Calculus Review – Day 1.

Limit Definition of the Derivative

for a function $f(x)$, the instantaneous rate of change / slope of the tangent at $x=$"a" is: $f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

} Use this if you only care about one value of x.

The Derivative $f'(x)$, a function to calculate the I.R.o.C. / m_{tan} at any point

is: $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

} Use this if you want a generic expression with x

Eg ①: a) Determine $f'(3)$ for $f(x) = 5(3x+1)^2$ using Limit Definition.

$$f'(3) = \lim_{h\to 0} \frac{f(3+h)-f(3)}{h}$$

$$= \lim_{h\to 0} \frac{45h^2+300h\cancel{+500}\cancel{-500}}{h}$$

$$= \lim_{h\to 0} (45\cancel{h}^{\,h=0}+300)$$

$\therefore f'(3) = 300$

$f(x) = 5(3x+1)^2$

$f(3+h) = 5(3(3+h)+1)^2$

$f(3+h) = 5(3h+10)^2$

$f(3+h) = 5(9h^2+60h+100)$

$f(3+h) = 45h^2+300h+500$

$f(3) = 5(3(3)+1)^2 = 500$

b) Verify using another method.

Using Differentiation (Power & Chain Rules),

$f(x) = 5(3x+1)^2$

$f'(x) = 10(3x+1)^1\cdot(3)$

$= 30(3x+1)$

$= 90x+30$

$\downarrow$

$f'(3) = 90(3)+30$

$= 270+30$

$= 300$ ✓

Power Rule

if $f(x)=x^n$

then $f'(x)=nx^{n-1}$

Chain Rule

if $h(x)=f(g(x))$

then $h'(x)=f'(g(x))\cdot g'(x)$

Product Rule

if $h(x)=f(x)\cdot g(x)$

then $h'(x)=f'(x)\cdot g(x)+f(x)\cdot g'(x)$

Eg ②: Use Limit Definition to determine $f'(x)$ for $f(x)=2\sqrt{3x-2}$.
Verify using differentiation.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Rationalization: Multiply by the Conjugate.

$$= \lim_{h \to 0} \left(\frac{2\sqrt{3(x+h)-2} - 2\sqrt{3x-2}}{h} \right) \cdot \frac{\left(2\sqrt{3(x+h)-2} + 2\sqrt{3x-2}\right)}{\left(2\sqrt{3(x+h)-2} + 2\sqrt{3x-2}\right)}$$

$$= \lim_{h \to 0} \frac{\left(2\sqrt{3(x+h)-2}\right)^2 - \left(2\sqrt{3x-2}\right)^2}{h\left[2\sqrt{3(x+h)-2} + 2\sqrt{3x-2}\right]}$$

$$= \lim_{h \to 0} \frac{\overset{12\cancel{h}}{4[\cancel{3x}+3h\cancel{-2}]} \cancel{-4}[\cancel{3x}\cancel{-2}]}{\cancel{h}\left[2\sqrt{3x+3h-2} + 2\sqrt{3x-2}\right]}$$

$$= \lim_{h \to 0} \frac{12}{2\sqrt{3x+\underset{h=0}{\cancel{3h}}-2} + 2\sqrt{3x-2}}$$

$$= \frac{12}{4\sqrt{3x-2}}$$

$$= \frac{3}{\sqrt{3x-2}} \quad \checkmark$$

Verification:

$$f(x) = 2\sqrt{3x-2}$$
$$= 2(3x-2)^{1/2}$$

$$f'(x) = 1(3x-2)^{-1/2} \cdot (3)$$
$$= \frac{3}{(3x-2)^{1/2}}$$