MAT135H1: Calculus 1(A) – University of Toronto

Sample Term Test – Fall 2017 – Fall 2017

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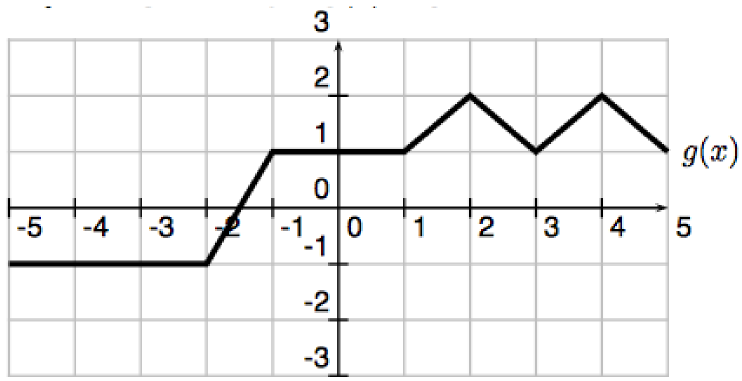
This test contains 9 pages (including this cover page) and 8 problems. Check to see if any pages are missing.

- **You have 1.5 hours to complete the test.**
- **Only the front of each page will be graded.** You may use the back of the pages as scratch paper.
- **Organize your work** in a reasonably neat and coherent way in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Unsupported answers will not receive full credit**. A correct answer without explanation will receive no credit unless otherwise noted; an incorrect answer supported by substantially correct calculations and explanations may receive partial credit.
- **Include units in your answer where appropriate.**
- You must use the methods learned in this course to solve all of the problems.

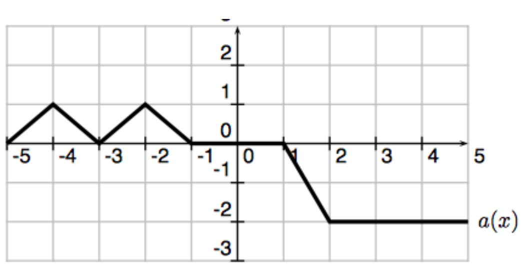
1. (10 points) The graph of $y = g(x)$ is shown below.



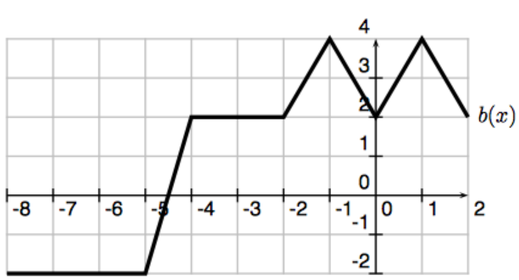
(a) (2 points) For what x in the interval $(-5, 5)$ is $g(x)$ not differentiable?

(b) (2 points) Find $\lim_{x\to -2} 3g(x) + 5$.

(c) (6 points) The graphs of 2 functions $a(x)$ and $b(x)$ are shown below. Write an algebraic expression for $a(x)$ and $b(x)$ in terms of the function g.



$a(x) =$ ______________________



$b(x) =$ ______________________

2. (12 points) According to ISED Canada, Canadian imports from Asia continued to grow between 2000 and 2016. Let $I(t)$ be the value (in billions of dollars) of merchandise that Canada imports from Asia t years after 2000. Interpret each of the following mathematical statements in practical terms. Do not use the word 'instantaneous' or the phrase 'rate of change' (i.e. your answer should be meaningful to a 12-year-old).

   (a) (2 points) $I(15) = 118$

   (b) (3 points) $I'(16) = 14$

   (c) (3 points) $I'(1) < I'(16)$

   (d) (4 points) $(I^{-1})'(61) > (I^{-1})'(78)$

3. (7 points) The *basal metabolic rate* of an animal is the rate of energy expenditure per unit time of an animal at rest. The larger an animal, the larger its basal metabolic rate. One model for the basal metabolic rate is Kleiber's Law, which states that the basal metabolic rate R of an organism is proportional to $M^{3/4}$, where M is the total mass of the animal. That is

$$R = kM^{3/4}$$

for some constant k.

Throughout this problem, energy is measured in kilocalories, mass is measured in kilograms, and time is measured in days.

(a) (4 points) An elephant weighs approximately 160,000 times as much as a mouse. Suppose that a resting mouse expends 3 kilocalories each day. If Kleibler's law holds, how much energy do you expect a resting elephant to expend in a day?

(b) (3 points) Suppose that the basal metabolic rate of a 55 kg person is 1010 kilocalories per day and that $\left.\frac{dR}{dM}\right|_{M=55} = 13.8$. Use this information and a local linearization to approximate the basal metabolic rate of a 58 kg person. .

4. (12 points) The table below gives several values of a continuous, invertible, differentiable function $p(x)$. Assume that the domain of both $p(x)$ and $p'(x)$ is the interval $(-\infty, \infty)$.

x	0	3	6	9	12	15	18	22	24
$p(x)$	-11.5	-9.5	-2	0	6	7	13	14	31

(a) (2 points) Compute the average rate of change of p on the interval $6 \leq x \leq 22$.

(b) (3 points) Approximate $p'(7)$.

(c) (3 points) Suppose that $p'(0) = 2$. Find an equation for the tangent line to the graph of $y = p(x)$ at $x = 0$.

(d) (4 points) Let $q(x) = p^{-1}(x)$. Estimate $q'(3)$.

5. (8 points) A typical electric oven is a metal box that is heated by a C- or U-shaped metal bar floating above the bottom of the oven. The metal bar is heated to about 1200 degrees Fahrenheit by an electrical current and glows red when it is on. The heating bar turns on when the oven's thermostat registers a temperature below some limit and turns off when the thermostat registers a temperature that rises above another limit. This causes the temperature of the oven to vary over time.

   (a) (4 points) *Consumer's Reports* is measuring the temperature of an oven using a thermometer set inside. They set the oven to 350 degrees Fahrenheit, but the temperature of the oven cycles between 335 degrees and 365 degrees. After preheating the oven, it takes exactly 11 minutes for the oven to go from its highest temperature to its lowest temperature. Suppose that $T(t)$ is a function that gives the temperature of the oven t minutes after the oven preheats to exactly 350 degrees.

   Which of the following equations best models $T(t)$?

   (a) $T(t) = 30\sin\left(\frac{\pi t}{11}\right) + 350$

   (b) $T(t) = 15\sin\left(\frac{\pi t}{11}\right) + 350$

   (c) $T(t) = 15\sin\left(\frac{2\pi t}{11}\right) + 350$

   (d) $T(t) = 15\cos\left(\frac{2\pi t}{11}\right) + 350$

   (e) $T(t) = 30\cos\left(\frac{\pi t}{11}\right) + 350$

   (b) (4 points) Suppose that when the oven is switched off, the oven's temperature in degrees Fahrenheit decreases by 17% each minute. Assuming that the oven is switched off when it is at 350 degrees, write a formula for the temperature T of the oven t minutes after it is switched off.

6. (6 points) Consider the limit

$$\lim_{h \to 0} \frac{(2+h)^3 - 8}{h}.$$

(a) (3 points) Find a function $f(x)$ and a number a such that this limit is equal to $f'(a)$.

(b) (3 points) Find the value of this limit using the limit laws. Be sure to justify your answer, and identify which limit laws you use when you use them.

7. (10 points) Compute the following limits algebraically using limit laws. Be sure to justify your answers, and clearly identify the limit laws that you use. If the limit does not exist, explain why.

   (a) (3 points) $\lim_{x\to 3} \dfrac{\sqrt{x+6}-x}{x^3-3x^2}$

   (b) (3 points) $\lim_{x\to \infty} \dfrac{\sqrt{x^2-9}}{2x-6}$

   (c) (4 points) $\lim_{x\to 5}(x-5)^2\cos(2/x^2+3e^{-x})+2$

8. (6 points) Determine whether each of the following statements is true or false. Clearly indicate your answer. No explanation is required.

   (a) (2 points) If an object moves with the same average velocity over every time interval, then its average velocity equals its instantaneous velocity at every time.

      A. True

      B. False

   (b) (2 points) The instantaneous speed of a moving object is the absolute value of the object's instantaneous velocity.

      A. True

      B. False

   (c) (2 points) Recall that there are 1000 grams in a kilogram. If $f(t)$ is the quantity in grams of a chemical produced after t minutes and $g(t)$ is the same quantity in kilograms, then $f'(t) = 1000g'(t)$.

      A. True

      B. False