

CLASS: PHY \_\_\_\_\_

STUDENT #: \_\_\_\_\_

NAME: \_\_\_\_\_

## Assignment 8: Kinematics and Dynamics

Assigned: Nov 4: Due: Nov 11 6 PM

1. A dive bomber has a velocity of 280 m/s at an angle  $\theta$  below the horizontal. When the altitude of the aircraft is 2.15 km, it releases a bomb, which subsequently hits a target on the ground. The magnitude of the displacement from the point of release of the bomb to the target is 3.25 km. Find the angle  $\theta$ .

When the bomb has fallen a vertical distance 2.15 km, it has traveled a horizontal distance  $x_f$  given by

$$x_f = \sqrt{(3.25 \text{ km})^2 - (2.15 \text{ km})^2} = 2.437 \text{ km}$$

$$y_f = x_f \tan \theta - \frac{g x_f^2}{2 v_i^2 \cos^2 \theta_i}$$

$$-2150 \text{ m} = (2437 \text{ m}) \tan \theta_i - \frac{(9.8 \text{ m/s}^2)(2437 \text{ m})^2}{2(280 \text{ m/s})^2 \cos^2 \theta_i}$$

$$\therefore -2150 \text{ m} = (2437 \text{ m}) \tan \theta_i - (371.19 \text{ m})(1 + \tan^2 \theta_i)$$

$$\therefore \tan^2 \theta - 6.565 \tan \theta_i - 4.792 = 0$$

$$\therefore \tan \theta_i = \frac{1}{2} \left( 6.565 \pm \sqrt{(6.565)^2 - 4(1)(-4.792)} \right) = 3.283 \pm 3.945.$$

Select the negative solution, since  $\theta_i$  is below the horizontal.

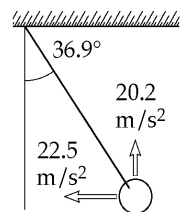
$$\therefore \tan \theta_i = -0.662, \quad \boxed{\theta_i = -33.5^\circ}$$

2. A ball swings in a vertical circle at the end of a rope 1.50 m long. When the ball is  $36.9^\circ$  past the lowest point on its way up, its total acceleration is  $(-22.5\hat{i} + 20.2\hat{j}) \text{ m/s}^2$ . At that instant, (a) sketch a vector diagram showing the components of its acceleration, (b) determine the magnitude of its radial acceleration, and (c) determine the magnitude and direction of the velocity of the ball. (with respect to horizontal)

- (a) See figure to the right.
- (b) The components of the 20.2 and the  $22.5 \text{ m/s}^2$  along the rope together constitute the centripetal acceleration:

$$a_c = (22.5 \text{ m/s}^2) \cos(90.0^\circ - 36.9^\circ) + (20.2 \text{ m/s}^2) \cos 36.9^\circ = \boxed{29.7 \text{ m/s}^2}$$

- (c)  $a_c = \frac{v^2}{r}$  so  $v = \sqrt{a_c r} = \sqrt{29.7 \text{ m/s}^2 (1.50 \text{ m})} = 6.67 \text{ m/s}$  tangent to circle
- $\mathbf{v} = \boxed{6.67 \text{ m/s at } 36.9^\circ \text{ above the horizontal}}$



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3A bag of cement of weight of  $F_g$  hangs from three wires as shown. Two of the wires make angles  $\theta_1$  and  $\theta_2$  with the horizontal. If the system is in equilibrium, show that the tension in the left-hand wire is  $\cos \theta_2 / \sin(\theta_1 + \theta_2)$

$$T_3 = F_g \quad (1)$$

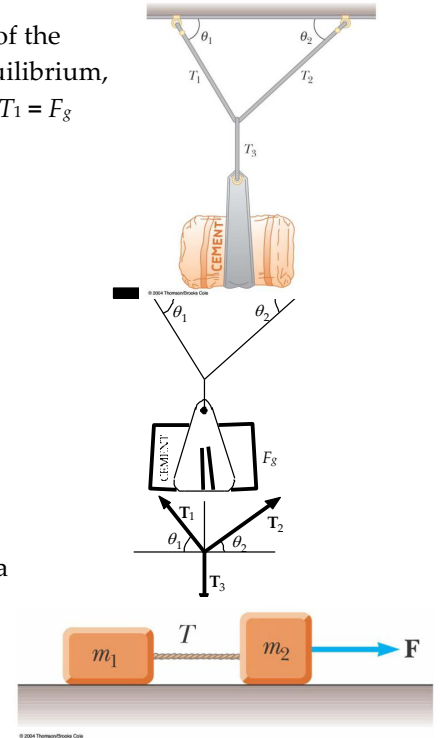
$$T_1 \sin \theta_1 + T_2 \sin \theta_2 = F_g \quad (2)$$

$$T_1 \cos \theta_1 = T_2 \cos \theta_2 \quad (3)$$

Eliminate  $T_2$  and solve for  $T_1$

$$T_1 = \frac{F_g \cos \theta_2}{(\sin \theta_1 \cos \theta_2 + \cos \theta_1 \sin \theta_2)} = \frac{F_g \cos \theta_2}{\sin(\theta_1 + \theta_2)}$$

- 4 Two blocks connected by a rope of negligible mass are being dragged by a horizontal force  $F$  (Fig. P5.45). Suppose that  $F = 68.0 \text{ N}$ ,  $m_1 = 12.0 \text{ kg}$ ,  $m_2 = 18.0 \text{ kg}$ , and the coefficient of kinetic friction between each block and the surface is 0.100. (a) Draw a free-body diagram for each block. (b) Determine the tension  $T$  and the magnitude of the acceleration of the system.



See Figure to the right

$$68.0 - T - \mu m_2 g = m_2 a \quad (\text{Block \#2})$$

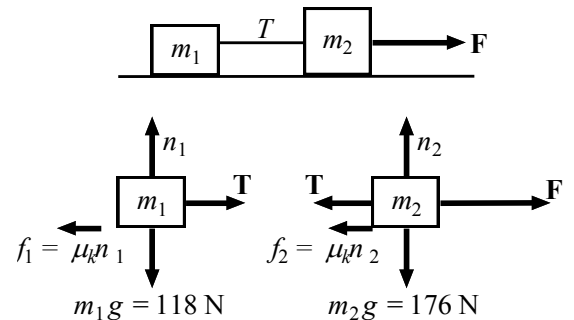
$$T - \mu m_1 g = m_1 a \quad (\text{Block \#1})$$

Adding,

$$68.0 - \mu(m_1 + m_2)g = (m_1 + m_2)a$$

$$a = \frac{68.0}{(m_1 + m_2)} - \mu g = \boxed{1.29 \text{ m/s}^2}$$

$$T = m_1 a + \mu m_1 g = \boxed{27.2 \text{ N}}$$



- 5 A crate of weight  $F_g$  is pushed by a force  $P$  on a horizontal floor. (a) If the coefficient of static friction is  $\mu_s$  and  $P$  is directed at angle  $\theta$  below the horizontal, show that the minimum value of  $P$  that will move the crate is given by

$$P = \frac{\mu_s F_g \sec \theta}{1 - \mu_s \tan \theta}$$

- (b) Find the minimum value of  $P$  that can produce motion when  $\mu_s = 0.400$ ,  $F_g = 100 \text{ N}$ , and  $\theta = 0^\circ, 15.0^\circ, 30.0^\circ, 45.0^\circ$ , and  $60.0^\circ$ .

The crate is in equilibrium, just before it starts to move.

Let the normal force acting on it be  $n$  and the friction force  $f_s$ .

Resolving vertically:  $n = F_g + P \sin \theta$

Resolving Horizontally:  $P \cos \theta = f_s$

But,  $f_s \leq \mu_s n$

i.e.,  $P \cos \theta \leq \mu_s (F_g + P \sin \theta)$

or  $P(\cos \theta - \mu_s \sin \theta) \leq \mu_s F_g$ .

Divide by  $\cos \theta$ :  $P(1 - \mu_s \tan \theta) \leq \mu_s F_g \sec \theta$ .

Then  $P_{\text{minimum}} = \frac{\mu_s F_g \sec \theta}{1 - \mu_s \tan \theta}$ .

(b)  $P = \frac{0.400(100 \text{ N}) \sec \theta}{1 - 0.400 \tan \theta}$

$\theta(\text{deg})$	0.00	15.0	30.0	45.0	60.0
$P(\text{N})$	40.0	46.4	60.1	94.3	260

If the angle were  $68.2^\circ$  or more, the expression for  $P$  would go to infinity and motion would become impossible.

$v \leq 14.3 \text{ m/s}$