CONCORDIA UNIVERSITY			**Department of Mathematics and Statistics**
Course **Mathematics**	Number **MAST 218**		Section(s) **A, B**
Examination **Final**	Date **December 2012**	Time **3 hours**	Pages **2**
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Instructions: Only approved calculators permitted. Answer all eight numbered questions. The value for each part is indicated in square brackets in the margin (out of a possible total of 100). Show all your steps. Write the complete solution on the right hand pages of your examination booklet only.

MARKS: marks for each problem are shown in front of the problems.

↓MARKS

12 **Problem 1 :** Consider the plane curve defined by the parametric equations:

$$x = t^3 - 3t, \quad y = t^3 - 3t^2 \ .$$

(a). Find d^2y/dx^2 in terms of t.

(b). Find the points on the curve where the tangent is horizontal or vertical.

12 **Problem 2 :** Consider the curves γ_1 and γ_2 defined by polar equations:

$$\gamma_1 : \quad r = 2 + \sin\theta, \quad 0 \le \theta \le 2\pi; \qquad \gamma_2 : \quad r = 3\sin\theta, \quad 0 \le \theta \le \pi$$

(a). Identify the curve γ_2 by finding its **Cartesian equations** in $(x, y)-$coordinates, and **sketch the polar curves** γ_1 and γ_2.

(b). Find the area A of the region that lies inside the first curve γ_1 and outside the second curve γ_2.

12 **Problem 3 :** Consider the space curve $\mathbf{r}(t) = \langle\, 2t^{3/2}, \, \cos(2t), \, \sin(2t) \,\rangle$.
Find the length of the curve for $t \in [1, 3]$.

12 **Problem 4 :** (a). Find an equation of the plane PL passing through the three points: $A\,(2,1,1)$, $B\,(-1,-1,10)$ and $C\,(1,3,-4)$.

(b). Find equations, expressed in implicit form, for the line passing through the point B that is perpendicular to the plane PL obtained in (a).

14 **Problem 5 :** Let the surface $z = f(x,y)$ be defined implicitly by the equation:

$$\sin(xyz) = x^2y^2 + z^2 - 1\ .$$

(a). Find the partial derivatives $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

(b). Find the equation of the tangent plane at the point $(1,1,0)$.

(c). In which direction does f increase most rapidly at this point?

12 **Problem 6 :** Find the limit, if it exists, or show that the limit does not exist:

$$(a).\quad \lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+2y^2}\ , \qquad\qquad (b).\quad \lim_{(x,y)\to(0,0)} \frac{x^4y^4\sin^2(x^2y^2)}{x^2+y^2}.$$

14 **Problem 7 :** Consider the function $f(x,y) = (y^2 - x^2)e^y$.

(a). Find all critical points of $f(x,y)$.

(b). Classify the critical points obtained in (a) as local minimum, local maximum, or saddle points.

(c). Find the absolute maximum and minimum values of f on the set

$$D = \big\{(x,y) : x^2 + y^2 \leq 4\big\}.$$

12 **Problem 8 :** Use **Lagrange Multipliers** to find the maximum and minimum values of $f(x,y) = e^{xy}$ subject to the constraint: $x^3 + y^3 = 16$.

GOOD LUCK !!!