Matrix Operations

Def.: Matrix Equality;

Two matrices are said to be equal if they have the same dimensions, and their corresponding entries are equal.

Def.: Matrix Addition;

Let A, B, be two matrices of the same dimensions. The sum of the matrices A, B, denoted $A + B$, is the matrix C, such that $C_{ij} = A_{ij} + B_{ij} \ \ \forall_{ij}$. The dimension of C is the same as that of A and B.

Def.: Matrix Subtraction;

Let A, B, be two matrices of the same dimensions. The difference (subtraction) of the matrices A, B, denoted $A - B$, is the matrix C, such that $C_{ij} = A_{ij} - B_{ij} \ \ \forall_{ij}$. The dimension of C is the same as that of A and B.

Def.: Matrix Scalar Product;

Let A be a matrix, c a scalar (number). The product cA is the matrix obtained by multiplying every element of A by c.

If $D = cA$, then $D_{ij} = cA_{ij} \quad \forall_{ij}$

Def.: Matrix Product;

Let A be a matrix of dimension $m \times p$.

Let B be a matrix of dimension $p \times n$.

The product of the matrices A, B, denoted AB, is the matrix C, of dimension $m \times n$, such that

$$C_{ij} = \sum_{k=1}^{p} A_{ik} B_{kj}$$

$$= A_{i1}B_{1j} + A_{i2}B_{2j} + A_{i3}B_{3j} + + A_{ip}B_{pj} \quad \forall_{ij}$$

Def.: Matrix Form of a Linear System;

$$\begin{aligned}
&a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n = b_1 \\
&a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots + a_{2n}x_n = b_2 \\
&a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \ldots + a_{3n}x_n = b_3 \\
&\ldots \\
&a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \ldots + a_{mn}x_n = b_m
\end{aligned}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & & a_{3n} \\ & \vdots & & \ddots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_m \end{bmatrix} \Rightarrow \quad AX = B$$

Def.: Transpose of a matrix;

Let A be an (m x n) matrix.

The Transpose of A, is the (n x m) matrix A^T ∋ $A^T_{ij} = A_{ji}$ $\forall_{ij}$.

The i_{th}. column of A^T is the i_{th}. row of A.

Def.: Trace of a Matrix;

Let A be an $n \times n$ matrix. The Trace of A, denoted $tr(A)$, is the sum of all the main diagonal elements of A.