Midterm 1

1) Matthew wants to buy a new car in four years from now. He expects to pay $15,000 for the purchase then. How much money should Matthew put in his savings account now if a bank pays 5% simple interest rate on this account?

A) $12,104

B) $12,500

C) $11,629

D) $14,286

E) $13,465

ANSWER: B

N = 4
Futrue value, F = 15,000
Simple interest rate, i = 5% = 0.05
Money Matthew needs to save now, the principal amount, P = ?
Formula for simple interest rate including the principal is:

$F = P + I$

$F = P + P\, i\, N$; where $I = P\, i\, N$

$15{,}000 = P\,(1 + 0.05 * 4)$

$P = 15000 / (1.2) = 12{,}500$

2) What is the dimension of interest rate? (Hint: Think in terms of what does a 9% interest mean.)

A) time period

B) currency/time period

C) currency/currency/time period

D) time period/currency

E) None of the above

ANSWER: C

9% interest means that for every $100 investment, $9 will be the interest per interest period. Hence, the dimension is: currency/currency/time period.

3) Janet kept $4,500 in a bank account that pays her 5% annual interest rate. After two years her friend Sam offered her an alternate but better investment deal for a two-year period. What is the minimum amount that Sam's offer must pay as a lump-sum payment at the end of the investment term for Janet to be interested in Sam's offer?

A) $3,100

B) $5,470

C) $5,500

D) $4,033

E) $5,169

ANSWER: B

Overall investment period is, N = 4
Principal, P = 4,500
Interest rate, i = 5% = 0.05
Futrue value, F = ?
Formula is: $F = P\,(1 + i)^N$

$F = 4{,}500\,(1 + 0.05)^4$

$F = 5{,}470$

4) If the effective equivalent annual interest rate is 20%, and interest is compounded quarterly, what is the corresponding nominal annual interest rate?

A) 11.49%

B) 17.32%

C) 19.95%

D) 18.65%

E) 13.19%

ANSWER: D

Effective annual interest rate, $i_e = 20\% = 0.2$
Interest period is quarterly, m = 4
Nominal interest rate, r = ?
Formula: $i_e = (1 + r/m)^m – 1$

$0.2 = (1 + r/4)^4 – 1$

$1.2 = (1 + r/4)^4$

$(1.2)^{1/4} = (1 + r/4)$

$r/4 = (1.2)^{1/4} – 1$

$r = 4 * (1.04663513939211 – 1) = 0.1865406 \approx 18.65\%$

5) The base unit over which an interest rate is calculated is called the

A) period

B) time period

C) base period

D) interest period

E) None of the above

ANSWER: D

6) It is known that the total interest paid over a 5-year period is $4,402.26. What was the principal amount borrowed at a 10% nominal interest rate compounded semi-annually (i.e., twice in a year)?

A) $4,000

B) $7,000

C) $3,000

D) $6,000

E) $5,000

ANSWER: B

Total interest amount, I = 4,402.26
Nominal interest rate, r = 10% = 0.1
Compounding period is semi-annual, m = 2
So, the effective interest rate for the semi-annual compounding period, $i_s = r/m = 0.1/2 = 0.05$
Note that the relationship among total interest amount, future value and principal amount is:
I = F – P
$I = P(1 + i)^N - P$; where N = 10 (why? – because, in 5-year, there are 10 semi-annual periods)
$4{,}402.26 = P[(1 + i)^N - 1]$
$4{,}402.26 = P[(1 + 0.05)^{10} - 1]$
$P = 4{,}402.26 / [(1.05)^{10} - 1] = 7{,}000$

Alternatively, we can use equivalent effective interest rate for an annual compounding period and use N = 5. In that case we have to use:
$i_e = (1 + r/m)^m - 1$
$i_e = (1 + 0.1/2)^2 - 1 = 0.1025 = 10.25\%$, this is the equivalent effective interest rate if the compounding period was annual (rather than semi-annual).

Now we use i_e with N = 5 in the following formula:
$I = P(1 + i_e)^N - P$
$I = P(1 + 0.1025)^5 - P$
$4{,}402.26 = P[(1.1025)^5 - 1]$

$P = 4{,}402.26 / [(1.1025)^5 - 1] = 7{,}000$

7) What is the conventional method of stating the annual (compound) interest rate?

A) Effective interest rate

B) Real interest rate

C) Compound interest rate

D) Nominal interest rate

E) None of the above

ANSWER: D

8) If you borrow $2,000 today at 5% interest rate for 5 years, what is the total interest that you have to pay?

A) $2,552.6

B) $2,500.0

C) $1,976.6

D) $552.6

E) $500.6

ANSWER: D

Principal, $P = 2{,}000$

$N = 5$

Interest rate, $i = 5\% = 0.05$

Total interest amount, $I = ?$

Formula is: $I = F - P$

$I = P(1 + i)^N - P$

$I = 2{,}000(1 + 0.05)^5 - 2{,}000$

$I = 2{,}552.6 - 2{,}000 = 552.6$

9) What does the term "market equivalence" imply?

A) the ability to exchange one cash flow for another at some cost

B) the ability to obtain a zero net cash flow

C) the existence of a mathematical relationship between time and money

D) indifference on the part of a decision maker among available cash flow choices due to zero transactional cost

E) the ability to exchange one cash flow for another at minimum cost

ANSWER: D

10) If the quarterly interest rate is 2.5%, what is the nominal interest rate?

A) 2.5%

B) 5.0%

C) 7.5%

D) 10.0%

E) 12.5%

ANSWER: D
Quarterly (effective) interest rate is, $i_s = 2.5\%$
Interest period is quarter, $m = 4$
Nominal interest rate, $r = ?$
Formula: $i_s = r/m$
$r = i_s * m = 2.5\% * 4 = 10\%$

11) How long will it take for any sum to triple when the interest rate is 5%, compounded annually?

A) approx. 14.5 years

B) approx. 17.0 years

C) approx. 22.5 years

D) approx. 12.0 years

E) approx. 18.5 years

ANSWER: C
Interest rate, $i = 5\% = 0.05$
Let principle amount be, P
So, future value, $F = 3P$
$N = ?$
Formula: $F = P (1 + i)^N$
$3P = P (1 + 0.05)^N$
$3 = (1.05)^N$
$\ln(3) = N \ln(1.05)$
$N = \ln(3) / \ln (1.05) = 22.5$ years

12) A one-year project has $105 million as initial investment and generates a net savings of $122 million in a year. What is the project's IRR?

A) 12.1%

B) 14.1%

C) 10.1%

D) 16.2%

E) 11.1%

ANSWER: D

Project period, N = 1 year
Initial investment (i.e., project cost) = 105 ml
Savings after one year (i.e., project benefit) = 122 ml

So, present worth of project cost, PW(cost) = 105
and present worth of project benefit, PW(benefit) = $122/(1 + i)^1$; where i = IRR

By definition of IRR: PW(cost) = PW(benefit)

$105 = 122/(1 + i)$

$1 + i = 122/105$

$i = 1.1619 - 1 = 0.1619 \approx 16.2\%$

13) The following table summarizes information for five projects:

Project	Frist Cost (in $)	IRR on Overall Investment	IRR on Incremental Investment Compared with Projects (%)			
			1	2	3	4
1	105,000	12%				
2	185,000	13%	11%			
3	215,000	9%	8%	7%		
4	265,000	14%	12%	14%	15%	
5	310,000	17%	13%	17%	12%	9%

The data can be interpreted in the following way: The IRR on the incremental investment between project 1 and project 2 is 11%.

If all projects are independent and the company has at least $1,080,000 to invest, which projects should be undertaken if the MARR is 10%?

A) only 3.

B) 1, 2, 3, 4 and 5.

C) 1, 2, 3 and 5.

D) 1, 2, 4 and 5.

E) 1, 3 and 5.

ANSWER: D

Since projects are <u>independent</u>, all projects with IRR ≥ MARR will be chosen if sufficient funds are available. From the first costs we observe that the investor has sufficient funds (i.e., $1,080,000).

Now note that all projects have IRR > MARR=10% except for project 3. Hence, all but project 3 will be chosen.

14) In general, the IRR comparison method and the PW comparison method

A) produce the same results for independent projects but not for mutually exclusive projects.

B) produce different results for both independent projects and mutually exclusive projects.

C) produce the same results for independent projects and mutually exclusive projects with unequal lives.

D) produce the same results for independent projects and mutually exclusive projects with equal lives.

E) produce the same results for mutually exclusive projects but not for independent projects.

ANSWER: D

15) I have 3 possible choices for a microwave oven. They have expected working lives of 2, 3 and 4 years. If I expect oven technologies to be stable for the foreseeable future, over what period of time should I compare the equivalent uniform annual costs of the three choices?

A) 45 years

B) 12 years

C) 30 years

D) 20 years

E) 65 years

ANSWER: B

This is a question where repeated lives method has to be applied. The least common multiplier for 2, 3 and 4 is 12, which is the answer.

16) A project requires no initial investment. It costs $4,000 a year from now and earns $6,000 two years from now. What is its internal rate of return?

A) 100%

B) 50%

C) 75%

D) 24%

E) 141%

ANSWER: B

Present worth of project cost, PW(cost) = 4,000
and present worth of project benefit, PW(benefit) = $6{,}000/(1 + i)^1$; where i = IRR

Note that in this case we are comparing project costs and benefits at the 1 year mark.

Now, by definition of IRR: PW(cost) = PW(benefit)

$4{,}000 = 6{,}000/(1 + i)$

$1 + i = 6{,}000/4{,}000$

$i = 1.5 – 1 = 0.5 = 50\%$

17) A 18-year project requires initial investment of $15,000. If it generates an annual revenue of $1,800 but also requires $300 in annual maintenance costs, what is the payback period when the MARR is 5%?

A) 7.9 years

B) 10 years

C) 7.5 years

D) 9.2 years

E) 8.4 years

ANSWER: B

Note that both project revenue and maintenance costs are annualized. Hence, can simply use those figures to find out annual savings, which is $1,800 – $300 = $1,500

By definition: Payback period = Initial cost / Annual savings
that is, payback period is the amount of time (usually in years) it requires for a project to recoup its initial investment.

So, Payback period = $15,000 / $1,500 = 10

18) The minimum acceptable rate of return (MARR) is

A) the least interest rate among all alternative projects.

B) an interest rate that allows an investor to recoup the investment.

C) an interest rate, which is equal to a current bank interest rate.

D) a highest interest rate among all alternative projects.

E) an interest rate that must be earned for a project to be accepted.

ANSWER: E

19) Robin is looking at a 20% annual interest rate, compounded quarterly. She wants to find out how much she should be saving on a quarterly basis if she wants to reach her savings target of $12,000 in 4 years.

A) $242.14

B) $322.37

C) $407.14

D) $507.24

E) $670.78

ANSWER: D

Nominal interest rate, r = 20% = 0.2
Compounding period is quarterly, m = 4

Note that since the question is asking for quarterly savings target, we can simply find out the effective interest rate for quarterly compounding period, i_s, by: $i_s = r/m = 0.2/4 = 0.05$

Also note that the quarterly savings target takes the form of an annuity, A, where the future value, F = 12,000. Hence, we need to use the sinking fund factor (A / F , i, N)

Formula: $A = F * (A / F , i, N)$
where $i = i_s = 5\%$ and N = 16 (why? Quarterly compounding in 4 years gives N = 16 compounding periods)

Hence, $A = 12{,}000 * (A / F , 0.05, 16)$
$A = 12{,}000 * 0.04227$; where $(A / F , 0.05, 16) = 0.04227$ taken from the interest table
$A = \$507.24$

20) The internal rate of return (IRR) is negative if

A) IRR cannot be negative.

B) a project is a simple investment.

C) a project just breaks even.

D) a cash inflow exceeds a cash outflow.

E) a project is losing money.

ANSWER: E

21) You want to have a million dollars in the bank when you retire. You think you can save $8,000 a year in a bank that offers you 5% interest. If you make your first deposit in a year's time, how many years will it be from now before you can retire?

A) 60.7

B) 70.2

C) 30.3

D) 50.5

E) 40.6

ANSWER: E

Future value, F = 1,000,000
Savings is done in the form of an annuity, A = 8,000
Interest rate, i = 5% = 0.05
N = ?
Formula will involve using the uniform series compound amount factor:
$(F/A, i, N) = [(1 + i)^N – 1] / i$

Hence:
$F = A * (F/A, i, N)$
$F = A * [(1 + i)^N – 1] / i$
$1,000,000 = 8,000 * [(1 + 0.05)^N – 1] / 0.05$
$1,000 = 8 * [(1.05)^N – 1] / 0.05$; dividing both sides by 1,000
$[(1,000/8) * 0.05] = (1.05)^N – 1$
$[125 * 0.05] + 1 = (1.05)^N$
$6.25 + 1 = (1.05)^N$
$\ln (7.25) = N \ln (1.05)$
$N = \ln (7.25) / \ln (1.05) = 40.6$ years

22) The internal rate of return (IRR) is

A) the interest rate that ensures the positive cash flow of a project.

B) the interest rate that measures the return from operating costs

C) the interest rate that allows an investor to recoup the initial investment.

D) the interest rate that breaks even a project's costs and benefits.

E) the interest rate that is set up by an investor to guarantee that the return on investment will be higher than from a bank interest rate.

ANSWER: D

23) A set of equal disbursements (or receipts) over a sequence of periods is referred to as

A) the arithmetic gradient series.

B) the annuity.

C) the geometric gradient series.

D) the present worth.

E) the annual worth.

ANSWER: B

24) The following table summarizes information for five projects:

Project	Frist Cost (in $)	IRR on Overall Investment	IRR on Incremental Investment Compared with Projects (%)			
			1	2	3	4
1	100,000	19%				
2	175,000	15%	9%			
3	200,000	18%	17%	23%		
4	250,000	16%	12%	17%	13%	
5	300,000	17%	14%	11%	17%	16%

The data can be interpreted in the following way: The IRR on the incremental investment between project 1 and project 2 is 9%.

If all projects are mutually exclusive and the company has at least $380,000 to invest, which projects should be undertaken if the MARR is 16%?

A) only 5.

B) 1, 2, 3, 4 and 5.

C) only 3.

D) 1, 2, 4 and 5.

E) only 1.

ANSWER: A
To answer this question, we first we need to arrange the project in an ascending order of first cost (for later pairwise comparison). Note that the table is organized as such.

Secondly, since projects are <u>mutually exclusive</u> (implying that only one of the projects can be chosen), we need find out the subset of projects that has IRR ≥ MARR (why? Because any project with IRR < MARR implies that that project will lose money or have a negative return compared to the MARR). A careful observation of column three (from the left), identifies that all but project 2 meet that criterion. Hence, for the next step we need to consider only the subset of project {1, 3, 4 and 5}.

Thirdly, for this subset of projects, we need to pairwise compare projects (that is, compare two projects at a time based on our ranking) which meet the criterion *Incremental* IRR ≥ MARR (why? Because this would ensure that, between a pair of projects, the additional investment required for the project with higher first cost earns a return at least equal to the MARR). This is done using information from column 4, 5 , 6 & 7 as follows:

<u>Comparing project 1 & 3</u>:
We first arbitrarily decide that project 1 is the best one (why? No specific reason although it can be loosely argued that it is based on the fact that project 1 requires the least amount of capital or the first cost money).

Then we compare project 1 to 3. We observe that the *Incremental* IRR between project 1 & 3 is 17%, which is greater than MARR = 16%. Hence project 3 is chosen over project 1.

<u>Comparing project 3 & 4</u>:
We observe that the *Incremental* IRR between project 3 & 4 is 13%, which is lower than MARR = 16%. Hence project 4 is rejected in favour of project 3 (and project 3 is retained as still the best option).

<u>Comparing project 3 & 5</u>:
We observe that the *Incremental* IRR between project 3 & 5 is 17%, which is greater than MARR = 16%. Hence project 5 is chosen over project 3 (and project 3 is now rejected).

Thus, in this mutually exclusive set of projects, project 5 is the best option to choose.

25) Octavia has a small house on a small street in a small town on which she currently has $80,000 as mortgage loan. She is now considering whether to sell the house now or one year later. If she sells the house now, she has an offer of $110,000. If she waits for one year, she has another offer of $120,000. If she sells the house now, she can invest the money in a one-year guaranteed growth bond that pays 5% interest, compounded monthly. If she keeps the house, the interest on the mortgage payment is 5% compounded daily. What should Octavia do?

A) Keep the house.

B) Sell the house.

ANSWER: A

Note that the question implicitly assumes that Octavia pays off the mortgage as soon as she sells the house (either now or a year later). Consider the following:

Selling the house:

Mortgage amount = $80,000

Current offer = $110,000

So, free cash to invest in a one-year guaranteed growth bond = $110,000 – $80,000 = $30,000

Note that this investment pays 5% (nominal) interest, compounded monthly. Hence, r = 5% = 0.05 and m = 12.

The effective monthly interest rate is, $i_s = r/m = 0.05 / 12 = 0.0041667$

So, by investing $30,000, Octavia would earn:

$F_{selling} = 30{,}000\ (1 + i_s)^m$

$F_{selling} = 30{,}000\ (1 + 0.0041667)^{12} = \$\ 31{,}534.86$

Alternatively, we can find out the effective annual interest rate, i_e using:

$i_e = (1 + r/m)^m - 1$

$i_e = (1 + 0.05/12)^{12} - 1 = 0.051162$, this is the equivalent effective interest rate if the compounding period was annual (rather than monthly). Hence, N = 1

So, by investing $30,000, Octavia would earn:

$F_{selling} = 30{,}000\ (1 + i_e)^N$

$F_{selling} = 30{,}000\ (1 + 0.051162)^1 = \$31{,}534.86$

Keeping the house:

Mortgage amount = $80,000

Future offer = $120,000

By deciding to keep the house, Octavia has to pay mortgage interest on $80,000 for a year. Note that the interest on the mortgage payment is 5% (nominal interest) compounded daily. Hence, r = 5% = 0.05 and m = 365.

The effective <u>daily</u> interest rate is, $i_s = r/m = 0.05 / 365 = 0.000137$
So, by keeping the mortgage of $80,000 now, a year later Octavia would have to pay a mortgage amount of:
$F_{keeping} = 80{,}000\ (1 + i_s)^m$
$F_{keeping} = 80{,}000\ (1 + 0.000137)^{365} = \$\ 84{,}101.40$

<u>Alternatively</u>, we can find out the effective <u>annual</u> interest rate, i_e using:
$i_e = (1 + r/m)^m - 1$
$i_e = (1 + 0.05/365)^{365} - 1 = 0.0512675$, this is the equivalent effective interest rate if the compounding period was annual (rather than daily). Hence, N = 1

So, by keeping the mortgage of $80,000, a year later Octavia would have to pay a mortgage of:
$F_{keeping} = 80{,}000\ (1 + i_e)^N$
$F_{keeping} = 80{,}000\ (1 + 0.05116275)^1 \approx \$84{,}101.40$

Hence, by keeping the house for a year and paying off the mortgage then, Octavia would have free cash = $120,000 – $84,101.40 = $35,898.60

<u>Decision:</u>
Sell the house and make $31,534.86
Or keep the house and make $35,898.60

Thus, Octavia would decide to keep the house.

Version 3 of MT1 had interest rate 8%. Following is the solution with interest rate 8%

Note that the question implicitly assumes that Octavia pays off the mortgage as soon as she sells the house (either now or a year later). Consider the following:

<u>Selling the house</u>:
Mortgage amount = $80,000
Current offer = $110,000
So, free cash to invest in a one-year guaranteed growth bond = $110,000 – $80,000 = $30,000

Note that this investment pays 8% (nominal) interest, compounded monthly. Hence, r = 8% = 0.08 and m = 12.
The effective <u>monthly</u> interest rate is, $i_s = r/m = 0.08 / 12 = 0.0066667$
So, by investing $30,000, Octavia would earn:

$F_{selling} = 30,000 (1 + i_s)^m$
$F_{selling} = 30,000 (1 + 0.0066667)^{12} = \$32,489.99$

<u>Alternatively</u>, we can find out the effective <u>annual</u> interest rate, i_e using:
$i_e = (1 + r/m)^m – 1$
$i_e = (1 + 0.08/12)^{12} – 1 = 0.083$, this is the equivalent effective interest rate if the compounding period was annual (rather than monthly). Hence, N = 1

So, by investing \$30,000, Octavia would earn:
$F_{selling} = 30,000 (1 + i_e)^N$
$F_{selling} = 30,000 (1 + 0.083)^1 = \$32,489.99$

<u>Keeping the house</u>:
Mortgage amount = \$80,000
Future offer = \$120,000

By deciding to keep the house, Octavia has to pay mortgage interest on \$80,000 for a year. Note that the interest on the mortgage payment is 8% (nominal interest) compounded daily. Hence, r = 8% = 0.08 and m = 365.

The effective <u>daily</u> interest rate is, $i_s = r/m = 0.08 / 365 = 0.0002192$
So, by keeping the mortgage of \$80,000 now, a year later Octavia would have to pay a mortgage amount of:
$F_{keeping} = 80,000 (1 + i_s)^m$
$F_{keeping} = 80,000 (1 + 0.0002192)^{365} = \$86,662.21$

<u>Alternatively</u>, we can find out the effective <u>annual</u> interest rate, i_e using:
$i_e = (1 + r/m)^m – 1$
$i_e = (1 + 0.08/365)^{365} – 1 = 0.0832776$, this is the equivalent effective interest rate if the compounding period was annual (rather than daily). Hence, N = 1

So, by keeping the mortgage of \$80,000, a year later Octavia would have to pay a mortgage of:
$F_{keeping} = 80,000 (1 + i_e)^N$
$F_{keeping} = 80,000 (1 + 0.0832776)^1 \approx \$86,662.21$

Hence, by keeping the house for a year and paying off the mortgage then, Octavia would have free cash = \$120,000 – \$86,662.21 = \$33,337.79

<u>Decision:</u>
Sell the house and make \$32,489.99
Or keep the house and make \$33,337.79

Thus, Octavia would decide to keep the house.