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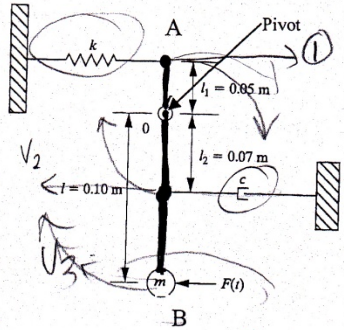
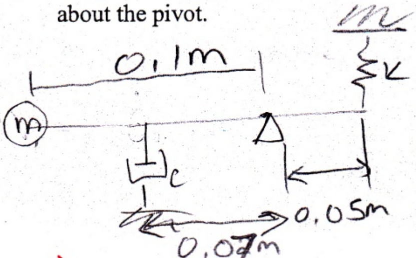
DEPARTMENT OF MECHANICAL & INDUSTRIAL ENGINEERING
CONCORDIA UNIVERSITY
MECH 375: Mechanical Vibrations

CLASS TEST 16 February 2011
DURATION: 70 Minutes 5 PAGES Max. Mark: 30
STATE YOUR ASSUMPTIONS CLEARLY

QUESTION # 1 (10)

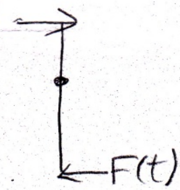
The pendulum mechanism, shown in Fig. 1, is pivoted at point O. Assuming the link AB to be rigid and small rotation of AB about the pivot, formulate the governing equation of motion in angular motion about the pivot.





$l = 0.15$m

$\cancel{0.1(40+40kg)\ddot{\theta} + 0.07c\dot{\theta} + 0.05k\theta} = 0\ F(t)\ l^x$



How? → FBD

1.5/5

For $F(t)$ = 0 N, m = 40 kg, k = 4000 N/m, mass of link AB = 40 kg, and mass moment of inertia of link AB about its mass centre= 0.075 kg.m^2, determine the damping coefficient c so as to achieve system damping ratio of 0.2.

$\zeta = 0.2 \rightarrow$ underdamped $\frac{c}{c_c} = \frac{c_{eq}}{2 m_{eq} \omega_n}$

2.5/5

$\omega_n = \cancel{\sqrt{\frac{K}{M+m}} = \sqrt{\frac{4000}{80kg}}}$

$\omega_n = 7.07$

$C_c = 2m_{eq}\omega_n$

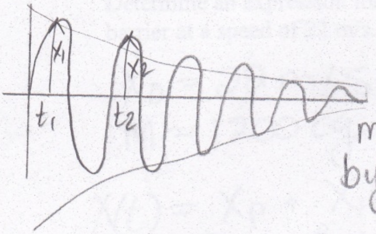
$C_c = 2(M+m)(\cancel{7.07})$

$C_c = 2(80)(\cancel{7.07}) = 1131.2$

$0.2 = \cancel{\frac{c}{1131.2}} \rightarrow \boxed{C = 226.24}$

$$\xi = \frac{c}{2m\omega_n}$$

$$T = \tfrac{1}{2} m v^2 = \tfrac{1}{2} m \dot{x}^2$$



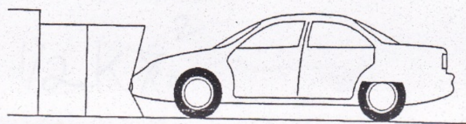
multiplied by $e^{-\xi\omega_n t}$

$$\omega_d = \omega_n\sqrt{1-\xi}$$

$$T_d = \frac{2\pi}{\omega_d}$$

$$\omega_d = \frac{2\pi}{1.5s} = 4.19 \text{ rad/s}$$

QUESTION # 2 (10)
A highway crash barrier is designed to absorb a vehicles kinetic energy, as shown. The deflection response of the barrier to a very low speed vehicle impact revealed exponentially decaying harmonic oscillations with an oscillation period of 1.5 s. The ratio of amplitudes of two consecutive peaks was measured as 4.7. For the vehicle mass of 1200 kg, determine the stiffness and damping coefficients of the barrier.



$\tau = 1.5s$

$M = 1200kg$

$$\left(\frac{X_n}{X_{n+1}}\right) = 4.7$$

$$X(t) = X_0 e^{-\xi\omega_n t}\cos(\omega_d t + \phi) + e^{-\xi\omega_n t}\frac{1}{\omega_d}\sin(\omega_d t + \phi).$$

Kinetic energy becomes potential

so $\tfrac{1}{2} m_{car} V_{car}^2 = \tfrac{1}{2} K x^2$

$$\tfrac{1}{2}(1200kg)V^2 = \tfrac{1}{2} K x^2$$

$$\ln(4.7) = \tau\xi$$

0/

Determine an expression for the barrier deflection response when 1200 kg vehicle impacts the barrier at a speed of 22 m/s.

$\dot{X}_0 = 22\ m/s$

$M = 1200\ kg$

$X(t) = X_p + X_t$

$X(t) = X_0 e^{-\zeta\omega_n t}\cos(\omega_d t + \phi) + \frac{e^{-\zeta\omega_n t}}{\omega_d}\sin(\omega_d t + \phi)$

$\cancel{\frac{1}{2}(1200)(22)^2 = \frac{1}{2}K X_0^2}$

$\cancel{290{,}400 = T}$ ~~becomes~~ $\cancel{U}$

~~fill in values for~~ $\cancel{\omega_n = \sqrt{\frac{K_{eq}}{m}}}$

$\cancel{X_0 = \int \dot{X}_0}$

$\cancel{\omega_d = \omega_n\sqrt{1-\zeta^2}}$

1/4

$\frac{rev}{s} \cdot \frac{2\pi\, rad}{rev} = 7539.8\, rad/s = \omega$

QUESTION # 3 **(10)**

A machine of mass m comprises an eccentric rotating mass m_u, and is supported on damped elastic mount, as shown. A stroboscope showed the position of the rotating mass at the top (aligned vertically) when the eccentric mass is rotating at a speed of 1200 rpms and the machine structure is moving upward through its static equilibrium with peak amplitude of 20 mm.

$x_p = \frac{1}{\omega_d} e^{-\zeta\omega_n t_p}$

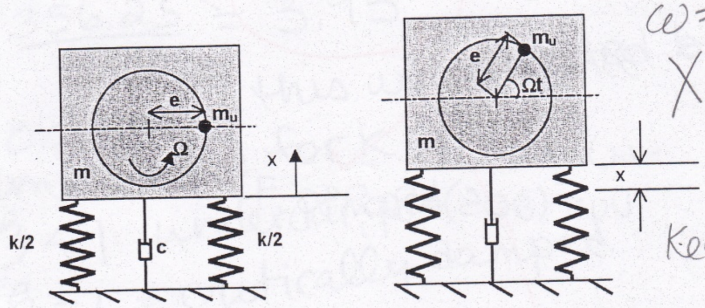
$t_p = \frac{1}{4}T$

$0.02 = \frac{1}{\omega_d} e^{-\zeta\omega_n t_p}$

$\omega = 1200 \frac{rev}{ms}$

$X_p = 20mm$

$K_{eq} = K$



(a) At time t = 0, in equilibirum

(b) After time t

$M\ddot{\theta} + C\dot{\theta} + K\theta = me\omega^2 \sin\omega t$

For m= 200 kg; m_u= 5 kg; e= 0.05 m;

Determine the natural frequency and damping ratio of the machine.

$\omega_n = \sqrt{\frac{K}{m+m_u}}$

$X_p = \frac{me\omega^2}{\sqrt{(K-M\omega^2)^2 + (C\omega)^2}}$??

$F_o = me\omega^2 = (5)(0.05)(7539.8\, rad/s) = 1884.95$

$X_p = \frac{1884.95}{\sqrt{(K-200(7539.8)^2)^2 + ((C)(7539.8))^2}} = 0.02 \Rightarrow$ solve fo C

assume $K=10 \Rightarrow \boxed{\omega_n = \sqrt{\frac{10}{205}} = 0.22}$ C=

$\zeta = \frac{C}{C_c}$ $\quad C_c = 2(M+m)\omega_n = 2(205)(0.22)$

$C_c = 90.2 \Rightarrow$ flip page

Determine the amplitude of the machine mass vibration and angular position of the eccentric mass at a rotational speed of 900 rpm.

$900\, rpm = 5654.9\, rad/s = \omega$