STA 220H1F

Summary of Inferential Procedures

<table>
<tr><th colspan="2">Situation</th><th>$\boldsymbol{H_0}$, test statistic and its distribution under $\boldsymbol{H_0}$</th><th>Confidence Interval</th></tr>
<tr><td colspan="2">One-sample inference
for the mean</td><td>$$H_0: \mu = \mu_0$$ $$t^* = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$ has a t distribution with $n-1$ df</td><td>For the mean μ: $$\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}$$</td></tr>
<tr><td rowspan="2">Two dependent samples
Inference for the difference

better known as

Matched pairs

a.k.a.

One sample inference for the differences</td><td>Case 1:
Differences are approximately normal
(or the CLT applies).
Inference is for the mean of the differences.</td><td>$$H_0:\ \mu_d = 0$$ $$t^* = \frac{\bar{x}_d}{s_d/\sqrt{n}}$$ where d is the difference between each pair of observations, $\bar{x}_d$ is the sample mean, and s_d is the sample standard deviation of the observed differences.
Has a t distribution with $n-1$ df</td><td>For the mean of the differences μ_d: $$\bar{x}_d \pm t_{\alpha/2}\frac{s_d}{\sqrt{n}}$$</td></tr>
<tr><td>Case 2:
No assumption about the distribution.
Inference is for the median of the differences.</td><td>Sign test (a non-parametric test)
X is the number of positive differences
X has a Binomial$(n, 0.5)$ distribution
H_0 is that the median of the differences is 0 (equivalent to $p = 0.5$ where p is the proportion of positive differences)</td><td></td></tr>
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Two-sample inference for the difference between two means Two independent samples of sizes n_1 and n_2	*Case 1:* $\sigma_1 \neq \sigma_2$ (Welch-Satterthwaite)	$H_0: \mu_1 = \mu_2$ $t^* = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$ has approximately a t distribution with df given by the Satterthwaite formula	For the difference in means $\mu_1 - \mu_2$: $(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}$
	Case 2: $\sigma_1 = \sigma_2$ (pooled variance)	$H_0: \mu_1 = \mu_2$ $t^* = \dfrac{\bar{x}_1 - \bar{x}_2}{s_P\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$ where s_P is the pooled estimate of the s.d. $s_P^2 = \dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$ Has a t distribution with df= $n_1 + n_2 - 2$	For the difference in means $\mu_1 - \mu_2$: $(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2} s_P\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}$

One sample inference for a proportion	*Case 1:* Small sample sizes	$H_0\colon\ p = p_0$ X, the number of successes, has a Binomial(n, p_0) distribution	(not covered in the course)
	Case 2: Large sample sizes	$H_0\colon\ p = p_0$ $z^* = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$ has a $N(0,1)$ distribution	For the population proportion p: $\hat{p} \pm z_{\alpha/2}\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$
Two-sample test for the difference in two proportions Two independent samples of sizes n_1 and n_2	Large sample test only	$H_0\colon p_1 = p_2$ $z^* = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$ where $\hat{p} = \dfrac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$ Has a $N(0,1)$ distribution	For the difference in proportions $p_1 - p_2$: $\hat{p}_1 - \hat{p}_2 \pm z_{\alpha/2}\times$ $\sqrt{\dfrac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \dfrac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$

Linear regression inference for the slope	$H_0: \beta_1 = 0$ $t^* = \frac{b_1}{S.E.(b_1)}$ has a t distribution with df $= n - 2$	For the slope, β_1: $b_1 \pm t_{\alpha/2} S.E.(b_1)$

Conditions for tests and confidence intervals to be valid:

- For all tests and confidence intervals: Observations are a simple random sample from a population or results from a randomized experiment.
- For all tests and confidence intervals: Observations are independent of each other (or the sample is at most 10% of the population, randomly chosen). Note that for the paired *t*-test (and the sign test for matched pairs) and paired confidence intervals, the pairs are independent of each other but the two observations in each pair are not independent of each other. (Sometimes the paired procedures are referred to as two dependent samples.)

- For tests and confidence intervals for two independent samples (for the mean or for a proportion): The two groups are independent. (Note that this is not true for the paired t-test (and sign test) and paired confidence interval.)
- For the pooled two-sample t-test: Both groups have the same standard deviation. *Rule of thumb:* OK to pool variances if the ratio of the largest to smallest sample standard deviation is < 2.
- For all tests and confidence intervals for means: The observations follow a normal distribution. As long as the sample size is sufficiently large and the deviations from normality are not extreme, this assumption is not critical because of the Central Limit Theorem.
- For tests and confidence intervals for proportions: *Rule of thumb:* To use the normal distribution approximation, need $np \geq 10$ and $n(1-p) \geq 10$ (for both groups for two-sample procedures).
- For linear regression: A linear relationship is an appropriate model.
- For linear regression: The error terms (estimated by the residuals) are normally distributed and the variability in the error terms is constant across the range of the independent variable (or, equivalently, across the range of the fitted values).