STAT 3502 (Fall Term 2016)
Solutions to Assignment 1

1. (a) $P(A \cap B \cap C) = P(A)P(B)P(C) = (.7)(.6)(.8) = .336.$

   (b) $P(\text{at least one flight is full}) = 1 - P(\text{no flight is full}) = 1 - P(A')P(B')P(C')$
   $= 1 - (.3)(.4)(.2) = 1 - .024 = .976.$

   (c) $P(A' \cap B' \cap C) = P(A')P(B')P(C) = (.3)(.4)(.8) = .096.$

2. (a) $P(OO) = P(O)P(O) = (.43)^2 = .1849.$

   (b) $P(AA) + P(BB) + P(ABAB) + P(OO) = (.4)^2 + (.12)^2 + (.05)^2 + (.43)^2 = .3618.$

3. (a) $P(A \cap B) = P(A) + P(B) - P(A \cup B) = .8 + .7 - .9 = .6.$

   (b) The event of interest is $(A' \cap B) \cup (A \cap B') = (A \cup B) - (A \cap B)$ and its probability is $P[(A' \cap B) \cup (A \cap B')] = P(A \cup B) - P(A \cap B) = .9 - .6 = .3.$

4. (a) The four possible events are (1,1), (1,2), (2,1), and (2,2), with understanding that the first component corresponds to the health plan (1 or 2), and the second component corresponds to the dental plan (1 or 2).

   (b) $P(2,2) = .35$. (c) $P(1,2) + P(2,2) = .14 + .35 = .49.$

5. (a) We want the number of ways of selecting 3 individuals out of 20 individuals, order being irrelevant. This number is

$$\binom{20}{3} = \frac{20 \times 19 \times 18}{3 \times 2} = 20 \times 19 \times 3 = 1140.$$

   (b) No crew should not contain any of the best three machinists. That is all three-member crews have to be selected out of the worst 17. The number of ways to do this is

$$\binom{17}{3} = \frac{17 \times 16 \times 15}{3 \times 2} = 17 \times 8 \times 5 = 680.$$

   (c) Thinking of the complement, we just need to take away crews that have <u>all</u> members from the best 10 machinists. The number of interest = the number of all possible crews − the number of crews formed by the worst 10 machinists =

$$\binom{20}{3} - \binom{10}{3} = 1140 - \frac{10 \times 9 \times 7}{6} = 5 \times 3 \times 7 = 1140 - 105 = 1035.$$

6. (a) Let G_1= the customer uses regular unleaded gas, G_2= the customer uses extra unleaded gas, and G_3=the customer uses premium unleaded gas. Let A=the customer fills their tank. Then we have

$$P(G_2 \cap A) = P(A \cap G_2) = P(A|G_2)P(G_2) = (.6)(.35) = .21$$

(b) Using total probability formula, we get

$$P(A) = P(A|G_1)P(G_1)+P(A|G_2)P(G_2)+P(A|G_3)P(G_3) = (.3)(.4)+(.6)(.35)+(.5)(.25) = .455$$

(c) Using Bayes rule, we get

$$P(G_3|A) = \frac{P(A|G_3)P(G_3)}{P(A)} = \frac{(.5)(.25)}{.455} = .275.$$

7. P(old pump fails)P(new pump fails)=(.15)(.05)=.0075.

8. (a) $P(X \leq 3) = .1 + .15 + .2 + .25 = .7.$

(b) $P(X < 3) = P(X \leq 2) = .1 + .15 + .2 = .45.$

(c) $P(X \geq 3) = 1 - P(X < 3) = 1 - .45 = .55.$

(d) $P(2 \leq X \leq 5) = P(X \leq 5) - P(X \leq 1) = 1 - .25 = .75.$