Curve Analysis & Curve Sketching

To fully analyze a curve for a function $f(x)$.

① Calculate intercepts (y & x) and restrictions on x (vertical asymptotes, holes, etc)

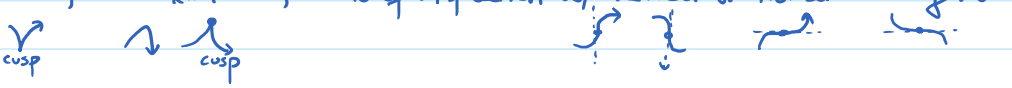
② Determine $f'(x)$ and use it to determine increasing/decreasing properties.

→ $f(x)$ is increasing when $f'(x) > 0$

→ " " decreasing when $f'(x) < 0$

→ a critical point occurs when $f'(x) = 0$ or when $f'(x)$ is Undefined.

↳ minimum, maximum, Point of Inflection w/ Vertical or Horizontal tangent



③ Determine $f''(x)$ and use it to determine concavity & other inflections:

→ $f(x)$ is concave up when $f''(x) > 0$

→ " " " down " $f''(x) < 0$

→ a point of inflection occurs when $f''(x) = 0$ or is Undefined, AND, when $f''(x)$ changes sign (ie. concavity changes at the point)

Eg ①: Do a full analysis and curve sketch for $f(x) = (x+3)^{\frac{1}{3}}(x-2)^3$

• y-int: $f(0) = \sqrt[3]{3}\cdot(-2)^3$ • x-int: $y=0$ when $x=-3$ and $x=2$.

$f(0) \doteq -11.54$

• $f'(x) = \frac{1}{3}(x+3)^{\frac{-2}{3}}(1)\cdot(x-2)^3 + (x+3)^{\frac{1}{3}}\cdot 3(x-2)^2(1)$

$= (x+3)^{\frac{-2}{3}}(x-2)^2\cdot\left[\frac{1}{3}(x-2) + 3(x+3)\right]$

$= (x+3)^{\frac{-2}{3}}(x-2)^2\left(\frac{10}{3}x + \frac{25}{3}\right)$

$\therefore f'(x) = \frac{5}{3}(x+3)^{\frac{-2}{3}}(x-2)^2(2x+5)$

$f'(x) = 0$ @ $x = 2, \frac{-5}{2}$.

$f'(x)$ is Undefined @ $x = -3$.

• $f''(x) = \frac{-10}{9}(x+3)^{\frac{-5}{3}}\cdot(x-2)^2(2x+5) + \frac{5}{3}(x+3)^{\frac{-2}{3}}\cdot 2(x-2)^1\cdot(2x+5) + \frac{5}{3}(x+3)^{\frac{-2}{3}}(x-2)^2\cdot(2)$

$= \frac{-10}{9}(x+3)^{\frac{-5}{3}}(x-2)^2(2x+5) + \frac{10}{3}(x+3)^{\frac{-2}{3}}(x-2)(2x+5) + \frac{10}{3}(x+3)^{\frac{-2}{3}}(x-2)^2$

$= \frac{10}{9}(x+3)^{\frac{-5}{3}}(x-2)\cdot\left[-(x-2)(2x+5) + 3(x+3)(2x+5) + 3(x+3)(x-2)\right]$

$= \frac{10}{9}(x+3)^{\frac{-5}{3}}(x-2)\cdot\left[-2x^2 - x + 10 + 6x^2 + 33x + 45 + 3x^2 + 3x - 18\right]$

$$= \frac{10}{9}(x+3)^{-\frac{5}{3}}(x-2)\cdot\left[-2x^2-x+10 + 6x^2+33x+45 + 3x^2+3x-18\right]$$

$$= \frac{10}{9}(x+3)^{-\frac{5}{3}}(x-2)\left(7x^2+35x+37\right)$$

$f''(x)$ is undefined @ $x=-3$.

$f''(x)=0$ @ $x=2$, $x \doteq -1.518$, -3.482

	$x<-3.482$	$x=-3.482$	$-3.482<x<-3$	$x=-3$	$-3<x<-2.5$	$x=-2.5$	$-2.5<x<-1.518$	$x=-1.518$	$-1.5<x<2$	$x=2$	$x>2$
f''	+	0	−	Undef.	+	+	+	0	−	0	+
f'	−	−	−	Undef.	−	0	+	+	+	0	+
f	↘	p.o.i	↘	p.o.i, x-int	↘	min	↗	p.o.i.	↗	p.o.i, x-int	↗