Calculus Review - Day 3

Derivatives of Sinusoidal and Exponential functions

function		Derivative	
$f(x) = \sin x$	$\rightarrow$	$f'(x) = \cos x$	(only when x is in Radian measure)
$g(x) = \cos x$	$\rightarrow$	$g'(x) = -\sin x$	(" " " " " ")
$f(x) = \sin(3x^2+1)$	chain $\rightarrow$	$f'(x) = \cos(3x^2+1)\cdot(6x)$	$\therefore f'(x) = 6x\cdot\cos(3x^2+1)$
$g(x) = \cos^2(3x^2+1) = [\cos(3x^2+1)]^2$	chain $\rightarrow$	$g'(x) = 2[\cos(3x^2+1)]^1\cdot[-\sin(3x^2+1)\cdot 6x]$	$\therefore g'(x) = -12x\cdot\cos(3x^2+1)\cdot\sin(3x^2+1)$

$f(x) = e^x$	$\rightarrow$	$f'(x) = e^x$	($e \doteq 2.718281828043\ldots$)
$g(x) = e^{3x^2}$	$\rightarrow$	$g'(x) = e^{3x^2}\cdot \ln e \cdot 6x$	$\therefore g'(x) = 6x\cdot e^{3x^2}$
$f(x) = 5^x$	$\rightarrow$	$f'(x) = 5^x\cdot \ln 5 \cdot 1$	
$g(x) = 10^{\sqrt{x}}$	$\rightarrow$	$g'(x) = 10^{\sqrt{x}}\cdot \ln 10 \cdot \frac{1}{2\sqrt{x}}$	

Challenge:

$h(x) = \cos^2(e^{\sin x^2}) = [\cos(e^{\sin x^2})]^2$

$\downarrow$

$h'(x) = 2[\cos(e^{\sin x^2})]^1\cdot[-\sin(e^{\sin x^2})\cdot e^{\sin x^2}\cdot \ln e\cdot \cos x^2\cdot 2x]$

$\therefore h'(x) = -4x\cos(e^{\sin x^2})\sin(e^{\sin x^2})\cdot e^{\sin x^2}\cdot\cos x^2$

Application Problems:

① for an object with position function $s(t) = 4\pi\sin(\pi t)$, $0 \le t \le 4$,

a) When is the object at rest? * rest $\rightarrow v(t) = 0$.

b) When does the object change direction of motion? * $v(t) = 0$ AND changes sign.

c) Is the object speeding up or slowing down at $t = 1.4$?

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$v(t)$ & $a(t)$ have same sign [or $v(t) \times a(t) > 0$]

$v(t)$ & $a(t)$ have opposite sign [or $v(t) \times a(t) < 0$]

$s(t) = 4\pi \cdot \sin(\pi t)$

$v(t) = 4\pi \cdot \cos(\pi t) \cdot \pi$ $\quad \therefore v(t) = 4\pi^2 \cos(\pi t)$

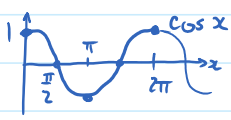
$a(t) = 4\pi^2 \cdot [-\sin(\pi t) \cdot \pi]$ $\quad \therefore a(t) = -4\pi^3 \cdot \sin(\pi t)$

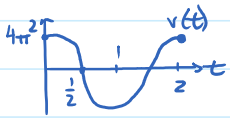
a) $v(t) = 0$ when $4\pi^2 \cdot \cos(\pi t) = 0$

$\cos(\pi t) = 0$

$\pi t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

$t = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$. (for domain $0 \le t \le 4$)





b) Since the velocity changes sign at each of the answers in (a), these times represent when the object is at rest AND when it changes direction.

c) $v(1.4) = 4\pi^2 \cos(\pi \times 1.4) \doteq -12.$

$a(1.4) = -4\pi^3 \sin(\pi \times 1.4) \doteq +117.95$

} The object is slowing down.