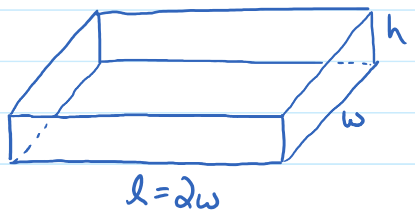
Calculus Review - Day 4

Optimization Process

1. Create an equation for the property that is to be optimized
   (eg. Maximum Volume, Minimum Area/Surface Area, min Cost, etc.)
2. Use known properties (eg. fixed Volume/Area) to make relationships between independent variables, and use it to reduce the # of variables in the equation from step 1. (hoping to have only <u>one</u> variable left)
3. Determine when the derivative of the resulting function is zero.
   → this gives potential max/min points
4. Justify whether the solutions represent max/min points
   → extremes, interval chart, graph, second derivative test.

<u>Eg ①</u>: An open-top box with base length twice the width has a Surface Area of $1000\,cm^2$. Determine the Maximum Volume of the box.



Volume, $V = lwh$

$V = (2w)wh$

$V = 2w^2h$

$SA = lw + 2lh + 2wh$

$1000 = (2w)w + 2(2w)h + 2wh$

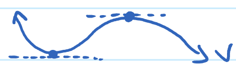
$1000 = 2w^2 + 6wh$

$1000 - 2w^2 = 6wh$

$\frac{1000 - 2w^2}{6w} = h$

$V = \cancel{2}w^{\cancel{2}}\left(\frac{1000 - 2w^2}{\cancel{6}_{3}\cancel{w}}\right)$

$V = \frac{1000}{3}w - \frac{2}{3}w^3$



cubic w/ -ve leading coefficient.

$\frac{dV}{dw} = \frac{1000}{3} - 2w^2$

$\frac{dV}{dw} = 0$ when $\frac{1000}{3} - 2w^2 = 0$

$\frac{1000}{3} = 2w^2$

$\frac{1000}{6} = w^2$

$w = \pm\sqrt{\frac{1000}{6}}$

$w \doteq \pm 12.9$

$w = -12.9$ is extraneous,
so $w = 12.9$ is the width for a Maximum Volume to occur.
(based on the shape of the graph of Volume function)

$\therefore$ Volume $= 1000\left(\sqrt{1000}\right) - 2\left(\sqrt{1000}\right)^3$

$$\therefore \text{Volume} = \frac{1000}{3}\left(\sqrt{\frac{1000}{6}}\right) - \frac{2}{3}\left(\sqrt{\frac{1000}{6}}\right)^3$$

$$\doteq 2869 \text{ cm}^3$$