Assignment 1 Solutions – Ravi Mateti

FINA 455

<u>You can thank me later</u>

1. Is LIOBR generally higher, lower, or the same as the repo rate?
   A. Higher
   B. Lower
   C. Same

   Solution:

   LIBOR is the rate for unsecured loans (no collateral). This is the rate that banks are willing to pay for their borrowings in the London Money Market.

   In a repo transaction, the borrower always deposits a collateral with the repo dealer (lender). Since the interest rate on unsecured loans is higher than on secured loans, LIBOR is higher than repo rate.

2. What is the 1-month return on capital for a trader who entered into a one-month repo where $P_t = 98.5,\ P_T = 99.01$, repo rate = 5%, and haircut = 2.00?
   A. 4.60%
   B. 4.80%
   C. 5.10%
   D. 5.40%
   E. 6.20%

   Solution:

   At time t, the trader borrows P_t - haircut = 98.5 – 2.00 = $96.5 from the repo dealer. The borrower adds $2 of his own money to this borrowing and purchases the bond for $98.5. He then deposits the bond as a collateral with the repo dealer.

   At time T, which is one month later, the trader takes the bond from the repo dealer and sells it in the market for $P_T = \$99.01$. From this, he has to repay the repo dealer the borrowing, P_t - haircut, with interest. The trader is left with {$P_T - (P_t - haircut) - repo$ interest}. In other words, the trader invests haircut at time t and gets back {$P_T - (P_t - haircut) - repo$ interest} at time T. The return is $[P_T - (P_t - haircut) - repo$ interest] –[haircut] = $P_T - P_t - repo$ interest.

   The return on capital = ($P_T - P_t - repo$ interest) / haircut

We have to find the repo interest to use the above formula to find the return on capital. The interest to be paid to the repo dealer = (P_t -haircut) × (repo rate × $\frac{n}{360}$) = (98.5 – 2) × (0.05 × $\frac{30}{360}$) = \$0.4021.

The return on capital = (99.01 – 98.5 – 0.4021) / 2 = 0.054 = 5.4% (this is the monthly return)

3. What is the profit for a trader who entered into a one-week reverse repo where $P_t = 99.40$, $P_T = 99.48$, and repo rate = 6%?

A. 0.0339
B. 0.0341
C. 0.0343
D. 0.0355
E. 0.0359

Solution:

In a reverse repo transaction, at time t the trader borrows a bond from the repo dealer and short sells it at the market price of P_t. He then deposits this money, P_t, as a collateral with the repo dealer. At time T, the trader buys back the bond from the market at the market price of P_T and gives it back to the repo dealer. The repo dealer in turn returns the money deposited by the trader as collateral, P_t, and also pays interest on this deposit. (Note that in a repo transaction, the trader pays interest to the repo dealer. However, in a reverse repo transaction, the repo dealer pays interest to the trader.)

At time t, there is no cash flow accruing to the trader because what he gets from the short sale is deposited with the repo dealer.

At time T, the trader pays P_T to buy back the bond from the market but gets P_t with interest from the repo dealer. (The reverse repo interest = $P_t \times \frac{n}{360}$)

The profit of the trader = P_t + reverse repo interest - P_T

= 99.40 + (99.40 × $\frac{7}{360}$) – 99.48 = \$0.0359

4. Discount factors depend on compounding frequency?
   A. True
   B. False

   Solution:

   No, discount factors do not depend on compounding frequency. Discount factors are the most basic entities that reflect time value of money. A discount factor tells you how much a dollar to be received at a specific time in the future is worth today. From this, we can express the associated interest rate (spot rate) in terms of any compounding frequency.

5. The higher the inflation, the __________ the discount factors.
   A. higher
   B. lower
   C. cannot say

   Solution:

   Interest rates reflect the expected rate of inflation. If the inflation is expected to be high, lenders will demand a high interest rate to protect the purchasing power of their money.

6. Answer questions 6 to 10 using the following information: the price of zero coupon bond maturing in 6 months is $P_Z(0,0.5)=99.20,$ the price of a coupon bond paying 3% quarterly and maturing in 3 months is $P(0,0.25)=100.5485,$ the price of a coupon bond paying 6% quarterly and maturing in 9 months is $P(0,0.75)=103.1655,$ and the price of a coupon bond paying 5% semiannually and maturing in 1 year is $P(0,1)=103.0325$.
   What is $Z(0,0.50)$?
   A. 0.9900
   B. 0.9910
   C. 0.9920
   D. 0.9930
   E. 0.9940

   Solution:

   We are given that $P_Z(0,0.5)=99.20.$ The subscript z indicates that this is the price of a zero coupon bond. This zero coupon bond with 0.50 years to maturity has a price of $99.20. This means that the face value of $100 to be received after 0.50 years is worth $99.20 now. Therefore, $100\times Z(0,0.50)=99.20.$ *Hence,* $Z(0,0.50)=99.20/100=0.9920.$

7. What is $Z(0,0.75)$?

A. 0.9850
B. 0.9860
C. 0.9870
D. 0.9880
E. 0.9890

Solution:

First, we have to find the 0.25-year discount factor, $Z(0,0.25)$. *It is given that the price of 3% quarterly bond maturing in 0.25 years,* $P(0,0.25)$, *is $100.5485. The cash flows expected from this bond after 0.25 year are the coupon, 100 x 3%/4 = 0.75, and the face value of $100. Therefore,* $100.75 \times Z(0,0.25) = 100.5485$. *Hence,* $Z(0,0.25) = 0.9980$.

We will now use the information about the 6% quarterly coupon bond maturing in 0.75 year. The cash flows from this bond for the next three quarters are $1.50, $1.50, ($1.50 + $100). The price of this bond is:

$$P(0,0.75) = 103.1655 = 1.50 \times Z(0,0.25) + 1.50 \times Z(0,0.50) + 101.50 \times Z(0,0.75).$$

The only term not known in the above equation is $Z(0,0.75)$. *Solving, we get* $Z(0,0.75) = 0.9870$.

8. What is $Z(0,1)$?
A. 0.9780
B. 0.9790
C. 0.9800
D. 0.9810
E. 0.9820

Solution:

Proceed along the same lines as in the above question.

9. What is the price of a 1-year coupon bond paying 4% quarterly?
A. 102.0580
B. 102.8530
C. 103.1100
D. 103.0410
E. 104.2200

Solution:

You have all the discount factors needed to price this bond from questions 6, 7, and 8. Multiply the quarterly cash flows of $1, $1, $1, and $101 by their corresponding discount factors to arrive at the price of the bond.

10. What is the price of a 9-month coupon bond paying 5% semiannually?
    A. 101.9315
    B. 102.6120
    C. 102.9800
    D. 103.6625
    E. 103.8315

    Solution:

    The cash flows from this bond are $2.50 after 3 months and ($2.50 + $100) after 9 months. (Make sure you understand this. Also see question 4 of chapter 2 from your homework.) Use the $Z(0,0.25)$ *and* $Z(0,0.75)$ *discount factors to arrive the present value of these cash flows, which is the price of the bond.*

11. What is the price of a 5.75-year floating rate bond that pays a semiannual coupon (no spread). Use the following information: (i) The price of a 3% quarterly coupon with 3 months to maturity is $P(0,0.25)=100.0448$, (ii) 3 months ago the 6-month LIBOR was 3%.
    A. 99.7641
    B. 100.1244
    C. 100.3217
    D. 100.5198
    E. 100.7895

    Solution:

    Since it is a 5.75-year floating rate bond with semiannual payments, the next coupon will be after 0.25 year (make sure you understand this. Also see question 4 of chapter 2 from your homework). The next coupon, which was determined 3 months ago and which will be paid 3 months later (a total of 6 months) will be 3% / 2 x 100 = $1.50.

    After the coupon is paid, the price of the floating rate bond is its face value, $100. Therefore, the total value from the bond after 3 months is 1.50 + 100 = $101.50. The present value of this amount is the price of the floating rate bond.

    To find this present value, we need the 0.25-year discount factor. This can be determined from the price of the 3% quarterly coupon bond with 0.25 year to maturity. The coupon is 3% / 4 x 100 = $0.75. Therefore, the cash flow from this bond after 3 months is 0.75 + 100 (face value) =

\$100.75. The price of the bond is given by $100.75 \times Z(0, 0.25) = 100.0448$ *(this information is given above). Therefore,* $Z(0, 0.25) = 0.9930.$

Therefore, the price of the floating rate bond =
$101.50 \times Z(0, 0.25) = 101.50 \times 0.9930 = \$100.7895.$

12. What is the price of a 0.5-year floating rate bond that pays a quarterly coupon equal to the floating rate plus 1% spread. Use the following information: (i) $P_Z(0.0.25) = 99.80,$ (ii) The price of a 2% quarterly coupon bond with 6 months to maturity is $P(0, 0.5) = 100.3960$.
    A. 100.1875
    B. 100.2270
    C. 100.3315
    D. 100.4980
    E. 100.5625

    Solution:

    To price a floating rate bond with a spread, break down the cash flow from the bond into two parts: (i) cash flows from a simple floating rate bond with no spread, and (ii) cash flows from the spread.

    Since the floating rate bond has quarterly coupons and there is 0.50 year to maturity, we are at a coupon day. Therefore, the price of the simple floating rate bond is \$100.

    To compute the value of the spread, which is 1% / 4 x 100 = 0.25 every quarter until the maturity of the bond, we need the 0.25-year and 0.50-year discount factors. Since $P_Z(0.0.25) = 99.80,$

    $Z(0, 0.25) = 99.80 / 100 = 0.9980.$

    To find $Z(0, 0.50)$, *we will use the information about the 2% quarterly 0.50-year maturity bond.*

    $P(0, 0.5) = 100.3960 = 0.50 \times Z(0, 0.25) + (0.50 + 100) \times Z(0, 0.50)$

    The only unknown in the above equation is $Z(0, 0.50)$. *Solving, we get* $Z(0, 0.50) = 0.9940.$

    Now, we can find the present value of the spread. It is 0.25 x 0.9980 + 0.25 x 0.9940 = 0.4980.

    Therefore, the price of the floating rate bond, which is the sum of the two components, is 100+0.4980 = \$100.4980.

13. What is the price of a 0.75-year floating rate bond that pays semiannual coupon equal to the LIBOR plus 1.50% spread? Use the following information: (i) $P_Z(0,0.25) = 99.70,$ (ii) $P_Z(0,0.50) = 99.20,$ (iii) There is a coupon bond paying 3% quarterly $P(0,0.75) = 101.7380,$ (iv) the 6-month LIBOR 3 months ago was 5%
   A. 103.69
   B. 103.83
   C. 103.98
   D. 104.11
   E. 104.47

   Solution:

   First let us determine the discount factors:

   Since the price of a zero coupon bond with maturity after 0.25 year is $P_Z(0,0.25) = 99.70,$ *the 0.25-year discount factor* $Z(0,0.25) = 0.9970.$ *Similarly, from the given information, we can determine the 0.50-year discount factor* $Z(0,0.50) = 0.9920.$

   To determine the 0.75-year discount factor, we will use the information about 0.75-year bond paying 3% quarterly coupon. The cash flows from the bond for the next three quarters are 0.75 (=3% / 4 x 100), 0.75, and 100.75 (=0.75 + 100). The price of the bond is given by the present value of these cash flows. That is,

$$101.7380 = [0.75 \times Z(0,0.25)] + [0.75 \times Z(0,0.50)] + [100.75 \times Z(0,0.75)]$$
$$= (0.75 \times 0.9970) + (0.75 \times 0.9920) + (100.75 \times Z(0,0.75)$$

   Solving for the 0.75-year discount factor from the above equation gives $Z(0.0.75) = 0.9950.$

   (Note: this discount factor is inconsistent with the other discount factors since the 0.75-year discount factor cannot be greater than the 0.50-year discount factor (0.9920). The given bond price of $101.7380 is too high and gives us this inconsistent discount factor. However, we will ignore this minor problem with the data and continue)

   We will treat the floating rate bond as having two components: (i) simple floating rate bond that makes (LIBOR/2 x face value) payments on coupon payment days plus the face value on maturity, and (ii) cash flow from spread on coupon payment days, that is (1.50%/2 x face value). Note that we use LIBOR/2 and 1.50%/2 because coupons are paid semiannually.

Look at the table below for the cash flows from the two components:

Time	*0.25 year*	*0.50 year*	*0.75 year*
Cash flow from simple floating rate bond	5% / 2 x 100 = $2.50 + Face Value		
Cash flow from the spread	1.50% /2 x 100 = $0.75		1.50% / 2 x 100 = $0.75

Let us first try to find the value of the simple floating rate bond. The cash flows from the simple floating rate bond have been shown only until 0.25 year. The cash flows for 0.75 year has not been shown. This is because once the coupon is paid, the price of a simple floating rate bond is equal to its face value ($100), which essentially reflects the futures cash flows from the simple floating rate bond. Therefore, since the face value of $100 is shown at 0.25 year, showing anymore cash flows from the bond will amount to double counting cash flows.

Today is not a coupon payment day since there is 0.75 year remaining and coupons are paid semiannually. The last coupon was paid 0.25 year ago and the next coupon will be paid, again, 0.25 year later. But 0.25 year later, immediately after the coupon is paid (last LIBOR / 2 x face value = 5% / 2 x 100 = $2.50), the value of the simple floating rate bond is equal to the face value, that is $100. Therefore, the value of the floating rate bond today

= ($2.50 + $100) x $Z(0,0.25) = 102.50 \times 0.9970 = \$102.1925$

What remains is to find the present value of the spread. It is

0.75 x Z(0, 0.25) + 0.75 x Z(0, 0.75) = 0.75 x 0.9970 + 0.75 x 0.9950 = $1.4940

For the remaining questions, use the following discount factors when necessary.

t	***0.25***	***0.50***	***0.75***	***1.00***	***1.25***	***1.50***	***1.75***	***2.00***
Z(0,t)	0.9840	0.9680	0.9520	0.9360	0.9190	0.9040	0.8880	0.8730
t	***2.25***	***2.50***	***2.75***	***3.00***	***3.25***	***3.50***	***3.75***	***4.00***
Z(0,t)	0.8587	0.8445	0.8308	0.8175	0.8047	0.7924	0.7806	0.7691

14. What is the duration of 2-year bond paying a fixed coupon of 5% quarterly?
    A. 1.91
    B. 1.98
    C. 2.27
    D. 2.83

E. 2.94

Solution:

Time	*0.25 year*	*0.50 year*	*0.75 year*	*1.00 year*	*1.25 years*	*1.50 years*	*1.75 years*	*2.00 years*
Cash flow (CF)	1.25	1.25	1.25	1.25	1.25	1.25	1.25	101.25
Discount factor	0.9840	0.9680	0.9520	0.9360	0.9190	0.9040	0.8880	0.8730
PV of CF	1.2300	1.2100	1.1900	1.1700	1.1488	1.1300	1.1100	88.3913
Weight	0.0127	0.0125	0.0123	0.0121	0.0119	0.0117	0.0115	0.9152

Note: The sum of PV of CF is $96.58. Using this, weights are found in the last column.

Multiply the time by weight and add the product to arrive at the duration, which is 1.91.

15. What is the duration of a 1.25-year floating rate bond that pays LIBOR + 50 bps semiannually? You know that the last quarter semiannual rate was 6.4%.
    A. 0.2411
    B. 0.2469
    C. 0.2532
    D. 0.2574
    E. 0.2633

Solution:

First, let us break down the cash flows from the floating rate bond into two parts: (i) cash flows from a simple floating rate bond, and (ii) cash flows from the spread.

Note that the cash flows from the simple floating rate bond have been shown only until 0.25 year. The cash flows for 0.75 year and 1.25 years have not been shown. This is because once the coupon is paid, the price of a simple floating rate bond is equal to its face value ($100), which essentially reflects the futures cash flows from the simple floating rate bond. Therefore, since the face value of $100 is shown at 0.25 year, showing anymore cash flows from the bond will amount to double counting cash flows.

Then calculate the present value of the total cash flows at each coupon date and add these present values. You will find it is $102.2626. This is the price of the floating rate bond.

Then find the weights.

Finally, multiply the time by weights and add the products. You will get 0.2532. This is the duration of the floating rate bond with spread.

The calculations are shown in the table below.

Time	*0.25 year*	*0.75 year*	*1.25 years*
Cash flow from simple floating rate bond	6.4%/2 x 100 = 3.20 + Face value ($100)		
Spread	0.005/2 x 100 = 0.25	0.005/2 x 100 = 0.25	0.005/2 x 100 = 0.25
Total cash flow	103.45	0.25	0.25
Discount factor	0.9840	0.9520	0.9190
PV of total cash flows	101.7948	0.2380	0.2298
Sum of PV of cash flows (price of floating rate bond)			Sum of above row = 102.2626
Weight	101.7948/102.2626 = 0.9954	0.2380/102.2626 = 0.0023	0.2298/102.2626 = 0.0022
Time x Weight	0.25 x 0.9954 = 0.2489	0.75 x 0.0023 = 0.0017	1.25 x 0.0022 = 0.0028
Duration			**0.2532**

16. What is the duration of the following portfolio?
    i. 5 units of a 2-year fixed rate bond paying 6% quarterly.
    ii. 2 units of a 1.75-year floating rate bond paying LIBOR + 80 bps semiannually. You know that the LIBOR was 6.5% three months ago.
    iii. 6 units of a 1-year zero coupon bond
    iv. 5 units of a 1.5-year floating rate bond with no spread paid semiannually.

    A. 0.98
    B. 1.02
    C. 1.06
    D. 1.10
    E. 1.14

Solution:

First, we calculate the price and duration of the 2-year fixed rate bond paying 6% quarterly.

Time	*0.25 yr*	*0.50 yr*	*0.75 yr*	*1.00 yr*	*1.25 yrs*	*1.50 yrs*	*1.75 yrs*	*2.00 yrs*
CF	1.50	1.50	1.50	1.50	1.50	1.50	1.50	101.50
Disc.Factor	0.9840	0.9680	0.9520	0.9360	0.9190	0.9040	0.8880	0.8730

PV of CF	1.476	1.452	1.428	1.404	1.3785	1.356	1.332	88.6095
Bond Price								**98.436**
Weight	0.0150	0.0148	0.0145	0.0143	0.0140	0.0138	0.0135	0.9002
Time x wt.	0.0037	0.0074	0.0109	0.0143	0.0175	0.0207	0.0237	1.8003
Duration								**1.8985**

Next, we calculate the price and duration of the 1.75-year floating rate bond paying LIBOR + 80bps semiannually.

Time	*0.25 year*	*0.75 year*	*1.25 year*	*1.75 year*
CF from simple floating bond	3.25 + 100 = 103.25			
Spread	0.40	0.40	0.40	0.40
Total CF	103.65	0.40	0.40	0.40
Disc. Factor	0.9840	0.9520	0.9190	0.8880
PV of CF	101.9916	0.3808	0.3676	0.3552
Price of fl.bond				**103.0952**
Weight	0.9893	0.0037	0.0036	0.0034
Time x wt.	0.2473	0.0028	0.0045	0.0060
Duration				**0.2606**

The price of the 1-year zero coupon bond is 100 x 0.9360 = $93.60. The duration, of course, is 1 year.

The price of the 1.5-year floating rate bond with no spread and the coupon paid semiannually is $100, the face value, because today is a coupon day (why?). The duration is 0.5 year (again, why?).

Using the prices of the four types of bonds and the number of units of each bond, we find that the value of the portfolio = (98.436 x 5) + (103.0952 x 2) + (93.60 x 6) + (100 x 5) = $1,759.97

The weights of the four bonds are 0.2796, 0.1171, 0.3191, and 0.2841.

The duration of the portfolio = (1.8985 x 0.2796) + (0.2606 x 0.1171) + (1 x 0.3191) + (0.50 x 0.2841) = 1.022.

17. What is the *dollar duration* of the following portfolio?
    i. Long 1.5-year zero coupon bond
    ii. Short 2-year fixed coupon bond paying 1% quarterly

    A. 39.15
    B. -39.45
    C. 39.75

D. -41.05
E. 41.85

Solution:

The price of the 1.5-year zero coupon bond = 100 x 0.9040 = $90.40. The duration of this bond is 1.50. Therefore, the dollar duration is 1.50 x 90.40 = 135.60.

You will find that the price of the 2-year, 1% quarterly bond is $89.156 and the duration is 1.981. (Try this; you have already been shown enough number of times how to solve problems like this.) The dollar duration is 1.981 x 89.156 = 176.6462.

The dollar duration of a portfolio is simply the sum of the dollar durations of the components of the portfolio. Make sure that you take into account the number of units of each component (here in this problem it is just 1 each), and those components that are shorted have a negative dollar duration.

Thus, the dollar duration of this portfolio = 135.60 – 176.6462 = -41.05.

18. What is the dollar duration of the following portfolio?
    i. Long 1-year fixed coupon bond paying 4% quarterly.
    ii. Long 1.75-year floating rate bond paying LIBOR plus 80 bps semiannually. You know that the reference rate was set at 6% three months ago.
    iii. Short a 2-year zero coupon bond.

    A. -49.73
    B. 50.05
    C. -50.26
    D. 51.47
    E. -51.82

Solution:

First calculate the price and duration of the 1-year bond paying 4% quarterly. You will find that the price is $97.44 and the duration is 0.9850. The dollar duration is 0.9850 x 97.44 = 95.98.

Then find the price and duration of the floating rate bond. The price is $102.849 and the duration is 0.2606. The dollar duration is 0.2606 x 102.849 = 26.80

Finally, the price of the 2-year zero coupon bond is 100 x 0.8730 = $87.30 and the duration is 2. The dollar duration is 2 x 87.30 = 174.60.

The dollar duration of the portfolio is 95.98 + 26.80 – 174.60 = -51.82.

19. What is the convexity of a 3-year fixed rate bond paying 4% coupon semiannually?
    A. 8.11

B. 8.20
C. 8.38
D. 8.73
E. 8.85

Solution:

The convexity of a fixed coupon paying bond is calculated just like the duration of the fixed coupon paying bond. We consider each coupon as a zero coupon bond, and the convexity of a zero coupon bond is the maturity-squared.

The table below shows the calculations.

Time	0.50 year	1 year	1.50 years	2 years	2.50 years	3 years
CF	2	2	2	2	2	102
Disc.Factor	0.9680	0.9360	0.9040	0.8730	0.8445	0.8175
PV of CF	1.936	1.872	1.808	1.746	1.689	83.385
Bond Price						**92.436**
Weight	0.0209	0.0203	0.0196	0.0189	0.0183	0.9021
Time-sq	0.25	1	2.25	4	6.25	9
Time-sq x weight	0.0052	0.0203	0.0440	0.0756	0.1142	8.1188
Convexity						**8.38**

20. What is the convexity of a 3-year floating rate bond with no spread paid quarterly?
    A. 0.25
    B. 0.0625
    C. Neither of the above

    Solution:

    For duration and convexity purposes, a simple floating rate bond with no spread is treated as a zero coupon bond with maturity equal to the time left until the next coupon. For the above floating rate bond, the time left until the next coupon is 0.25 year. Therefore, the duration is 0.25 and the convexity is 0.25 x 0.25 = 0.0625.

21. Calculate the convexity of the following portfolio.
    i. 2 units of a 1.5-year fixed rate bond paying 6% quarterly.
    ii. 4 units of a 1.75-year floating rate bond paying LIBOR + 80 bps semiannually. You know that the reference rate was 7% three months ago.
    iii. 6 units of a 2-year zero coupon bond.
    iv. 1 unit of a 1.5-year floating rate bond with no spread paid semiannually.

A. 1.50
B. 1.62
C. 1.87
D. 1.92
E. 2.07

Solution:

The price of 1.5-year fixed rate bond paying 6% quarterly is $98.89 and the convexity is 2.136.

The price of the 1.75-year floating rate bond paying LIBOR + 80 bps semiannually is $103.34 and the convexity is 0.08.

The price of the 2-year zero coupon bond is $87.30 and the convexity is 4.

The price of the 1.5-year floating rate bond with semiannual coupon and no spread is $100 and the convexity is 0.5 x 0.5 = 0.25.

The value of the portfolio = $1,234

Convexity of the portfolio is 2.07.

22. Consider the 5-year inverse floater $18\% - r_1(t-1)$. This inverse floater is equivalent to a portfolio of
   A. 1 long 5-year floating rate bond plus 1 short 5-year zero coupon bond plus 1 short 5-year 18% coupon fixed rate bond
   B. 1 long 5-year zero coupon bond plus 1 long 5-year 18% coupon bond plus 1 short 5-year floating rate bond
   C. 1 long 5-year 18% coupon bond plus 1 short 5-year zero coupon bond plus 1 short 5-year floating rate bond
   D. 1 long 5-year 18% coupon bond plus 1 long 5-year floating rate bond plus 1 short 5-year zero coupon bond
   E. None of the above

   Solution:

   See your class notes.

23. Consider the 4-year inverse floater $12\% - r_1(t-1)$. The coupons are paid annually. What is the duration of this inverse floater?
   A. 5.82

B. 6.08
C. 6.27
D. 6.49
E. 6.63

Solution:

The price of the 4-year, 12% annual coupon bond is $117.66 and the duration is 3.45.

The price of the 4-year zero coupon bond is $76.91 and the duration is 4 years.

The price of the 4-year floating rate bond is $100 and the duration is 1 year.

The price of the inverse floater is 117.66 + 76.91 – 100 = $94.57.

Find the weights of the three components of the inverse floater and calculate the weighted average of the durations. You will find it is 6.49. This is the duration of the inverse floater.

24. Consider the 4-year leveraged inverse floater $30\% - 3 \times r_1(t-1)$. The coupons are paid annually. What is the duration of this leveraged inverse floater?
A. 10.454
B. 10.653
C. 10.746
D. 10.852
E. 10.957

Solution:

The given leveraged inverse floater can be broken down as:

3 long 4-year zero coupon bond + 1 long 4-year 30% annual coupon bond – 3 short 4-year floating rate bond

Proceed along the same lines as in the above question, but you will have to take into account the number of bonds of each type to get the correct weights. You will find that the duration of the leveraged inverse floater is 10.746.

See the spreadsheet "Assignment 1 Solutions – Questions 24, 26-28 and 32-35."

25. Consider the Nelson-Siegel model for modeling the continuously compounded spot rates.

$$r(0,T) = \theta_0 + \theta_1 \left(\frac{1 - \exp(-\frac{T}{\lambda})}{\frac{T}{\lambda}} \right) + \theta_2 \left(\frac{1 - \exp(-\frac{T}{\lambda})}{\frac{T}{\lambda}} - \exp(-\frac{T}{\lambda}) \right)$$

$$= \theta_0 + (\theta_1 + \theta_2) \left(\frac{1 - \exp(-\frac{T}{\lambda})}{\frac{T}{\lambda}} \right) - \theta_2 \left(\exp(-\frac{T}{\lambda}) \right)$$

Which of the following statements true?

i. The short-term spot rate is given by θ_0.

ii. The long-term spot rate is given by θ_1.

A. i only
B. ii only
C. Both of the above
D. None of the above

Solution:

θ_0 *is the long-term spot rate.*

θ_1 *is the difference between the short-term spot rate and the long-term spot rate. To put it differently,* θ_0 + θ_1 *= short-term spot rate.*

θ_2 *has no influence on either the short-term spot rate or the long-term spot rate and affects the term structure only at medium term spot rates. Roughly speaking, when* $\theta_2 > 0$ *the term structure is concave, and when* $\theta_2 < 0$ *the term structure is convex.*

In the Nelson-Siegel model, $\lambda > 0$, *always. If the term structure has a hump,* λ *indicates roughly at what maturity the hump occurs. Large values of* λ *produce an upward sloping term structure, low values of* λ *produce a humpy term structure.*

26. Assume annual coupons for the following bonds.

	Annual Coupon(%)	Maturity (Years)	Price
Bond 1	8	1	105
Bond 2	7	2	103
Bond 3	5	3	101

Bond 4	8	4	108
Bond 5	7	5	100

Derive the spot curve until 5-year maturity. Use matrix multiplication and matrix inverse in Excel.

What is the 1-year <u>discount</u> factor?

(Knowing the discount factors is just like knowing the spot rates; one can be converted to the other very easily)

A. 0.9841

B. 0.9722

C. 0.9665

D. 0.9438

E. None of the above

Solution:

See the spreadsheet "Assignment 1 Solutions – Questions 24, 26-28 and 32-35."

27. What is the 3-year discount factor?

A. 0.8216

B. 0.8553

C. 0.8727

D. 0.8938

E. None of the above

Solution:

See the spreadsheet "Assignment 1 Solutions – Questions 24, 26-28 and 32-35."

28. What is the 5-year discount factor?

A. 0.7209

B. 0.7167

C. 0.7029

D. 0.6914

E. None of the above

Solution:

See the spreadsheet "Assignment 1 Solutions – Questions 24, 26-28 and 32-35."

29. A semi-annual 7% coupon bond has 3 years to maturity. The price of the bond is $943.31, the duration is 2.75, and the convexity is 7.96. What is the drop in the bond price when there is a parallel shift in the yield curve by 75 bps?
    A. $19.24
    B. $20.31
    C. $21.06
    D. $22.80
    E. $23.12

    Solution:

    Apply the formula $\frac{dP}{P} = -D \times dr + \frac{1}{2} \times C \times dr^2$

    or, multiplying both sides by P, we have

$$dP = -D \times P \times dr + \frac{1}{2} \times C \times P \times dr^2$$

30. Calculate the annualized expected returns for a 30-year zero coupon bond when E[dr] = 0 and E[dr^2] = 7 x 10^{-7} (on a daily basis) .
    A. 5.93%
    B. 6.52%
    C. 7.94%
    D. 8.12%
    E. 9.38%

    Solution:

    We know that:

$$\frac{dP}{P} = -D \times dr + \frac{1}{2} \times C \times dr^2$$

    Applying the expectations operator to the above equation:

$$E\left(\frac{dP}{P}\right) = -D \times E(dr) + \frac{1}{2} \times C \times E(dr^2)$$

    During a very short interval of time, like a day, the expected change in interest rates, $E(dr)$, is zero. $E(dr^2)$ in the above equation is the daily variance of interest rates (make sure you understand why).

    The duration of a 30-year zero coupon bond is 30 and the convexity is 30 x 30 = 900.

The daily expected return = $= -30 \times 0 + \frac{1}{2} \times 900 \times (7 \times 10^{-7}) = 0.000315$

The annualized return (assuming 252 trading days in a year) = 0.000315 x 252 = 0.0794 = 7.94%.

31. You currently hold a 7-year fixed rate bond paying 5% annually. You would like to hedge against changes in the level and slope of the yield curve by using 1-year and 7-year zero coupon bonds. How many 1-year and 7-year zero coupon bonds should be sold short? Make use of the table below.

maturity	β_1	β_2	$Z(t, T)$
1.00	1.1150	-0.2540	0.9800
2.00	0.9940	-0.3010	0.9600
3.00	0.9640	-0.1470	0.9300
4.00	0.9330	0.0080	0.8900
5.00	0.9300	0.1620	0.8500
6.00	0.9260	0.3160	0.8100
7.00	0.9270	0.4230	0.7700
8.00	0.9270	0.5300	0.7300

A. 0.4651 and 1.1231
B. 0.5112 and 1.1845
C. 0.6038 and 1.2249
D. 0.7422 and 1.3172
E. 0.7716 and 1.3961

Solution:

Step 1: Find the price of the 7-year, 5% annual coupon paying bond. It is $107.95.

Step 2: Find the weights of PV of cash flows every year: 0.0454, 0.0444, 0.0430, 0.0412, 0.0393, 0.0375, and 0.7489.

Step 3: Just as for ordinary duration, each coupon is treated as a zero coupon bond for finding factor durations. We will first find the factor duration of the bond with respect to the level factor. To do this, multiply the time of each coupon (this is the simple duration of the zero coupon bond) by the corresponding β_1 *coefficient and then multiply by the weight in Step 2, and then add all these products. You will find that the factor duration with respect to the level factor is 5.6689.*

Step 4: Repeat Step 3 with respect to the slope factor. You will find that the factor duration with respect to the slope factor is 2.2647.

Step 5: Using the discount factors, we know that the prices of the 1-year and 7-year zero coupon bonds are $98 and $77. Also find the factor durations of the 1-year and 7-year zero *coupon bonds. The factor durations of the 1-year zero coupon bond are 1.1150 and -0.2540. The factor durations of the 7-year zero coupon bond are 6.489 and 2.961.*

Step 6: Finally, make the factor durations of the level and slope factors equal to zero by equating their weighted averages to zero. This is just like immunizing a portfolio against changes in interest rates by equating the weighted averages of duration and convexity equal to zero. You will find that the number of 1-year zero coupon bonds to be shorted are 0.4651 and the number of 7-year zero coupon bonds to be shorted are 1.1231.

For questions 32 to 35, use the interest rates data in the Excel file "Factor Analysis – Data for Questions 32-35." Use this data and do factor analysis with respect to factors Level, Slope, and Curvature.

32. What is the coefficient of the 3-year rate with respect to the level factor?
    A. 0.9763
    B. 0.9812
    C. 0.9900
    D. 1.0063
    E. 1.0291

    Solution:

    See the spreadsheet "Assignment 1 Solutions – Questions 24, 26-28 and 32-35."

33. What is the coefficient of the 20-year rate with respect to the slope factor?
    A. -0.3903
    B. -0.4724
    C. 0.7335
    D. 0.9138
    E. None of the above

    Solution:

    See the spreadsheet "Assignment 1 Solutions – Questions 24, 26-28 and 32-35."

34. What is the coefficient of the 5-year rate with respect to the curvature factor?
    A. 0.1836
    B. 0.2112
    C. 0.3727
    D. -0.2764
    E. -0.2936

F. None of the above

Solution:

See the spreadsheet "Assignment 1 Solutions – Questions 24, 26-28 and 32-35."

35. Which of the interest rates has the smallest coefficient with respect to the level factor?
    A. 1-year rate
    B. 3-year rate
    C. 5-year rate
    D. 10-year rate
    E. 20-year rate

    Solution:

    See the spreadsheet "Assignment 1 Solutions – Questions 24, 26-28 and 32-35."