

Calculus Review - Day 1.

Limit Definition of the Derivative

For a function $f(x)$, the instantaneous rate of change / slope of the tangent

at $x="a"$ is: $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$ } Use this if you only care about one value of x .

The Derivative $f'(x)$, a function to calculate the I.R.O.C. / m_{tan} at any point

is: $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ } Use this if you want a generic expression with x

Eg ① a) Determine $f'(3)$ for $f(x) = 5(3x+1)^2$ using Limit Definition.

$$\begin{aligned} f'(3) &= \lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h} \\ &= \lim_{h \rightarrow 0} \frac{45h^2 + 300h + 500 - 500}{h} \\ &= \lim_{h \rightarrow 0} (45h + 300) \end{aligned}$$

$\therefore f'(3) = 300$

$$\begin{aligned} f(x) &= 5(3x+1)^2 \\ f(3+h) &= 5(3(3+h)+1)^2 \\ f(3+h) &= 5(3h+10)^2 \\ f(3+h) &= 5(9h^2 + 60h + 100) \\ f(3+h) &= 45h^2 + 300h + 500 \\ f(3) &= 5(3(3)+1)^2 = 500 \end{aligned}$$

b) Verify using another method.

$$f(x) = 5(3x+1)^2$$

Using Differentiation (Power & Chain Rules), $f'(x) = 10(3x+1)^1 \cdot (3)$

$$\begin{aligned} &= 30(3x+1) \\ &= 90x + 30 \end{aligned}$$

Power Rule
if $f(x) = x^n$
then $f'(x) = nx^{n-1}$

Chain Rule
if $h(x) = f(g(x))$
then $h'(x) = f'(g(x)) \cdot g'(x)$

$$\begin{aligned} \downarrow \\ f'(3) &= 90(3) + 30 \\ &= 270 + 30 \\ &= 300 \end{aligned}$$

Product Rule
if $h(x) = f(x) \cdot g(x)$
then $h'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

Eg ②: Use Limit Definition to determine $f'(x)$ for $f(x) = 2\sqrt{3x-2}$.
Verify using differentiation.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Rationalization: Multiply by the Conjugate.

$$= \lim_{h \rightarrow 0} \frac{(2\sqrt{3(x+h)-2} - 2\sqrt{3x-2}) \cdot (2\sqrt{3(x+h)-2} + 2\sqrt{3x-2})}{h (2\sqrt{3(x+h)-2} + 2\sqrt{3x-2})}$$

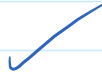
$$= \lim_{h \rightarrow 0} \frac{(2\sqrt{3(x+h)-2})^2 - (2\sqrt{3x-2})^2}{h [2\sqrt{3(x+h)-2} + 2\sqrt{3x-2}]}$$

$$= \lim_{h \rightarrow 0} \frac{4[\cancel{3x+3h-2}] - 4[\cancel{3x-2}]}{h [2\sqrt{3x+3h-2} + 2\sqrt{3x-2}]}$$

$$= \lim_{h \rightarrow 0} \frac{12}{2\sqrt{3x-2} + 2\sqrt{3x-2}}$$

$$= \frac{12}{4\sqrt{3x-2}}$$

$$= \frac{3}{\sqrt{3x-2}}$$



Verification:

$$f(x) = 2\sqrt{3x-2} = 2(3x-2)^{1/2}$$

$$f'(x) = 1(3x-2)^{-1/2} \cdot (3) = \frac{3}{(3x-2)^{1/2}}$$