

FINAL EXAM-MATH 1300
FALL TERM, 2011

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Name(Print LEGIBLY)_____

I.D. Number_____

Instructions-This final examination consists of 10 multiple choice questions worth 4 points each. Your answers to the multiple choice questions must be clearly marked in the squares below. There are also 5 long answer questions worth a total of 60 points. For the long answer questions, you must show your work **on the exam itself** and clearly display your answers. **Do not unstaple these pages.**

NO CALCULATORS. NO BOOKS. NO NOTES.

**TURN OFF YOUR CELL PHONES AND
PUT THEM AWAY.**

Multiple Choice Answers:

#1

#2

#3

#4

#5

#6

#7

#8

#9

#10

Question 1- Calculate the following one-sided limit:

$$\lim_{x \rightarrow -1^+} \frac{x}{x^2 - 1}$$

- A) 0 B) 1 C) ∞ D) $-\infty$ E) -1

Question 2- Suppose that the variable y is implicitly defined as a function of x by the equation $(x^2 - y)^2 = 2y$. Find $\frac{dy}{dx}$ at the point $(2,2)$.

- A) $\frac{1}{2}$ B) 3 C) -1 D) $\frac{8}{3}$ E) $\frac{5}{2}$

Question 3- Let $h(x) = x \ln(x)$. Find the equation of the tangent line at $x = e^2$.

- A) $y = ex - 1$ B) $y = 3x - 2e^2$ C) $y = 3x - e^2$ D) $y = 3x - e$ E) $y = ex + 2$

Question 4- Over what interval is the function $f(x) = xe^{2x}$ decreasing?

- A) $(\frac{1}{2}, \infty)$ B) $(-\infty, -\frac{1}{2})$ C) $(-1, 1)$ D) $(-1, \infty)$ E) $(0, 1)$

Question 5- Consider the function:

$$f(x) = \frac{5x + 2}{x - 3}$$

Which of the following statements are correct?

- A) The asymptotes of f occur at $y = 5, x = 3$.
- B) The asymptotes of f occur at $y = 3, x = 5$.
- C) $x = 3$ is the only asymptote of f .
- D) $y = 3$ is the only asymptote of f .
- E) The function has no asymptotes.

Question 6- Evaluate:

$$\int_{\ln(2)}^{\infty} e^{-x} dx$$

- A) 1 B) $\frac{1}{2}$ C) $\ln(2)$ D) $2\ln(2)$ E) divergent

Question 7- Suppose $f'(x) = 3x^2 + 4x - 1$ and $f(1) = 3$. Find $f(2)$.

- A) 13 B) 15 C) 19 D) 37 E) 51

Question 8- Suppose that for a certain product, the demand function is given by $D(x) = 8 - x^2$ and the supply function is given by $S(x) = 2x$. Calculate the consumer surplus.

- A) 3 B) $\frac{11}{3}$ C) $\frac{16}{3}$ D) 1 E) $\frac{25}{7}$

Question 9- If $f(x, y) = y \ln(xy)$, what is $f_{xy}(2, 2)$?

- A) $\frac{4}{3}$ B) $\frac{-1}{4}$ C) $\frac{3}{2}$ D) $\frac{1}{2}$ E) $\frac{e}{3}$

Question 10- If $f(x, y) = x^3 - 4x + y^2 - 3y + 1$, how many critical points does $f(x, y)$ have?

- A) 0 B) 1 C) 2 D) 3 E) 4

Long Answer Question 1 (10 points)

Suppose that 10,000 dollars is put into a bank account which earns 4% interest compounded continuously.

- How much money will there be in the account after 6 years?
- How many years will it take until there is 40,000 dollars in the account?

Long Answer Question 2 (12 points)

Calculate the following two indefinite integrals:

$$\int x e^{3x} dx$$

$$\int \frac{\ln(x)}{x^2} dx$$

Long Answer Question 3 (12 points)

A store has been selling skateboards at \$40 per board. At this price they sell 50 boards per month. For each 1 dollar increase in price, they sell 2 fewer boards. If each board costs the store \$25, what price will maximize profit?

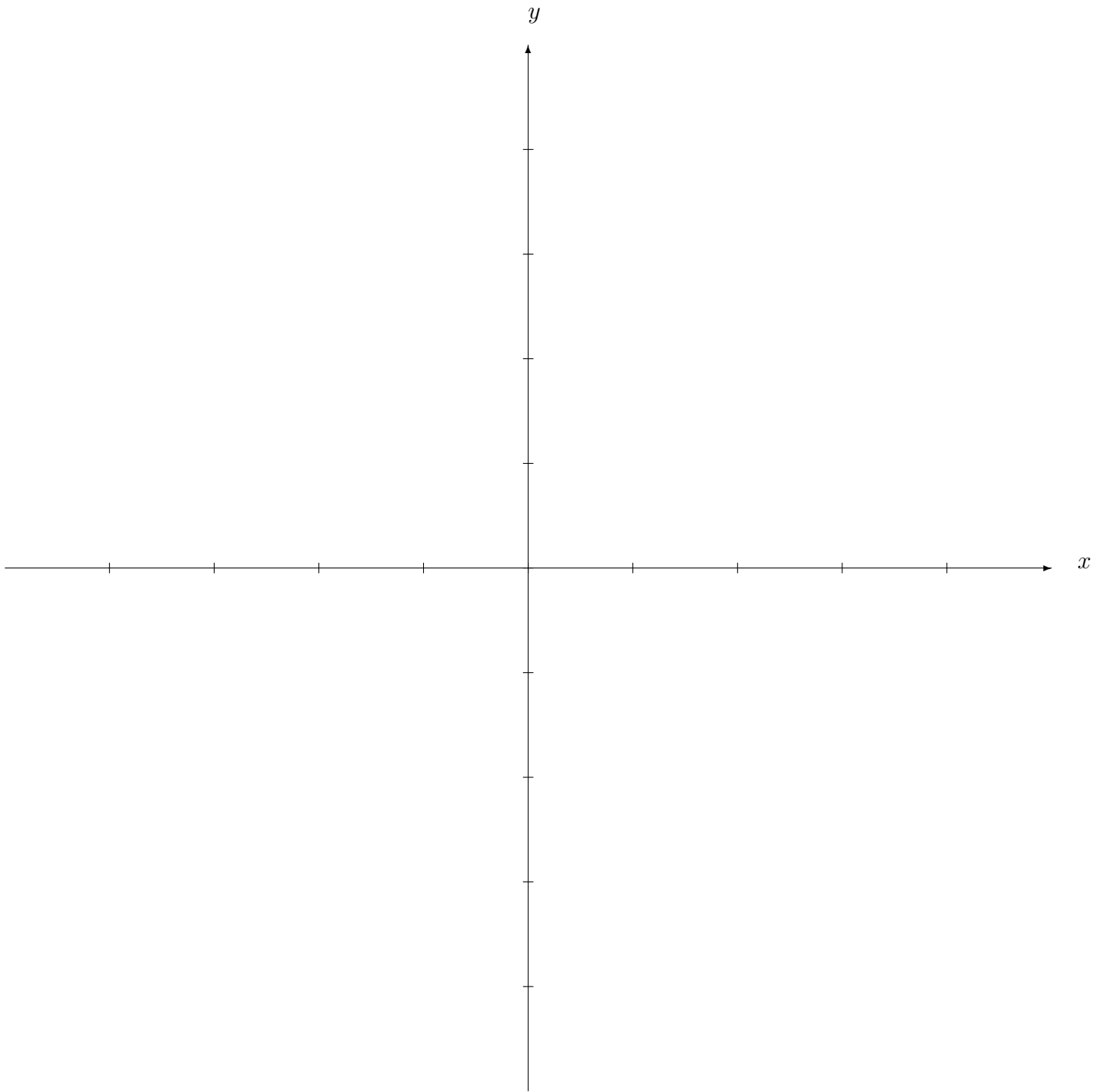
Be sure to explain why your answer is an absolute maximum.

Long Answer Question 4 (14 points)

Consider the two functions:

$$f(x) = 5 - x \quad \text{and} \quad g(x) = 3 + x^2$$

- (a) **(4 points)** Find the intersection points of the graphs of the two functions.
- (b) **(6 points)** On the next page, graph these functions, and shade the region between the graphs of f and g for $x \in [0, 4]$.
- (c) **(6 points)** Find the area of the shaded region.



Long Answer Question 5 (12 points)

Consider the function of two variables

$$f(x, y) = x^2 - 4x + 6xy + 2y^3 - 6.$$

- (a) **(3 points)** Calculate the first-order partial derivatives.
- (b) **(3 points)** Find all critical points.
- (c) **(6 points)** Identify what type of critical points they are (local max, local min or saddle point).

Extra page for additional work