# COMP 232 Tutorial 1

September 12, 2017

## Exercise 1.1 Question 4

What is the negation of each of these propositions?

a) Jennifer and Teja are friends.
Jennifer and Teja are not friends.

b) There are 13 items in a baker's dozen.
There are not 13 items in a baker's dozen.

c) Abby sent more than 100 text messages every day.
Abby didn't send more than 100 text messages every day.

d) 121 is a perfect square.
121 is not a perfect square.

## Question 6

Suppose that Smartphone A has 256 MB RAM and 32 GB ROM, and the resolution of its camera is 8 MP; Smartphone B has 288 MB RAM and 64 GB ROM, and the resolution of its camera is 4 MP; and Smartphone C has 128 MB RAM and 32 GB ROM, and the resolution of its camera is 5 MP. Determine the truth value of each of these propositions.

a) Smartphone B has the most RAM of these three smartphones.
True

b) Smartphone C has more ROM or a higher resolution camera than Smartphone B.
True

c) Smartphone B has more RAM, more ROM, and a higher resolution camera than Smartphone A.
False

d) If Smartphone B has more RAM and more ROM than Smartphone C, then it also has a higher resolution camera.
False

e) Smartphone A has more RAM than Smartphone B if and only if Smartphone B has more RAM than Smartphone A.
False

## Question 9

Let $p$ and $q$ be the proposition "Swimming at the New Jersey shore is allowed" and "Sharks have been spotted near the shore," respectively. Express each of these compound propositions as an English sentence.

a) $\neg q$
Sharks have not been spotted near the shore.

b) $p \wedge q$
Swimming at the New Jersey shore is allowed, and sharks have been spotted near the shore.

c) $\neg p \vee q$
Swimming at the New Jersey shore is not allowed, or sharks have been spotted near the shore.

d) $p \rightarrow \neg q$
If swimming at the New Jersey shore is allowed, then sharks have not been spotted near the shore.

e) $\neg q \rightarrow p$
If sharks have not been spotted near the shore, then swimming at the New Jersey shore is allowed.

f) $\neg p \rightarrow \neg q$
If swimming at the New Jersey shore is not allowed, then sharks have not been spotted near the shore.

g) $p \leftrightarrow \neg q$
Swimming at the New Jersey shore is allowed if and only if sharks have not been spotted near the shore.

h) $\neg p \wedge (p \vee \neg q)$
Swimming at the New Jersey shore is not allowed, and either swimming at the New Jersey shore is allowed or sharks have not been spotted near the shore.
(Note that we were able to incorporate the parentheses by using the word "either" in the second half of the sentence.)

## Question 14

Let $p, q$, and $r$ be the propositions
$p$: You get an A on the final exam.

$q$: You do every exercise in this book.
$r$: You get an A in this class.
Write these propositions using $p, q$, and $r$ and logical connectives (including negations).

a) You get an A in this class, but you do not do every exercise in this book.
$r \wedge \neg q$

b) You get an A on the final, you do every exercise in this book, and you get an A in this class.
$p \wedge q \wedge r$

c) To get an A in this class, it is necessary for you to get an A on the final.
$r \rightarrow p$

d) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.
$p \wedge \neg q \wedge r$

e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.
$(p \wedge q) \rightarrow r$

f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.
$r \leftrightarrow (p \vee q)$

## Question 19

For each of these sentences, determine whether an inclusive or, or an exclusive or, is intended. Explain your answer.

a) Coffee or tea comes with dinner.
Presumably the diner gets to choose only one of these beverages, so this is an exclusive or.

b) A password must have at least three digits or be at least eight characters long.
This is probably meant to be inclusive, so that long passwords with many digits are acceptable.

c) The prerequisite for the course is a course in number theory or a course in cryptography.
This is surely meant to be inclusive. If a student has had both of the prerequisites, so much the better.

d) You can pay using U.S. dollars or euros.
At first glance one might argue that no one would pay with both currencies simultaneously, so it would seem reasonable to call this an exclusive or.

There certainly could be cases, however, in which the patron would pay a portion of the bill in dollars an the remainder in euros. Therefore, an inclusive or seems better.

## Question 31

Construct a truth table for each of these compound propositions.

a) $p \wedge \neg p$

b) $p \vee \neg p$

| $p$ | $\neg p$ | $p \wedge \neg p$ | $p \vee \neg p$ |
|---|---|---|---|
| T | F | F | T |
| F | T | F | T |

c) $(p \vee \neg q) \rightarrow q$

| $p$ | $q$ | $\neg q$ | $p \vee \neg q$ | $(p \vee \neg q) \rightarrow q$ |
|---|---|---|---|---|
| T | T | F | T | T |
| T | F | T | T | F |
| F | T | F | F | T |
| F | F | T | T | F |

d) $(p \vee q) \rightarrow (p \wedge q)$

| $p$ | $q$ | $p \vee q$ | $p \wedge q$ | $(p \vee q) \rightarrow (p \wedge q)$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | T | F | F |
| F | T | T | F | F |
| F | F | F | F | T |

e) $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$

| $p$ | $q$ | $p \rightarrow q$ | $\neg q \rightarrow \neg p$ | $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | F | T |
| F | T | T | T | T |
| F | F | T | T | T |

f) $(p \rightarrow q) \rightarrow (q \rightarrow p)$

| $p$ | $q$ | $p \rightarrow q$ | $q \rightarrow p$ | $(p \rightarrow q) \rightarrow (q \rightarrow p)$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | T | T |
| F | T | T | F | F |
| F | F | T | T | T |

## Exercises 1.2 Question 8

Express these system specifications using the propositions $p$ "The user enters a valid password," $q$ "Access is granted," and $r$ "The user has paid the subscription fee" and logical connectives (including negations).

a) "The user has paid the subscription fee, but does not enter a valid password."
$r \wedge \neg p$

b) "Access is granted whenever the user has paid the subscription fee and enters a valid password."
$(p \wedge r) \rightarrow q$

c) "Access is denied if the user has not paid the subscription fee."
$\neg r \rightarrow \neg q$

d) "If the user has not entered a valid password but has paid the subscription fee, then access is granted."
$(\neg p \wedge r) \rightarrow q$

## Question 10

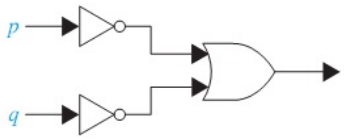
Are these system specification consistent? "Whenever the system software is being upgraded, user cannot access the file system. If users can access the file system, then they can save new files. If users cannot save new files, then the system software is not being upgraded."
consistent

## Question 12

Are these system specifications consistent? "If the file system is not locked, then new messages will be queued. If the file system is not locked, then the system is functioning normally, and conversely. If new messages are not queued, then they will be sent to the message buffer. If the file system is not locked, then new messages will be sent to the message buffer. New messages will not be sent to the message buffer."
consistent

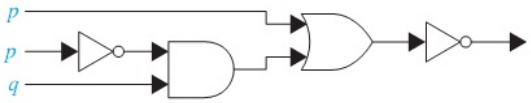
## Question 40

Find the output of each of these combinatorial circuits.




a)

$\neg p \vee \neg q$




b)

$\neg(p \vee (\neg p \wedge q))$

## Question 42

Construct a combinatorial circuit using inverters, OR gates, and AND gates that produces the output $(p \wedge \neg r) \vee (\neg q \wedge r)$ from input bits $p, q$, and $r$.