

Lecture 6-1348

①

~~Last~~ Last time

Predicate logic, especially English translation
English \rightarrow Pred Logic

Note - Expect a batch of simple test questions on website over weekend.

Good email question

Translation for "Every comedian is funny"

W's $\forall x (P(x) \Rightarrow Q(x))$

Does the following also work?

$$\neg \exists x (P(x) \wedge \neg Q(x))$$

"It is not the case that there is a comedian who is not funny"

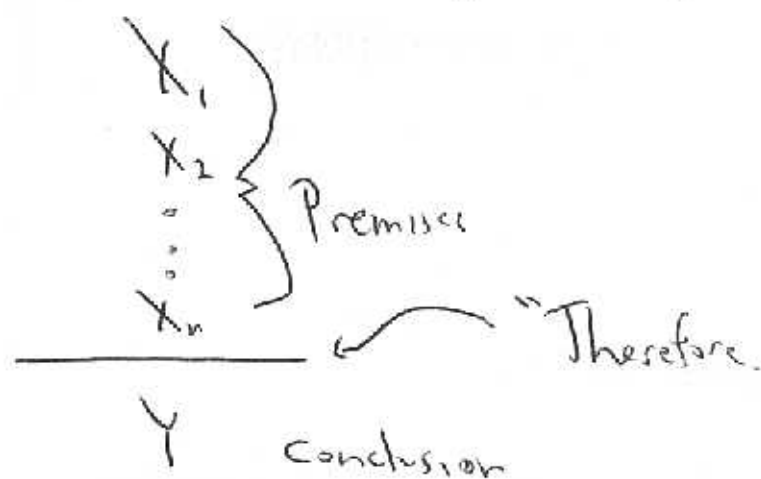
Uses the fact that

$$\forall x C(x) \equiv \neg \exists x \neg C(x)$$

This is a special case of de Morgan. Why?

Also last time: Arguments.

In Prop logic (or any logic), an argument looks like:



Example from last class

$$\begin{array}{c} (P \wedge T) \Rightarrow A \\ A \Rightarrow D \\ \neg D \wedge T \\ \hline \neg P \end{array}$$

Is this valid? Yes. But need a defn of validity.

Def'n 1 p 64 (first part only)

An argument in prop logic is valid if

For every row of the truth table in which
all premises are true, the conclusion is also true.

Ex:

$$\frac{A \vee \neg B}{B} \\ A$$

It ~~was~~ sunny or I ~~did~~ not walk to work.
I walked to work. Therefore, it was sunny.

| | A | B | B | $A \vee \neg B$ | A |
|---|---|---|---|-----------------|---|
| ① | T | T | T | T | T |
| ② | T | F | F | T | T |
| ③ | F | T | T | F | F |
| ④ | F | F | F | T | F |

premises conclusion

Is the argument valid? YES. The only row that matters is the first row. In rows 2,3,4, at least one of the premises is false.

Ex 2

$$\begin{aligned} & A \wedge (B \Rightarrow C) \\ & \neg A \Rightarrow \neg B \\ & \neg C \vee A \\ \hline & C \Rightarrow B \end{aligned}$$

Consider:

| A | B | C | $A \wedge (B \Rightarrow C)$ | $\neg A \Rightarrow \neg B$ | $\neg C \vee A$ | $C \Rightarrow B$ | ④ |
|---|---|---|------------------------------|-----------------------------|-----------------|-------------------|---|
| T | F | T | T | T | T | F | |

In this row, all the premises are true & conclusion is false. So the argument is not valid. The other rows are irrelevant.

To prove an argument not valid, just find one such row. How did I find this one? Calculate as follows:

I am trying to find a row where all premises are true, and conclusion is false. So $C \Rightarrow B$ must be false. So I must have

$$C = T, B = F$$

Also must have $A = T$ to make first formula true. This is the only possible row.

So check it.

This is the short cut method.

Ex 3

$$B \Rightarrow (C \wedge \neg A)$$

$$\underline{C \Rightarrow A}$$

$$A \vee \neg B$$

Again, try to find a row where conclusion is false, and premises all true.

In this case, I must have

$$A = F \text{ and } B = T \text{ to make conc false.}$$

Since we must have $C \Rightarrow A$ true, and A is false. We have to have $C = F$. Why?

We also have to make $B \Rightarrow (C \wedge \neg A)$ true.

But I now have $A = F, B = T, C = F$, and

this makes $B \Rightarrow (C \wedge \neg A)$ false. So it is impossible to make both premises true, and conclusion false. So argument is valid.

Note - Each dec is not forced, no arbitrary choices.

EX 4 Is the following valid?

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$$\neg A \Rightarrow (C \wedge B)$$

$$C \Rightarrow (B \wedge A)$$

$$A \wedge \neg B$$

Goal - make conc. false and premises true.

Two ways to make $A \wedge \neg B$ false

① Make $A = F$

But this then forces $C = T$ and $B = T$, to make

$\neg A \Rightarrow (C \wedge B)$ true.

And this makes $C \Rightarrow (B \wedge A)$ false.

So this guess didn't work.

② Make $B = T$

Then, make $A = C = T$. This makes both premises true

So argument is not valid.

This is all we cover in 1.5.

On to 1.6 "Introduction to proofs"

Have to prove things in this course.

So, you have to know what a proof looks like.

Ex 1, p 77

If $\underbrace{n \text{ is an odd integer}}_P$, then $\underbrace{n^2 \text{ is odd}}_Q$.

$$P \Rightarrow Q$$

So, assume P . Assume n is an odd integer.

This means $n = 2m + 1$, where m is an integer.

This is key.

After writing out precisely what your assumptions are, the proof is easy.