**UNIVERSITY OF TORONTO**
**Joseph L. Rotman School of Management**

**Oct. 25, 2016** **Burke/Corhay/Kan**
**RSM332** **MID-TERM EXAMINATION** **Geoffrey/Wang**

---

**DURATION - 2 hours**

**Aid Allowed: Silent electronic calculator and one 1-sided $8\frac{1}{2}$"×11" crib sheet**

**Name:**_____ **Student Number:** _____

Circle the section that you are registered in:

| | | |
|---|---|---|
| Burke | Corhay (Thu. 9–11a.m.) | Corhay (Thu. 11a.m.–1p.m.) |
| Corhay (Thu. 2–4p.m.) | Kan (Mon. 11a.m.–1p.m.) | Kan (Mon. 2p.m.–4p.m.) |
| Kan (Tue.) | Geoffrey (Mon.) | Geoffrey (Thu.) Wang |

# Instructions

1. Write all your answers on the examination paper.
2. Answer five out of six questions. Each question is worth 20 marks. Do not answer all six questions! In the table below, cross out the question that you choose not to answer.

| Question | Marks |
|---|---|
| 1 | _____ |
| 2 | _____ |
| 3 | _____ |
| 4 | _____ |
| 5 | _____ |
| 6 | _____ |
| Total | _____ |

1. (a) You obtain a $500,000 mortgage loan from Royal Bank to buy a house. The mortgage has a 5-year fixed rate of 4%/year (using Canadian mortgage convention), and the amortization period of the mortgage is 30 years. What is the monthly mortgage payment? (4 marks)

   (b) Suppose instead of making monthly payment, you choose to pay your mortgage on a bi-monthly basis. What is the bi-monthly mortgage payment? (4 marks)

(c) For the 10th bi-monthly payment in part (b), how much of it is for interest, and how much of it is for principal repayment? (4 marks)

(d) After four years, you need some additional money. Suppose at that time, the appraised value of your house is \$650,000. Royal Bank gives you the option to obtain a second mortgage by borrowing up to 80% of the appraised value of your home, minus the remaining mortgage balance. What is the maximum amount that you can borrow at that time? (4 marks)

(e) Instead of borrowing the maximum amount in part (d), you decide to borrow only \$50,000, and you plan to pay it off in 10 years by making monthly payments. Royal bank offers you a fixed rate of 5%/year (using Canadian mortgage convention). Instead of making the same payment every month, you would like to increase your monthly mortgage payment at a rate of 0.5%/month. What should be your first monthly mortgage payment for this new mortgage? (4 marks)

2. Suppose you have the following utility function

$$U(C_0, C_1) = C_0^{\frac{1}{2}} C_1^{\frac{1}{2}},$$

where $C_0$ is today's consumption and $C_1$ is tomorrow's consumption, measured in bushels of corn. You are endowed with $Y_0 = 1200$ bushels of corn today and $Y_1 = 200$ bushels of corn tomorrow.

Your friend has a farm that allows you to invest $I_0$ bushels of corns today to produce $40I_0^{\frac{1}{2}}$ bushels of corn tomorrow.

(a) Suppose capital market does not exist. If your friend gives you the farm for free, what is the optimal level of investment in the farm? (10 marks)

(b) Suppose a capital market exists with an interest rate of 10%. What is the maximum amount of corn that you are willing to pay for the farm? (5 marks)

(c) Suppose you pay the maximum amount to obtain the farm. What would be your optimal consumption decisions today and tomorrow? (5 marks)

3. You win $5 million in a lottery and decide to invest the money and live on the investment returns. You will invest the entire $5 million amount in return for a stream of annual payments. You will invest the money on your 22nd birthday and receive payments each year starting on your 23rd birthday, with the final payment on your 64th birthday.

   There are two firms, ABC and DEF, offering an investment product that suits your needs. ABC's annualized percentage rate (APR) is 4.95%, with monthly compounding, and DEF's APR is 4.90% with continuous compounding.

   (a) Which firm should you choose for your investment? (3 marks)

   (b) Given your choice in part (a), what constant annual payment will you receive? (5 marks)

After five years you notice your buying power is shrinking each year due to inflation. You decide to restructure your investment so that the remaining payments will grow each year by 2% to keep pace with inflation.

(c) What is the remaining value of your investment after the 5th payment? (5 marks)

(d) What is the dollar amount of the first payment in the restructured investment plan? (5 marks)

(e) What is the dollar amount of the payment on your 64th birthday? (2 marks)

4. The following information is for parts (a) and (b). Four bonds are available in the market:

   B1: 1-year zero-coupon bond, face value of \$100; current price of \$92.59;

   B2: 2-year zero-coupon bond, face value of \$100; current price of \$85.34;

   B3: 3-year zero-coupon bond, face value of \$1,000; current price of \$782.91;

   B4: 4-year zero-coupon bond, face value of \$10,000; current price of \$7,149.62.

   There is no transaction cost for trading any of the bonds and one can buy or sell (short-sell) any fraction of any of the bonds. There is no default risk with any of the bonds.

   (a) Calculate the spot rates $r_1$, $r_2$, $r_3$, and $r_4$ (for the periods of one, two, three, and four years, respectively). Also, calculate the forward rates $f_2$, $f_3$, and $f_4$ for year 2, year 3, and year 4. (6 marks)

(b) You find out that a financial institution offers you an opportunity to either borrow or lend \$10,000 at the end of year 3 for one year at an interest rate of 9%/year. Suppose that such a contract can be entered today and it is completely risk-free.

Given the availability of the four bonds (B1 through B4), is there any arbitrage opportunity?

If yes, explain carefully your approach to earn a risk-free profit. (8 marks)

(c) This part is independent of parts (a) and (b). Demonstrate that a forward rate cannot be negative. You can focus on $f_2$, in other words, if $f_2 < 0$, show clearly that there exists a risk-free arbitrage opportunity. (6 marks)

5. You work as a fixed income trader at TD securities. You obtain information on three Canadian government bonds. All have a principal of $1000, and are assumed to pay coupon annually (if applicable):

| Security | Maturity | Coupon rate | Yield-to-maturity |
|---|---|---|---|
| Bond 1 | 1 year | 8% | 5% |
| Bond 2 | 2 year | 10% | 6.43% |
| Bond 3 | 3 year | 0% | 7% |

(a) What is the term structure of spot rates implied by the three bonds, $r_1$, $r_2$ and $r_3$? (4 marks)

(b) The Canadian government is about to issue a new level-coupon bond maturing in three years that promises a coupon rate of 6% (paid annually, with the first payment starting a year from now). The face value of the bond is $1000.

(i) Assuming there is no arbitrage opportunities, what should be the price of this bond? (2 marks)

(ii) Once issued, the government bond trades at $950. Is there an arbitrage opportunity? If so, describe your trading strategy carefully, i.e., which bond you buy/short, and in what quantity. What is your profit per trade? (6 marks)

(c) You decide to buy Bond 2 (at the current market price) and plan to sell it in a year. What future values of the 1-year spot rate (at the end of year 1) would ensure you earn a positive holding period return on your investment? (4 marks)

(d) A fund manager would like to buy a 4-year zero coupon bond with a \$1000 face value, but none are currently being traded. You call around and find a bank offering a forward rate of $f_4 = 8\%$ for the 4th year. Describe how you can create a 4-year zero coupon bond using the government bonds and the forward contract. What is the price of the synthetic four-year zero coupon bond? (4 marks)

6. The following information is relevant to parts (a)–(d).

   Gluskin Sheff and Associates (GS) trades on the Toronto Stock Exchange. Historically, GS has a retention ratio of 10.5% and pays regular dividends on a quarterly basis; both of these traits are expected to continue indefinitely. The most recent dividend was \$0.25. The market consensus is that future dividends will grow by 1.6%/quarter forever, with the next dividend being paid three months from today. You believe using an effective annual discount rate of 13.25% is appropriate for analysing GS.

   (a) According to the Dividend Discount Model, what is the fair price of one share of GS? (6 marks)

(b) Use the Dividend Discount Model to calculate the expected price of GS one year from today after its dividends have been paid. (3 marks)

(c) Calculate the price earnings ratio of GS. When calculating the price earnings ratio, use the forecasted earnings for the next four quarters instead of the earnings from the trailing twelve months. (6 marks)

(d) You notice that the price earnings ratio you calculated for GS is lower than the sector average price earnings ratio. Assuming all else equal, provide and explain two reasons why this may be the case. (2 marks)

(e) Facebook (FB) started trading on the Nasdaq Exchange on November 6, 2013. To date it has never paid a dividend and the company has implied that it has no intention to pay a dividend. You have learned that the dividend valuation model suggests that the value of a companys stock is the present value of its expected future dividends. Does the fact that FB does not trade at $0.00 imply the dividend valuation model has been contradicted? Explain why or why not. (3 marks)