Answers to Assignment 1

1. In perfect competition all firms produce at minimum average cost (MAC). This defines the long run industry supply curve. We have $C = 25 + q^2$, so

$$AC = \frac{25}{q} + q$$

and to find min AC we set

$$\frac{\partial AC}{\partial q} = -\frac{25}{q^2} + 1 = 0$$

Solving for q we obtain $q^* = 5$. So min $AC = 10$ and $P^* = 10$. From the demand curve we obtain $Q^* = 30$, and hence 6 firms produce in the perfectly competitive equilibrium.

2. The marginal cost for a fringe firm = q_f . The supply curve of each fringe firm is given by $q_f = p$. Total fringe supply is given by $\sum_{i=1}^{50} q_f^i = 100p$. Thus residual demand for the Dominant Firm is: $Q^D = Q^M - Q^f = 1000 - 50p - 100p = 1000 - 150p$. Since the dominant firm has zero marginal costs, profits are:

$$\pi^D = (1000 - 150p)p$$

Choosing p to maximize π^D yields $p = 3.3$.

ANSWERS TO QUESTION 3

The monopolist can pursue THREE different strategies.

A. *Sell to both types in period 1.*

Type 1 values good at $1 + \delta$ in period 1

Type 2 values good at $0.5(1 + \delta)$ in period 1.

So for this strategy the monopolist would have to set

$$P_1 = 0.5(1 + \delta)$$

To induce both types to buy in period 1.

We can compute profits from this strategy as

$$\pi = 2(0.5 + 0.5\delta - c)$$

$$= 1 + \delta - 2c$$

B. *Sell to type 1 in period 1 and type 2 in period 2.*

Cannot set $P_1 = (1 + \delta)$ because type 1 would get zero surplus from buying in period 1 but positive surplus from "waiting" until period 2. So have to choose a price P_1 that just allows type 1 the same surplus (or epsilon more) from buying in period 1 and from buying in period 2. We know that in order to sell to type 2 in period 2 we must have $P_2 = 0.5$ So the price P_1 must satisfy

$$1 + \delta - P_1 \geq \delta(1 - 0.5)$$

Or

$$P_1 \leq 1 + 0.5\delta$$

Which means the monopolist will set

$$P_1 = 1 + 0.5\delta$$

So that now we can compute profits as

$$\pi = (P_1 - c) + \delta(P_2 - c)$$

$$= (1 + 0.5\delta - c) + \delta(0.5 - c)$$

$$= 1 + \delta - c(1 + \delta)$$

C. *Sell only to type 1 in period 1*

(one way to do this is to set $P_2 = \infty$)

So $P_1 = (1+\delta)$ and $\pi = 1 + \delta - c$

And by comparing expressions, we can see that

$$\pi_C > \pi_A$$

And

$$\pi_C > \pi_B$$

So strategy C is optimal!!

(b) C is not available now, because the monopolist cannot commit to not selling in period 2 (and everyone knows that $P_2 = 1/2$. But, since

$$\pi_B > \pi_A$$

Then strategy B dominates strategy A.

BUT there is another option available to the monopolist, a version of the Pacman Strategy. Charge $P_1 = (1+\delta)$ in period 1, where the type 1 consumer knows that even though they are getting zero surplus for purchasing in period 1, if they wait, the monopolist will charge $P_2 = 1$ in period 2, and only consumer 1 will purchase and still get zero surplus. $P_2 = 1$ in period 2 is optimal because the profits from doing so $P - c$ are greater than the profits from selling to both (in period 2) = $2(1/2 - c)$ = $1 - 2c$.

Knowing this, the type 1 consumer will buy in period 1 and the monopolist will charge $P_2 = \frac{1}{2}$ in period 2 for a total profit of

$$\pi = ((1+\delta) \quad - c) + \delta(1/2 - c)$$

$$= 1 + \frac{3}{2}\delta - c(1+\delta)$$

This obviously dominates (the no commitment version of) strategy B, because the monopolist is getting a higher price in period 1.

(c) neither answer depends on δ and c (actually, if $\delta = 1$ then $\pi_B = \pi_A$)