Family Name……………. First Name…………………………..[3]

Solve the differential equation $y' - y = te^t sint, \quad y(0) = 0$
using Laplace Transforms. Hint: Derivative of transform rule [7]

. . *Take the Laplace of the equation*

$\mathcal{L}\{y'\} - \mathcal{L}\{y\} = \mathcal{L}\{te^t sint\}$ . . . .

1. *Find Laplace Transform* $\mathcal{L}\{te^t sint\}$ . . . .
2. *Laplace of sint* $\mathcal{L}\{sint\} = \frac{1}{s^2+1}$ *Table 7.* . . . .
3. *Apply shift of transform Rule* $\mathcal{L}\{e^{at} f(t)\} = F(s-a)$. . . . . .
4. $\mathcal{L}\{e^t sint\} = F(s-1)$ *where* $F(s) = \mathcal{L}\{sint\}$, $\mathcal{L}\{e^t sint\} = \frac{1}{(s-1)^2+1}$
5. *Apply derivative of transform rule* $\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d}{ds} F(s)$
6. $\mathcal{L}\{te^t sint\} = (-1)\frac{d}{ds}\left[\frac{1}{(s-1)^2+1}\right] = \frac{2(s-1)}{[(s-1)^2+1]^2}$ . . . .
7. $\mathcal{L}\{y'\} - \mathcal{L}\{y\} = \mathcal{L}\{te^t sint\} \rightarrow s\mathcal{L}\{y\} - \mathcal{L}\{y\} = \frac{2(s-1)}{[(s-1)^2+1]^2}$. . . . .

. . . $\mathcal{L}\{y\} = \frac{2}{[(s-1)^2+1]^2}$. [4]

Another Way

1. *Find Laplace Transform* $\mathcal{L}\{te^t sint\}$ . . . .
2. *Laplace of sint* $\mathcal{L}\{tsint\} = \frac{2s}{s^2+1}$ *Table 22.* . . . .
3. *Apply shift of transform Rule* $\mathcal{L}\{e^{at} f(t)\} = F(s-a)$. . . . . .
4. $\mathcal{L}\{e^t tsint\} = F(s-1)$ *where* $F(s) = \mathcal{L}\{tsint\}$, $\mathcal{L}\{e^t tsint\} = \frac{2(s-1)}{(s-1)^2+1}$
5. $\mathcal{L}\{y'\} - \mathcal{L}\{y\} = \mathcal{L}\{te^t sint\} \rightarrow s\mathcal{L}\{y\} - \mathcal{L}\{y\} = \frac{2(s-1)}{[(s-1)^2+1]^2}$. . . . .

. . . $\mathcal{L}\{y\} = \frac{2}{[(s-1)^2+1]^2}$. [4]

$Find\ the\ Inverse\ of\ \frac{2}{[(s-1)^2+1]^2}$ . .

$Table\ 25\ \ \mathcal{L}\{sinkt - ktcoskt\} = \frac{2k^3}{[s^2+k^2]^2}$,

$k = 1,\ \mathcal{L}\{sint - tcost\} = \frac{2}{[s^2+1]^2}$ . . . .

. $Apply\ shift\ of\ transform\ \ \mathcal{L}\{e^{at}f(t)\} = F(s-a)$ . . . .

$\mathcal{L}\{e^t sint - te^t cost\} = \frac{2}{[(s-1)^2+1]^2}$

$Solution\ y(t) =\ e^t sint - te^t cost$ . . . . [3]