Name (please print):
Signature:
Student number: SOLUTIONS

FACULTY OF SCIENCE
YORK UNIVERSITY
DEPARTMENT OF PHYSICS AND ASTRONOMY

PHYS 1800 03 – Engineering Mechanics
Fall 2014

Final examination

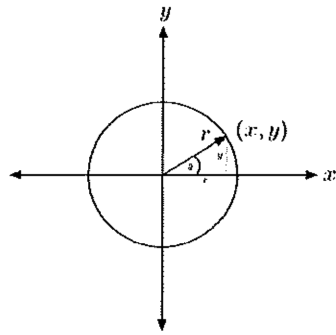
Instructions:

1. Print your name and student number on this page in the upper left corner.
2. Check that your exam contains 7 questions.
3. The total number of marks is 100.
4. Simple scientific calculators are allowed as aid.
5. Answer all questions in the space provided on this exam paper.
6. Formulae are provided on the last page.

1.	
2.	
3.	
4.	
5.	
6.	
7.	
Total	

1. (10 marks)

An object (mass m = 0.20 kg) is moving with constant speed in a circle in the xy plane. The position of the object is given by the following vector: **r** = 0.8 cos $(2.5\,t)$**i** + 0.8 sin $(2.5\,t)$**j**, where r is in meters and t is in seconds.



a. Determine the velocity (magnitude and direction) of the object at t = 2.0 s.

v = $\frac{dr}{dt}$ = -2.0 sin $(2.5\,t)$**i** + 2.0 cos $(2.5\,t)$**j**,
v(t = 2.0 s) = -2.0 sin [(2.5)(2.0)]**i** + 2.0 cos [(2.5)(2.0)]**j** = 1.92**i** + 0.567**j** (m/s)
v = $\sqrt{(1.92)^2 + (0.567)^2}$ = 2.0 m/s
tan θ = $\frac{0.567}{1.92}$ θ = $16.45^0 \approx 16^0$

b. Determine the acceleration (magnitude and direction) of the object at t = 2.0 s.

a = $\frac{dv}{dt}$ = -5.0 cos $(2.5\,t)$**i** - 5.0 sin $(2.5\,t)$**j**,
a(t = 2.0 s) = -5.0 cos [(2.5)(2.0)]**i** - 5.0 sin [(2.5)(2.0)]**j** = -1.42**i** + 4.79**j** (m/s)
a = $\sqrt{(-1.42)^2 + (4.79)^2}$ = 5.0 m/s^2
tan θ = $\frac{4.79}{-1.42}$ θ = $-73.48^0 \approx -74^0$

c. What is the angle between velocity and acceleration vectors at t = 2.0 s?

90 degrees

d. How long does it take the object to make 5.0 revolutions?

ω = 2.5 Hz $\frac{2\pi}{T} = 2.5$ T = 2.51 s

Time of 5 rev = 5(2.51) = 12.6 s ≈ 13 s

e. What is the magnitude of the force keeping the object in circular motion?

F = ma = (0.20 kg)(5.0 m/s^2) = 1.0 N

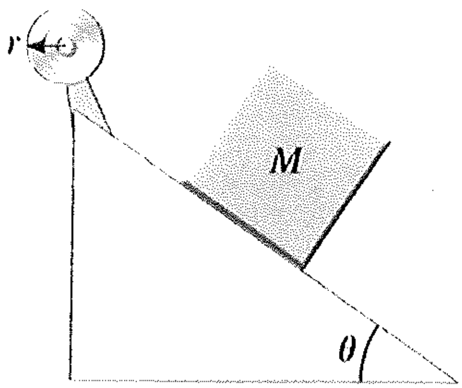
2. (12 marks)

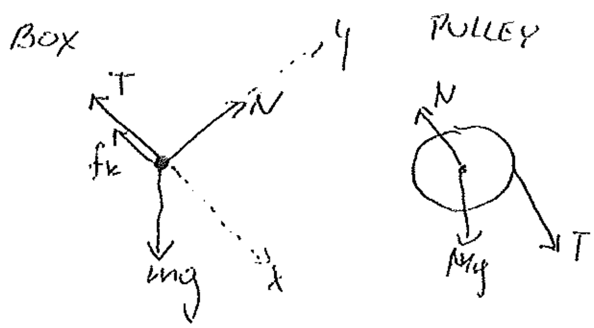
A box of mass M = 12.0 kg is attached to a cord that is wrapped around a pulley whose moment of inertia is $I = 0.15\ kg\ m^2$ and radius r = 0.300 m. The coefficient of kinetic friction between the box and the incline surface is $\mu_k = 0.25$ and the angle $\theta = 30.0^0$.

a. Draw a free-body force diagram for the box and the pulley.

b. Determine the kinetic force of friction acting on the box.

c. Determine the acceleration of the box, angular acceleration of the pulley and the tension in the cord.





b) $f_k = \mu_k N = \mu_k mg\cos\theta = (0.25)(12.0)(9.81)\cos 30.0^\circ = 25.49\ N \approx 25\ N.$

c) BOX

$\Sigma F_x = ma_x$

$mg\sin 30.0^\circ - T - 25.49 = 12.0a$

$(12.0)(9.81)\sin 30.0^\circ - T - 25.49 = 12.0a$

$33.37 - T = 12.0a$

PULLEY

$\Sigma\tau = I\alpha$

$-Tr = I(-\frac{a}{r})$

$0.300T = (0.15)(\frac{a}{0.300})$

$T = 1.67a$

$33.37 - 1.67a = 12.0a$

$13.67a = 33.37$

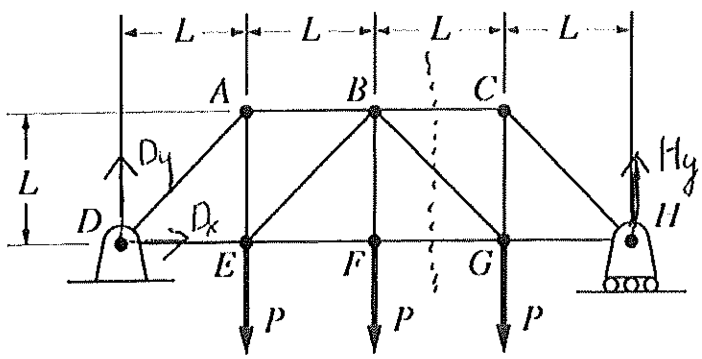
$a = 2.44\ m/s^2 \approx 2.4\ m/s^2$

$T = (1.67)(2.44) = 4.1\ N$

$\alpha = \frac{a}{r} = \frac{2.44}{0.300} = 8.1\ rad/s^2$

3. (12 marks)

Determine the forces in the following truss members: BC, BG and FG. State whether each member is in tension or compression. Assume that L = 3.00 m and P = 5000.0 N. The roller is frictionless.



$\Sigma F_x = 0$

$D_x = 0$

$\Sigma F_y = 0$

$D_y + H_y - 15000.0 = 0$

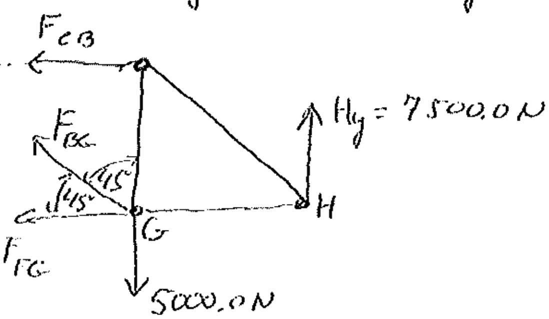
$\Sigma M_D = 0$

$\Sigma M_D = -(5000.0)(3.00) - (5000.0)(6.00) - (5000.0)(9.00) + H_y(12.0) = 0$

$H_y = 7500.0\ N.$

$D_y = 15000.0 - H_y = 15000.0 - 7500.0 = 7500.0\ N$

SECTION (1)



$\Sigma M_G = (7500.0)(3.00) + F_{CB}(3.00) = 0 \Rightarrow F_{CB} = -7500.0\ N$ (COMPRESSION ~~TENSION~~)

$\Sigma F_x = -F_{BG}\cos 45° - F_{FG} - F_{CB} = 0$

$F_{FG} = +7500.0 - F_{BG}\cos 45°$

$\Sigma F_y = F_{BG}\sin 45° + H_y - 5000.0 = 0$

$F_{BG}\sin 45° = 5000.0 - 7500.0$

$F_{BG} = -3535\ N$ (COMPRESSION)

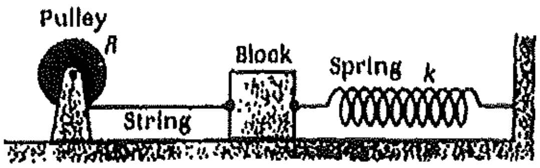
$F_{FG} = +7500.0 - (-3535)\cos 45°$

$F_{FG} = +5000.0\ N$ (COMPRESSION)

10 000 N (TENSION)

4. (10 marks)

A pulley of radius R = 0.070 m and the moment of inertia I = 0.01 kg m^2 is rotated clockwise until the spring is stretched d = 0.50 m, and then released from rest. Determine the speed of the block (mass m = 4.3 kg) when it passes through the position at which the spring is un-stretched. The spring constant is k = 235 N/m. The coefficient of kinetic friction between the block and the surface is μ_k = 0.15. The pulley is frictionless.



$$\Delta K + \Delta U = W_{nc}$$

$$\Delta K = K_f - K_i = \left(\tfrac{1}{2}mv^2 - 0\right) + \left(\tfrac{1}{2}I\omega^2 - 0\right) = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\frac{v^2}{R^2}$$

$$\Delta U_s = U_{sf} - U_{si} = 0 - \tfrac{1}{2}kd^2 = -\tfrac{1}{2}kd^2$$

$$W_{nc} = f_k d\cos 180^\circ = -\mu_k mgd$$

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}I\frac{v^2}{R^2} - \tfrac{1}{2}kd^2 = -\mu_k mgd$$

$$v^2\left(\frac{m}{2} + \frac{I}{2R^2}\right) = \tfrac{1}{2}kd^2 - \mu_k mgd$$

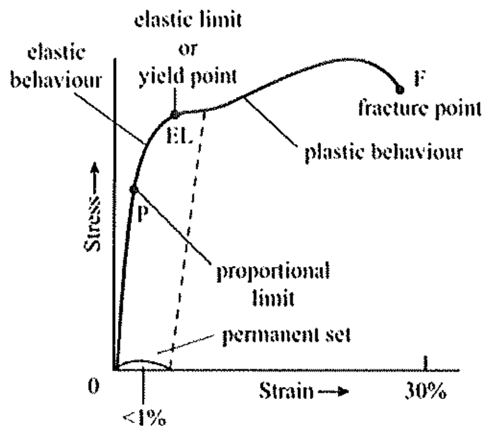
$$v = \sqrt{\frac{\tfrac{1}{2}kd^2 - \mu_k mgd}{\frac{m}{2} + \frac{I}{2R^2}}}$$

$$v = \sqrt{\frac{\tfrac{1}{2}(235)(0.50)^2 - (0.15)(4.3)(9.81)(0.50)}{\frac{4.3}{2} + \frac{0.01}{2(0.070)^2}}}$$

$$v = 2.9 \text{ m/s.}$$

5. (8 marks)

Draw a typical stress-strain dependence and explain the meaning of the following terms:



Proportional limit

The maximum stress for which stress-strain graph is linear (point P in the graph)

Yield point

The maximum stress for which the material remains elastic (when stress is removed the material returns to original length and shape).

Permanent set

The amount of deformation (strain) that remains when the stress is removed.

Plastic deformation

Permanent deformation that occurs when for stress applied is larger than yield point

Ultimate strength

The maximum stress on the stress-strain graph

Engineering stress

The force per unit area of the original cross-section of the material

True stress

The force per actual cross-section as the material is deformed. True stress continues to increase until the material fractures because the actual cross-section area decreases

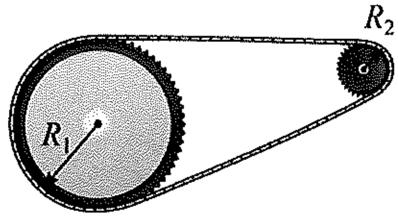
6. (18 marks)

Answer the following six questions.

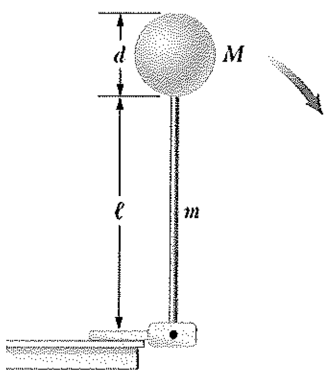
a. Two gears (R_1 = 52 cm and R_2 = 17 cm) mounted on fixed axes are connected by a belt. The smaller gear is driven by a motor that causes it to spin at 7.0 rad/s. The belt does not slip. Determine the angular speed of the larger gear.

$$\omega_1 R_1 = \omega_2 R_2$$

$$\omega_1 = \frac{\omega_2 R_2}{R_1} = \frac{(7.0)(0.17)}{(0.52)} = 2.3 \text{ RAD/s}$$

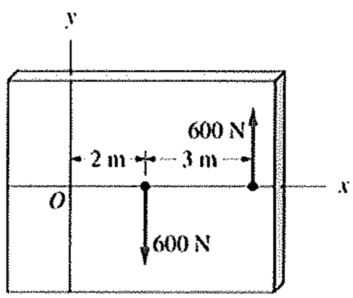


b. Determine the moment of inertia of the object (shown below) about an axis perpendicular to the page, running through the black dot. The object consists of a thin rod (mass m, length ℓ) and a solid sphere (mass M, diameter d). Express the moment of inertia in terms of m, ℓ, M and d.



$I = I_{rod} + I_{sphere} = (1/3)mL^2 + (2/5)\,M(d/2)^2 + M(d + L)^2$ (Attention: The parallel-axis theorem was omitted in 2015)

c. Determine the magnitude and direction of the moment of the couple for forces for the situation shown below.



$$M = -(600)(2) + (600)(5) = 1800 \text{ Nm}$$

DIRECTION: PARALLEL TO Z-AXIS

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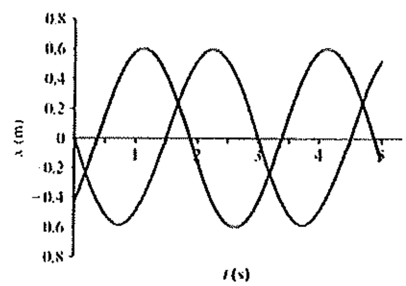
d. What is the phase difference between the two oscillators?



$$x = A\cos(\omega t + \phi)$$

(1) SUBSTITUTING $t = 0,\ x = 0$

$0 = 0.6\cos\phi_1$

$\phi_1 = +\frac{\pi}{2}$ RAD $(+90°)$

or $-\frac{3\pi}{2}$ RAD $(-270°)$

(2) SUBSTITUTING $t = 0,\ x = -0.4$

$-0.4 = 0.6\cos\phi_2$

$\phi_2 = -2.3$ RAD $(-132°)$

OR $+4.0$ RAD $(+228°)$

$$\Delta\phi = \phi_1 - \phi_2 = \frac{\pi}{2} - (-2.3) = 3.9 \text{ RAD } (222°)$$

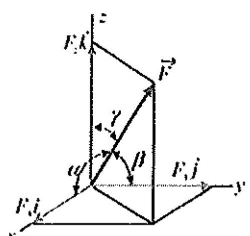
$$\text{OR } (2\pi - 3.9) = 2.4 \text{ RAD } (138°)$$

e. A travelling wave is described by the following equation: D(x,t) = 0.2 sin(0.30x - 5.0t + 0.4). What is the speed of the wave?

$$v = \frac{\omega}{k}$$

$$v = \frac{5.0}{0.2} = 25 \text{ m/s.}$$

f. Determine the angles α, β *and* γ between the force F = 1200i + 800j - 1500k and the x-, y-, and z-axes.



$$F = \sqrt{(1200)^2 + (800)^2 + (-1500)^2} = 2081$$

$$\cos\alpha = \frac{F_x}{F} = \frac{1200}{2081} \Rightarrow \alpha = 55°$$

$$\cos\beta = \frac{F_y}{F} = \frac{800}{2081} \Rightarrow \beta = 67°$$

$$\cos\gamma = \frac{F_z}{F} = \frac{-1500}{2081} \Rightarrow \gamma = 136°$$

7. (30 marks)

Answer the following five questions.

a. An engine pulls a 1000.0 kg crate from rest to 20.0 m/s in 5.00 s with constant acceleration. The force of kinetic friction between the crate and the ground is f_k = 500.0 N. How much work is done by the engine?

$$W = \Delta K$$

$$W_{ENGINE} + W_{FRICTION} = \frac{1}{2} m v^2$$

$$W_{ENGINE} = \frac{1}{2} m v^2 - f_k d \cos 180° = \frac{1}{2} m v^2 + f_k d$$

TO FIND d: $v = v_0 + at$, $v_0 = 0$

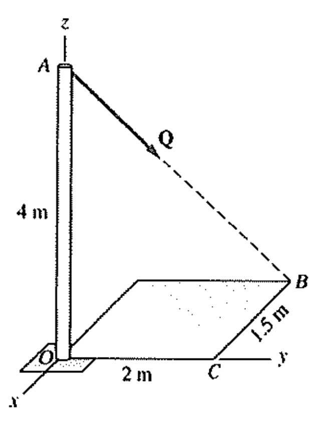
$$a = \frac{v}{t} = \frac{2.00}{5.00} = 0.40 \ m/s^2$$

$$d = \frac{1}{2} a t^2 = \frac{1}{2}(0.40)(5.00)^2 = 5.00 m$$

$$W_{ENGINE} = \frac{1}{2}(1000.0)(2.00)^2 + (220.0)(5.00) = 3100 J.$$

$$P = \frac{W_{ENGINE}}{t} = \frac{3100 J}{5.00 s} = 620 W.$$

b. Determine the moment of the force **Q** about point O. The magnitude of **Q** is 100.0 N.



$$\vec{Q} = Q \hat{u}_{AB}$$

$$\hat{u}_{AB} = \frac{\vec{AB}}{AB} = \frac{-1.5\hat{i} + 2\hat{j} - 4\hat{k}}{\sqrt{(-1.5)^2 + 2^2 + (-4)^2}} =$$

$$\hat{u}_{AB} = -0.318\hat{i} + 0.424\hat{j} - 0.847\hat{k}$$

$$\vec{Q} = 100.0(0.318\hat{i} + 0.424\hat{j} - 0.847\hat{k})$$

$$\vec{Q} = -31.8\hat{i} + 42.4\hat{j} - 84.7\hat{k}$$

$$M_o = \vec{r}_{OA} \times \vec{Q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 4 \\ -31.8 & 42.4 & -84.7 \end{vmatrix} = -169.6\hat{i} - 127.2\hat{j} + 0\hat{k}$$

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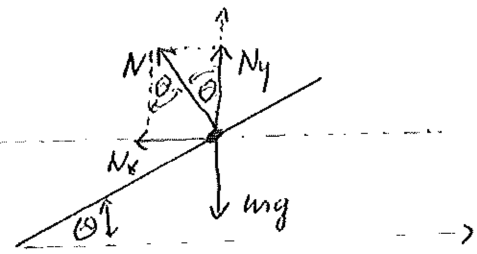
c. An engineer is given a task to calculate the slope of a curved banked portion of a highway to prevent cars from slipping off the road on icy day when friction is almost zero. Assume that the radius of the curved banked road is R = 190.0 m and that the speed limit is v = 100.0 km/h. Draw a free-body force diagram for the car on a banked road and determine the bank angle θ that prevents cars sliding off the road.

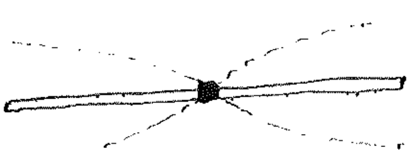


$$\Sigma F_x = m a$$
$$N_x = m \frac{v^2}{R}$$
$$N \sin\theta = m \frac{v^2}{R}$$
$$\frac{mg \sin\theta}{\cos\theta} = m \frac{v^2}{R}$$
$$\tan\theta = \frac{v^2}{R}$$

$$\Sigma F_y = 0$$
$$N \cos\theta - mg = 0$$
$$N = \frac{mg}{\cos\theta}$$

$$v = (60.0\ \text{km/h})\left(\frac{1\text{h}}{3600\text{s}}\right)\left(\frac{1000\text{m}}{1\text{km}}\right) = 16.7\ \text{m/s}$$
$$\tan\theta = \frac{(16.7)^2}{(190.0)(9.81)} \Rightarrow \theta = 8.5^\circ$$

d. An aluminum rod L = 1.60 m long is held at its center. It is hit with a hammer to set up a longitudinal vibration (like you did in the lab). The density of aluminum is ρ = 2700.0 kg/m^3 and Young's modulus Y = 7.0x10^{10} Pa. Calculate the speed of longitudinal waves in the rod and the wavelength and frequency of the fundamental standing wave (first harmonic).



$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{7.0\times10^{10}}{2700}} = 5091\ \text{m/s}$$
$$\frac{\lambda}{2} = L \Rightarrow \lambda = 2L = 2(1.60) = 3.20\text{m}$$
$$v = \lambda f \Rightarrow f = \frac{v}{\lambda} = \frac{5091}{3.20} = 1590\ \text{Hz}$$
$$\simeq 1600\ \text{Hz}$$

THE END

THE END

$x = x_0 + v_0t + ½ at^2$ $v = v_0 + at$ $v^2 = v_0^2 + 2a(x - x_0)$ $v_{avg} = ½(v_0 + v)$ $(x - x_0) = ½(v_0 + v)t$

$\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$ $\mathbf{v}_{avg} = \frac{\Delta r}{\Delta t}$ $\mathbf{v} = \frac{dr}{dt} = \frac{dx}{dt}\boldsymbol{i} + \frac{dy}{dt}\boldsymbol{j} + \frac{dz}{dt}\boldsymbol{k}$ $\mathbf{a}_{avg} = \frac{\Delta v}{\Delta t}$ $\mathbf{a} = \frac{dv}{dt} = \frac{dv_x}{dt}\boldsymbol{i} + \frac{dv_y}{dt}\boldsymbol{j} + \frac{dv_z}{dt}\boldsymbol{k}$

$\mathbf{F} = m\mathbf{a}$ $f = \mu N$ $a_c = \frac{v^2}{r}$ $F_g = mg$ $F_g = \frac{GMm}{r^2}$ $F = - kx$

$\theta = \frac{s}{r}$ $\omega = \frac{d\theta}{dt}$ $\alpha = \frac{d\omega}{dt}$ $\Delta\theta = \omega_0 t + ½ \alpha t^2$ $\omega = \omega_0 + \alpha t$ $\Delta\theta = \left(\frac{\omega_i + \omega_f}{2}\right)t$

$v = r\omega$ $\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$ $a_c = \frac{v^2}{r} = r\omega^2$ $\mathbf{a}_c = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$ $a_t = \frac{dv}{dt} = r\alpha$ $\mathbf{a}_t = \boldsymbol{\alpha} \times \mathbf{r}$

$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$ $\boldsymbol{\tau} = I\,\boldsymbol{\alpha}$ $I_{CM}^{rod} = (1/12)\ ML^2$ $I_{CM}^{disk} = (1/2)\ ML^2$ $I_{CM}^{sphere} = (2/5)\ ML^2$ $I = I_{cm} + Md^2$

$\mathbf{p} = m\mathbf{v}$ $\mathbf{F} = \frac{dp}{dt}$ $\Delta\mathbf{p} = \mathbf{F}_{av}\Delta t$ $\mathbf{L} = I\boldsymbol{\omega}$ $\boldsymbol{\tau} = \frac{dL}{dt}$ $\Delta\mathbf{L} = \boldsymbol{\tau}_{av}\Delta t$

$\sigma = Y\,\varepsilon$ $\frac{F}{A} = Y\frac{\Delta L}{L}$ $\frac{F}{A} = G\frac{\Delta L}{L}$ $\Delta p = -B\frac{\Delta V}{V}$ $Y = 2G(1 + \nu) = 3B(1 - 2\nu)$

$F = - kx$ $\tau = -D\,\theta$ $x = A\cos(\omega t + \phi)$ $x = (A_o e^{-t/\tau})\cos(\omega t + \phi)$

$\omega = \frac{2\pi}{T} = 2\pi f t$ $T = 2\pi\sqrt{\frac{m}{k}}$ $T = 2\pi\sqrt{\frac{I}{D}}$ $T = 2\pi\sqrt{\frac{I}{MgL}}$ $T = 2\pi\sqrt{\frac{L}{g}}$

$D(x,t) = A\sin(kx - \omega t + \phi)$ $D(x,t) = A\sin(kx)\cos(\omega t)$ $k = \frac{2\pi}{\lambda}$ $\omega = \frac{2\pi}{T} = 2\pi f t$ $v = \lambda f = \frac{\omega}{k}$ $v = \sqrt{\frac{T}{\mu}}$ $v = \sqrt{\frac{Y}{\rho}}$

$W = \mathbf{F \cdot \Delta x}$ $W = \int_{x_1}^{x_2} F(x)\,dx$ $\Delta K + \Delta U = W_{nc}$ $\Delta K + \Delta U = 0$ $K = ½ mv^2$ $K = ½ I\omega^2$

$\Delta U_g = mgh$ $\Delta U_g = -\frac{GMm}{R}$ $\Delta U_s = ½ kx^2$

$F = \sqrt{F_x{}^2 + F_y{}^2 + F_z{}^2}$ $F_x = F\cos\alpha,$ $F_y = F\cos\beta,$ $F_z = F\cos\gamma$ $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$g = 9.81\ m/s^2$ $G = 6.67\text{x}10^{-11}\ N\ m^2/kg^2$ $\bar{x} = \frac{\Sigma_i x_i}{N}$ $\sigma = \sqrt{\frac{\Sigma_i(x_i - \bar{x})^2}{N - 1}}$ $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}} = \sqrt{\frac{\Sigma_i(x_i - \bar{x})^2}{N(N-1)}}$

$\frac{d(x^n)}{dx} = nx^{n-1}$ $\frac{d(\cos x)}{dx} = -\sin x$ $\frac{d(\sin x)}{dx} = \cos x$