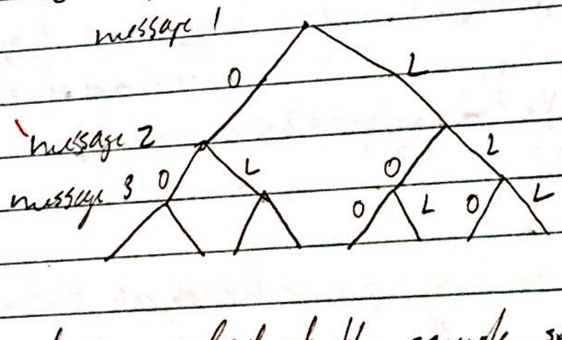
5/5

Engr 371 - 2.1 - 2.4. Assignment 1

random experiment: Can result in different outcomes, even though it is repeated in the same manner every time.

models can include variations → mathematical model (abstraction).
→ Newton's. approximately quantify performance of engineering products.
- when validated with measurements from systems
- understand, describe, quantify aspects → predict response to inputs.

Sample Space: The set of all possible outcomes of a random experiment is called the sample space of the experiment. The sample space is denoted as S.
sample space of the experiment set of outcomes
- Discrete: finite or countable infinite set of outcomes for camera conforming. $S: \{nn, yn, yyyn ...\}$ countably infinite
- $S = \{yes, no\}$ → set of two outcomes
- Continuous contains an interval (either finite or infinite) of real numbers.
- $S = R^+ = \{x \mid x > 0\}$
- Tree diagrams → steps represent each of n_1 ways of completing 1st step of branch
n_2: ways of completing second step → represents sample space of outcomes.



Event: An event is a subset of the sample space of a random experiment.
- union: all outcomes contained in either of the two events. $E_1 \cup E_2$
Intersection: all outcomes contained in both of the two events. $E_1 \cap E_2$
Complement: set of outcomes in sample space that are not in the event. E' or E^c
Ex. $S = \{yy, yn, ny, nn\}$ $E_1 = \{yy, yn, ny\}$ → at least one camera conforming.
$E_2 = \{nn\}$ → both cameras non conforming $E_3 = \emptyset$ → null set. $E_5 = \{yn, ny, nn\}$
$E_1 \cup E_5 = S$ $E_1 \cap E_5 = \{yn, ny\}$ $E_1' = \{nn\}$
Events are used to define outcomes of interest from a random experiment. Probabilities of specified over...

combination

Venn diagrams: represents a sample space and events in a sample space.
mutually exclusive: $E_1 \cap E_2 = \emptyset$

more complicated
counting techniques: counts of the number of outcomes in the sample space and various events.

- multiplication rule: operation has K steps (sequence)
  - # of ways of completing step 1 is n_1, ...
  - total # of ways: $n_1 \times n_2 \times \dots \times n_k$

Permutations: # of ordered sequences of the elements of a set.
ex: $S = \{a, b, c\}$ then abc, acb, bac, cba, bca, cab - permutations
# of permutations: $n! = n \times (n-1) \times (n-2) \times \dots \times 2 \times 1$. (2-1).
n different element.

$48 \times 47 \times 6$ b/c one less element every time.

linear permutation: selecting which element is first, 2nd, etc

Permutation of subsets: # of arrangements of some elements of a set.
Permutation of subsets: $P_r^n = n \times (n-1) \times (n-2) \times \dots \times (n-r+1) = \frac{n!}{(n-r)!}$ → # of spots available, → # of elements placed
of r elements selected (2-2)
from set of n different elements

$\frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1}$

Permutation of similar objects: counting # ordered sequences for objects that are not all different
$n = n_1 + n_2 + \dots + n_r$ n_1 are one type. n_2 another n_r are an r^{th} type.

$$\frac{n!}{n_1! n_2! n_3! \dots n_r!} \quad (2\text{-}3$$

3 knee surgeries 2 hip surgeries
$\frac{5!}{2!3!} = 10!$

① | | | | | bars: 5 = $\frac{5!}{2!3!}$

need two wide bars: B → b

② white space = 4 ⇒ $10 \times 4 = 40 - 1 = 39$ delimiter

Should it also be factorial? $\frac{4!}{3!1!} = 4$

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Combinations: # subsets of r elements that can be selected from set n elements.

$\binom{n}{r}$ $\quad C_r^n = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$ $\quad$ (2-4)

$\dfrac{8!}{5!\,3!}$ → locations of elements = 56 → # possible designs of all elements and places.
identical components ← $5!$; $3!$ → extra places.

Replacement and non replacement (w or w/o)

w/o Replacement: sample w/ 6 elements & 2 def. parts. when 50 ⇒ 3 def.

① Defective parts $\dfrac{3!}{2!\,1!}$ = Total # defective parts: 3 different ways of picking
2 defective ← $2!$; $1!$ → 1 left over (still defective)

② Non defective parts $\dfrac{47!}{4!\,43!}$ → non defective = 178,365 different ways of pickin[g]
for sample of 6 ← $4!$ (4 non-def); $43!$ → rest of non def

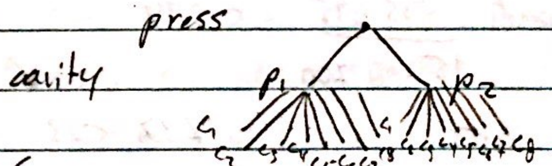
subsets of size 6 w/ 2 def. parts = 3 × 178,365 = 535,095

Total # different subsets size 6: $\binom{50}{6} = \dfrac{50!}{6!\,44!} = 15{,}890{,}700$

2.1 summary: For a random experiment, you can have different outcomes organized in a sample space which may be described as finite or infinite interval of discrete or continuous set of outcomes. A subset of that sample space is an event and those may be analyzed as an union, complement, intersection of each other. There are several techniques for counting the number of outcomes, using the multiplication rule, permutations, and combinations.

Exercises.

2.15: p_1 & p_2 & 8 cavities therefore for one part you have 2 machines × 8 cav[ities]



2 × 8 = 16 different outcomes.

$S = \{p_1c_1, p_1c_2, p_1c_3, p_1c_4, p_1c_5, p_1c_6, p_1c_7, p_1c_8, p_2c_1, p_2c_2, p_2c_3, p_2c_4, p_2c_5, p_2c_6, p_2c_7, p_2c_8\}$ subset A = {p
with p_1's

		B:	B':
		E F	
		E.	G.
Sur F	A: E	80	2
	A': G	10	8

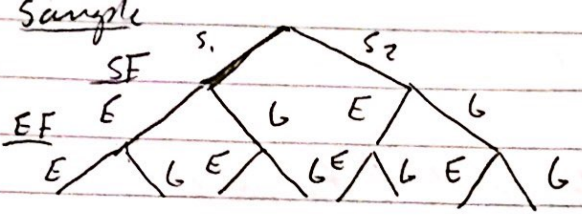
2.27.

a) A: Excellent {80, 2} B: {80, 10}

A': {10, 8}

$A' \cap B = 10.$ $A \cup B = 92$

b) sample



4+2 = 8 different outcomes

2.2. Interpretations and Axioms of probability.

probability w/ discrete sample spaces.

probability = 0→1.

Degree of belief: outcome will occur according to relative frequency.

Sum of all probabilities = 1.

Random probability means all outcomes have equal chance of being selected.

equal likely outcomes = probabilities are equal. Sample space of N possible outcomes equally likely, probability of each outcome is 1/N.

Ex. 30% of 100 meet req. E = 30 outcomes ∴ probability = 0.3.

$P(E) = 30(0.01) = 0.30$

Probability of an Event: For discrete sample spaces: probability of an event E (P(E)) equals the sum of the probability of the outcomes in E.

ratio of # outcomes in event to # of outcomes in sample space (For equally likely outcomes)

prob. sample w/ 2 def parts: $\frac{535,095}{15,890,700} = 0.034.$

prob. sample w/ 0 def. parts: $\frac{47!}{6!41!} = 10,737,573$

$\frac{10,737,573}{15,890,700} = 0.676$ ⇒ Sample size 6 likely to omit def parts

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axioms: rules probabilities in a random experiment must satisfy.

Probability is a number that is assigned to each member of a collection of events from a random experiment that satisfies the following properties:

S: sample space E = any event in a random experiment

1. $P(S) = 1$
2. $0 \leq P(S) \leq 1$
3. For two events E_1 & E_2 with $E_1 \cap E_2 = \emptyset$

$$P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

$P(\emptyset) = 0$ $\quad P(E') = 1 - P(E)$ $\quad P(E_1) \leq P(E_2)$ if E_1 contained in E_2.

Exercises.

2.66. 16 digits b/w 0 & 9. $16 \times 10 = 160$ outcomes for 1 number.

100,000,000 numbers available.

probability $\frac{160 \text{ outcomes}}{100{,}000{,}000 \times 160}$ $\boxed{1 \times 10^{-8}}$

2.70.

		Sh. R H	Sh. R L
Scr. R	H	70	9
	L	16	5

a) $P(A) = \frac{70+16}{100} = 0.86$ b) $P(B) = \frac{70+9}{100} = 0.79$ c) $P(A') = \frac{9+5}{100} = 0.14$

d) $P(A \cap B) = \frac{70}{100} = 0.70$ e) $P(A \cup B) = \frac{70+9+16}{100} = 0.95$ f) $P(A' \cup B) = \frac{70+9+5}{100} = 0.84$

Summary: Probability can be between 0 & 1, where you can have a random probability of all outcomes having equal chance of being selected, and depends on Probability of an event.

2.3 Addition Rules.

Probability of an union: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

If A & B are mutually exclusive: $P(A \cup B) = P(A) + P(B)$.

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$.

mutually exclusive events: collection of events is mutually exclusive if $E_i \cap E_j = \emptyset$

$P(E_1 \cup E_2 \cup \ldots \cup E_k) = P(E_1) + P(E_2) + \ldots P(E_k)$

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Exercises.

2.89

		V.T. S	V
I.	S	117	3
	U	8	2

a) $\frac{5}{130} + \frac{10}{130} - \frac{2}{130} = \frac{13}{130}$ b) $\frac{117}{130} = 0.90 \Rightarrow$ No.

2.87.

		B Roundness conforms Y	N
SF conforms A	Y	345	5
	N	12	8

$345 + 5 + 12 + 8$

a) $\frac{350}{370} = 0.945$ b) $P(A') + P(B) = \frac{345+5+12}{370} = \frac{362}{370} = 0.978$

c) $345 + 5 + 8$ $\frac{358}{370}$ d) $\frac{345}{370} =$

2.4. Conditional Probability.

Probability of B if outcome in event A: $P(B|A) \rightarrow$ conditional probability of B given A

400 parts. F = Flaws: 40 D = Defective: 10

with surface. $P(D|F) = \frac{10}{40} = 0.25$ $P(D|F') = \frac{18}{360} = 0.05$

Conditional Probability: of event B given an event A, denoted as $P(B|A)$ is

$$P(B|A) = P(A \cap B) / P(A) \quad \text{for } P(A) > 0$$

n total outcomes $P(A) = \frac{(\# \text{ of outcomes in } A)}{n}$

$$P(A \cap B) = \frac{(\# \text{ of outcomes in } A \cap B)}{n}$$

$$\frac{P(A \cap B)}{P(A)} = \frac{\# \text{ of outcomes in } A \cap B}{\# \text{ of outcomes in } A}$$

Random Samples: to select randomly implies that at each step of the sample, the items that remain in the batch are equally likely to be selected.

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2.92

a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\frac{1}{4} + \frac{4}{5} - \left(\frac{1}{4} \times \frac{4}{5}\right) = 0.85$$

b) $P(A \cup B') = P(A) + P(B') - P(A \cap B')$

$$= \frac{1}{4} + \frac{1}{5} - \left(\frac{1}{4} \times \frac{1}{5}\right) = 0.40$$

c) $P(A' \cup B') = P(A') + P(B') - P(A' \cap B')$

$$\frac{3}{4} + \frac{1}{5} - \left(\frac{3}{4} \times \frac{1}{5}\right) = 0.80$$

2.103

	B	D
		C W
	H	L
SR A H	12	16
C L	88	34

a) $P(A|B)$ $\frac{12}{100} = 0.12$ b) $P(B|A) = \frac{12}{28} = 0.429$ c) $P(D|C) = \frac{34}{122} = 0.279$

2.107 100, 20 are DF

a) $\frac{20}{100} = 0.2$ b) $\frac{19}{99} = 0.191$ c) $\frac{20}{100} \times \frac{19}{99} = \frac{19}{495} = 0.0384$ d) It would be equally likely.

2.114 A: visit to hospital 4 B: LWBS. at any hospital.

a) $P(A|B) = \frac{242}{953}$ b) $P(A'|B) = \frac{711}{953}$ c) $P(A|B') = \frac{242}{21,299}$ $P(B|A) = \frac{1}{243}$

Z.1 - 2.4 Summary

Outcomes from a random experiment can be organized into a sample space S which can have a discrete or continuous set of outcomes. A subset of these sample spaces can be described as an event, which can be related to other events in the set by union, complement, or intersection. In order to count the number of outcomes in an efficient way, counting techniques may be used. The probability of any of those events to occur is calculated from 0 to 1, and each outcome

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has an equally likely chance of occuring. Probability can be determined whether the events are mutually exclusive, or one event must occur before the other, which is conditional probability.

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