. Your retired Aunt in Florida purchased a new condo near Sarasota two years ago (October 1, 2006) at a price of $300,000. The interest rate for the first two years of the 15-year mortgage was fixed at 2.8% (APR, compounded semiannually) and no down payment was required. Beginning year 3 (October 1 2008), the rate adjusts to LIBOR plus 3% (also APR, compounded semiannually). Your Aunt has just noticed the ARM clause and has come to you for advice as she knows you are a business student and recently you told her that you are 'acing' the 3530 finance course.

(a) If LIBOR is currently at 4.25%, what will be the difference in payments between her September 1, 2008 and October 1, 2008 payments? (8 marks)

(b) Since the Florida real estate market has declined sharply, she recently had her condo appraised at $245,000. How much higher or lower is the new market value compared to the remaining principal amount on the mortgage as of October 1, 2008? (2 marks)

(c) What will be the total interest amount that your Aunt will pay over the 15-year mortgage period if LIBOR remains at 4.25%? (6 marks)

Solution

(a) First find the **original monthly mortgage payment**:

A semi-annually compounded rate of 2.8% is equivalent to a monthly rate i_m given by:

$EAR = (1 + i_m)^{12} - 1 = (1 + 2.8\%/2)^2 - 1 = 2.8196\%$
$\underline{i_m = 0.2320\%}$

The mortgage has originally 15 years or 180 months. The annuity formula for the condo price is:

300,000 = PMT x PVIFA(0.2320%,180)
PMT = $2,040.72

Next, find the **new monthly mortgage payment** beginning on Oct 1, 2008. You will have to first determine the remaining balance of the mortgage loan (or remaining principal) after 2 years (with 13 years remaining, or 156 months):

$PV_{Oct,\ 2008}$ = $2,040.72 x PVIFA(0.2320%,156)
= $266,850.51

Now we use this $PV_{Oct,\ 2008}$ to find the new monthly payment with the higher rate of 7.25% (i.e. 4.25% + 3%), semi-annually compounded. The new monthly rate is:

$EAR = (1 + i_m)^{12} - 1 = (1 + 7.25\%/2)^2 - 1 = 7.3814\%$
$\underline{i_m = 0.5952\%}$

The mortgage has now 13 years or 156 months. The annuity formula for the condo price gives us the new monthly payment as:

266,850.51 = PMT x PVIFA(0.5952%,156)
PMT = $2,630.63

The difference in payments is $2,630.63 – $2,040.72 = **$589.91 per month**

(b) New Market Value:	$245,000	
Remaining Principal at end of year 2:	$266,851.	(from part a)
	-$ 21,851	

So the new market value of her condo is $21,851 lower than the remaining principal amount of the loan.

(c) Interest paid during the first two years:

Total payments made: $2,040.72 x 24 payments = $48,977.28
Principal paid off: ($300,000 - $266,850.51) = $33,149.49
Interest paid in the first two years $15,827.79

Interest paid years 3 to 15:

Total payments made: $2,630.63 x 156 payments = $410,378.28
Principal paid off: = $266,850.51
Interest paid in the last 13 years: $143,527.77

Total interest paid over entire 15 years = $15,827.79 + $143,527.77
= $159,355.56

Question 5 (Bonds) (16 marks)

You purchased a 10-year bond on the basis of a current yield of 6.5%. The face value of the bond at maturity is $1,000 and the coupon rate is 6% (APR) payable semi-annually.

(a) What are the purchase price of the bond and its YTM? (6 marks)

(b) The day after you bought the bond, interest rates on similar bonds decreased by one full percentage. You were planning to sell the bond after 3 years. If interest rates don't change between now and then, what price would you get for your bond in 3 years? (4 marks)

(c) Instead, you sold your bond a year after purchasing it. Calculate the rate of return on your investment at yield to maturity of 5.5% and any interest income received on the bond is reinvested at 1% APR compounded semi-annually? (6 marks)

Solution

(a) The price of the bond is obtained as:

P = Coupon / Current Yield = $60 / 6.5% = $923.08

Knowing the price of the bond, one can solve for the YTM using the price equation:

$P = \$30 \times PVAF(y,20) + \$1{,}000 \times (1+y)^{-20} = \$923.08$

Solving for y, we have y = 3.5433%, and <u>YTM = 7.0867%</u>

(b) The yield on similar bonds has now dropped to 6.0867%. The price of the bond three years from now (with only 14 coupons remaining) will be:

$P = \$30 \times PVAF(3.0433\%,14) + \$1{,}000 \times (1+3.0433\%)^{-14}$
<u>P = $995.12</u>

(c) First we need to compute the bond price one year from now (with only 18 coupons remaining) at a yield to maturity of 5.5%:

$P = \$30 \times PVAF(2.75\%,18) + \$1{,}000 \times (1+2.75\%)^{-18} = \$1{,}035.12$

The rate of return over one year is given by:

ROR = (Interest Income + Final Price - Initial Price) / Initial Price

where Interest Income = $30 x (1 + 0.5%) + $30 = $60.15, and the capital gain/loss is 1,035.12 - $923.08 = $112.04

ROR = (60.15 + 112.04) / 923.08 = <u>18.6541%</u>

Question 6 (Bonds) (12 marks)

A company decides to issue 10,000 bonds with a 10-year maturity and a 6% coupon rate. The bond has a face value of $1,000 and pays semi-annual coupons. The market requires an <u>effective</u> yield to maturity on a bond with similar risk of 7%.

(a) What is the fair price of the bond? (6 marks)

In order to be able to pay the total face value back in 10 years, the company decides to use a sinking fund in which it will make semi-annual payments that would serve to accumulate the total face value. The sinking fund pays an interest rate of 3% compounded semi-annually.

(b) What is the semi-annual payment the company must make to the sinking fund? (4 marks)

(c) What is the total semi-annual cash outflow for the company? (2 marks)

Solution

(a) First, we need the semi-annual discount rate:

$$(1+r)^2 = 1+7\% \quad \Rightarrow \quad r = 3.4408\%$$

The bond price is:

$$P = C\times\left[\frac{1-(1+r)^{-n}}{r}\right] + F\times(1+r)^{-n}$$

$$P = \$30\times\left[\frac{1-(1+3.4408\%)^{-20}}{3.4408\%}\right] + \$1{,}000\times(1+3.4408\%)^{-20}$$

$$P = \$937.01$$

(b) To accumulate the total face value, $10 million, the company must make the following semi-annual payments to the sinking fund:

$$FV = PMT\times\left[\frac{(1+r)^n - 1}{r}\right]$$

$$PMT = \frac{FV}{\left[\frac{(1+r)^n-1}{r}\right]} = \frac{10{,}000{,}000}{\left[\frac{(1+1.5\%)^{20}-1}{1.5\%}\right]}$$

$$PMT = \$432{,}457.36$$

(c) Adding the total semi-annual coupon, that is $300,000, to the sinking fund payment, $432,457.36, results in a total semi-annual cash outflow of <u>$732,457.36</u> for the company.

Question 7 (Stocks) (12 marks)

Your broker offers to sell you some shares of XYZ Co. common stock that paid a dividend of $2 today. You expect the dividend to grow at the rate of 5% per year for the next 3 years, and then at 10% forever. The interest rate is 12%.

(a) Find the expected dividend for each of the next four years? (4 marks)

(b) What is the fair price of the stock three years from now? (4 marks)

(c) What is the fair price of the stock today? (4 marks)

Solution

(a) The expected dividends for each of the next four years are:

$D_1 = D_0 \times (1 + 5\%) = \$2.10$
$D_2 = D_1 \times (1 + 5\%) = \$2.21$
$D_3 = D_2 \times (1 + 5\%) = \$2.32$
$D_4 = D_3 \times (1 + 10\%) = \$2.55$

(b) The fair price of the stock three years from now is simply given by the constant growth rate formula:

$P_3 = D_4 / (r – g) = 2.55 / (12\% - 10\%)$
$P_3 = \$127.50$

(c) The fair price of the stock today is simply given by:

$P_0 = D_1 / (1+r) + D_2 / (1+r)^2 + D_3 / (1+r)^3 + P_3 / (1+r)^3$
$P_0 = 2.10 / (1.12) + 2.21 / (1.12)^2 + (2.32 + 127.50) / (1.12)^3$
$\underline{P_0 = \$96.03}$