# ECON4020

# Advanced Microeconomic Theory

# Answers to Problem Set Three

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**10.2** We use the following notations:

$\boldsymbol{p^0}$: Price vector in City 1

$\boldsymbol{p}'$: Price vector in City 2 (proposed change)

$m^{\boldsymbol{0}}$**:** Ellsworth's income in City 1

$m'$: Ellsworth's income if he moves to City 2

$\mathrm{u}^0$: Original utility level in City 1

$\mathrm{u}'$: The new utility level if move to City 2

The equivalent variation of moving to City 2 is

$EV = \mu(\boldsymbol{p^0}; \boldsymbol{p}', m') - \mu(\boldsymbol{p^0}; \boldsymbol{p^0}, m^0) = \mathrm{e}(\boldsymbol{p^0}, \mathrm{u}') - \mathrm{e}(\boldsymbol{p^0}, \mathrm{u}^0) = \mathrm{e}(\boldsymbol{p^0}, \mathrm{u}') - m^0.$

The compensating variation of moving to City 2 is

$CV = \mu(\boldsymbol{p}'; \boldsymbol{p}', m') - \mu(\boldsymbol{p}'; \boldsymbol{p^0}, m^0) = \mathrm{e}(\boldsymbol{p}', \mathrm{u}') - \mathrm{e}(\boldsymbol{p}', \mathrm{u}^0) = m' - \mathrm{e}(\boldsymbol{p}', \mathrm{u}^0).$

Given the utility function $u(x, y) = \min\{x, y\}$, the utility-maximizing solution has to satisfy $x = y$. Here, given $\boldsymbol{p^0} = (1,1)$, $m^0 = \$150$, we can solve for $x^0$ and $y^0$:

$$1 \cdot x^0 + 1 \cdot y^0 = 2x^0 = 2y^0 = \$150$$
$$\Longrightarrow x^0 = y^0 = u^0 = 75.$$

After moving to the other city where $\boldsymbol{p}' = (1,2)$, his consumption bundle would be $(x', y')$ such that

$$1 \cdot x' + 2 \cdot y' = 3x' = 3y' = \$150$$
$$\Longrightarrow x' = y' = u' = 50.$$

Hence,

$$EV = \mathrm{e}(\boldsymbol{p^0}, \mathrm{u}') - m^0 = 1 \cdot x' + 1 \cdot y' - 150 = -\$50.$$

That is, the move is equivalent to a wage cut of \$50, or, $A = 50$.

To maintain the original utility level at 75, Ellsworth needs to consume 75 units of $x$ and $y$ each.

$$CV = m' - \mathrm{e}(\boldsymbol{p}', \mathrm{u}^0) = 150 - (1 \cdot x^0 + 2 \cdot y^0) = 150 - 3 \cdot 75 = -\$75.$$

That is, he needs to get a raise of \$75, or, $B = 75$.

**13.4**

a) Each farmer solves the following profit maximization problem:

$$max_y\ py - c(y) = py - y^2.$$

From the first-order condition we find $p = c'(y) = 2y$. So the supply function of corn by each farmer is $y = p/2$.

b) The market supply is the total number of bushels supplied by all 100 identical farmers:

$$Y = n \cdot y = 100 \times \frac{p}{2} = 50p.$$

c) In equilibrium, market supply equals market demand:

$$D(p) = Y \Longrightarrow 200 - 50p = 50p \Longrightarrow p^* = 2.$$
$$Q^* = D(p^*) = 200 - 50 \times 2 = 100.$$

d) The rent on the land is the profit that a farmer earns from the land. When $p^* = 2$, each farmer's supply of corn is $y^* = p/2 = 1$. Therefore, the equilibrium rent on the land is:

$$p^* y^* - y^{*2} = 2 \times 1 - 1^2 = 1.$$

**13.8**

a) Aggregate supply of umbrellas is the sum of the supply for the representative U.K. and U.S. firm. Since each firm behaves competitively, the supply of each representative firm is determined by $p = c'(y) = y$. Then the aggregate supply is $Y = 2 \cdot y = 2p$

b) In equilibrium, market supply equals to market demand:

$$D(p) = Y \Longrightarrow 90 - p = 2p \Longrightarrow p^* = 30 \ \Longrightarrow Y^* = 60.$$

c) Let $p$ denote the price paid by domestic consumers. Because of the $3 tariff, the price received by the foreign firm is $p - 3$. The price received by the domestic firm is still $p$. Thus, the supply function of the U.K. firm becomes $y_{UK} = \ p - 3$, and the supply function of the U.S. firm is the same as before the tariff: $y_{US} = p$. The new market supply function is

$$Y = p + (p - 3) = 2p - 3.$$

In equilibrium:

$$D(p) = Y \Longrightarrow 90 - p = p + p - 3 \Longrightarrow p_t^* = 31.$$

That is, the price paid by the consumers is $31.

d) Domestic supply is $y_{US} = p_t^* = 31$ , and foreign supply is $y_{UK} = p_t^* - 3 = 28$.

**17.4**

This a general equilibrium question with two agents and two goods. One agent has Cobb-Douglas preference and the other has Leontief preference. First we calculate the Marshallian demand functions of these two agents.

Consumer A's utility maximization problem:

$$max_{x_A^1, x_A^2}\, u_A = a ln x_A^1 + (1 - a) ln x_A^2 \text{ subject to } p_1 x_A^1 + p_2 x_A^2 = m_A$$

where $m_A = p_1 \cdot 0 + p_2 \cdot 1 = p_2$. The Lagrange function is:

$$\mathcal{L}(\lambda, \boldsymbol{x_A}) = a ln x_A^1 + (1 - a) ln x_A^2 \ + \ \lambda\left(m_A - p_1 x_A^1 - p_2 x_A^2\right).$$

The first-order conditions are

$$\frac{\partial \mathcal{L}(\lambda, \boldsymbol{x_A})}{\partial x_1} = \frac{a}{x_A^1} - \lambda p_1 = 0 \Longrightarrow \frac{a}{x_A^1} = \lambda p_1 \quad (1)$$

$$\frac{\partial \mathcal{L}(\lambda, \boldsymbol{x_A})}{\partial x_2} = \frac{(1 - a)}{x_A^2} - \lambda p_2 = 0 \Longrightarrow \frac{(1 - a)}{x_A^2} = \lambda p_2 \quad (2)$$

$$\frac{\partial \mathcal{L}(\lambda, \boldsymbol{x_A})}{\partial \lambda} = m_A - p_1 x_A^1 - p_2 x_A^2 = 0.$$

(1) ÷ (2):

$$\frac{a\, x_A^2}{(1 - a) x_A^1} = \frac{p_1}{p_2} \Longrightarrow x_A^2 = \frac{(1 - a)}{a} \frac{p_1}{p_2} x_A^1. \quad (3)$$

Substitute (3) into the budget constraint:

$$p_1 x_A^1 + p_2 \frac{(1 - a)}{a} \frac{p_1}{p_2} x_A^1 = m_A \Longrightarrow \left[1 + \frac{(1 - a)}{a}\right] p_1 x_A^1 = m_A$$

$$\Longrightarrow x_A^1 = \frac{am_A}{p_1} = \frac{ap_2}{p_1} \qquad (4)$$

$$\Longrightarrow x_A^2 = \frac{(1-a)}{a}\frac{p_1}{p_2}\frac{am_A}{p_1} = \frac{(1-a)m_A}{p_2} = \frac{(1-a)p_2}{p_2} = 1-a \qquad (5)$$

Consumer B's utility maximization problem is:

$$max_{x_B^1,x_B^2}\, u_B = \min(x_B^1, x_B^2) \text{ subject to } p_1x_B^1 + p_2x_B^2 = m_B$$

where $m_B = p_1 \cdot 1 + p_2 \cdot 0 = p_1$. With a Leontief utility function, the utility-maximizing bundle must satisfy $x_B^1 = x_B^2$. Therefore, we have $x_B^1(p_1 + p_2) = m_B$

$$\Longrightarrow x_B^1 = x_B^2 = \frac{m_B}{p_1 + p_2} = \frac{p_1}{p_1 + p_2}.$$

In equilibrium, markets clear for good 1 and good 2:

$$x_A^1 + x_B^1 = {\omega_A}^1 + {\omega_B}^1 = 1$$
$$x_A^2 + x_B^2 = {\omega_A}^2 + {\omega_B}^2 = 1$$

That is,

$$\frac{ap_2}{p_1} + \frac{p_1}{p_1 + p_2} = 1$$
$$(1-a) + \frac{p_1}{p_1 + p_2} = 1$$

By Walras' law, we need to consider the market clearing condition of one good only. Let $p_1 = 1$ as a numeraire. Then we have

$$(1-a) + \frac{1}{1+p_2} = 1 \Longrightarrow \frac{1}{1+p_2} = a$$

$$\Longrightarrow \text{Equilibrium price: } p_2 = \frac{1}{a} - 1 = \frac{1-a}{a}.$$

Hence, the general equilibrium is described as below:

$$(x_A^1, x_A^2) = \left(ap_2, 1-a\right) = \left(a\frac{1-a}{a}, 1-a\right) = (1-a, 1-a)$$
$$(x_B^1, x_B^2) = (a, a)$$
$$(p_1, p_2) = \left(1, \frac{1-a}{a}\right), or\ equivalently, \frac{p_2}{p_1} = \frac{1-a}{a}.$$

**17.6** To keep the notations clear, we name the two consumers A and B, and rewrite their indirect utility functions as

$$v_A(p_1, p_2, y_A) = \ln y_A - alnp_1 - (1-a)lnp_2.$$
$$v_B(p_1, p_2, y_B) = \ln y_B - blnp_1 - (1-b)lnp_2.$$

We use the Roy's identity to derive the Marshallian demands of consumer A:

$$x_A^1 = -\frac{\partial v_A/\partial p_1}{\partial v_A/\partial y_A} = -\frac{-a/p_1}{1/y_A} = \frac{ay_A}{p_1}.$$

$$x_A^2 = -\frac{\partial v_A/\partial p_2}{\partial v_A/\partial y_A} = -\frac{-(1-a)/p_2}{1/y_A} = \frac{(1-a)y_A}{p_2}.$$

We use the same method to derive the Marshallian demands of consumer B:

$$x_B^1 = -\frac{\partial v_B/\partial p_1}{\partial v_B/\partial y_B} = -\frac{-b/p_1}{1/y_B} = \frac{b y_B}{p_1}.$$
$$x_B^2 = -\frac{\partial v_B/\partial p_2}{\partial v_B/\partial y_B} = -\frac{-(1-b)/p_2}{1/y_B} = \frac{(1-b) y_B}{p_2}.$$

The two consumers have the same endowments. Hence, $y_A = y_B = p_1 \times 1 + p_2 \times 1 = p_1 + p_2$.
The market clearing conditions

$$x_A^1 + x_B^1 = {\omega_A}^1 + {\omega_B}^1 = 1 + 1 = 2$$
$$x_A^2 + x_B^2 = {\omega_A}^2 + {\omega_B}^2 = 1 + 1 = 2.$$

By Walras' law, we need to consider only one of the two market clearing conditions. Let $p_1 = 1$ as a numeraire. Substitute $p_1 = 1$ and $y_A = y_B = p_1 + p_2$ into the market clearing condition of good 1:

$$x_A^1 + x_B^1 = \frac{a y_A}{p_1} + \frac{b y_B}{p_1} = (a+b)(1 + p_2) = 2.$$

$$\Rightarrow \text{The market clearing price: } p_2 = \frac{2}{a+b} - 1 = \frac{2-a-b}{a+b}.$$

**17.9** In a competitive equilibrium, each consumer maximizes his/her utility function, which implies the following condition for any consumer $i$:

$$\frac{\partial u_i/\partial x_i^1}{\partial u_i/\partial x_i^2} = \frac{p_1}{p_2}.$$

Applying this to consumer 3:

$$\frac{p_1}{p_2} = \frac{\partial u_3/\partial x_3^1}{\partial u_3/\partial x_3^2} = \frac{x_3^2}{x_3^1} = \frac{5}{10} = \frac{1}{2}.$$

**18.2** There are three goods in this economy, oil, butter and gun. By Walras' law, we need to consider the market clearing conditions of two goods only. Normalize the price of oil $p_x = 1$.
(a) Note that the production technologies exhibit constant returns to scale. Thus, the only price that is consistent with a competitive equilibrium is one that makes a firm's profit to be zero. For firm 1, this means

$$p_g \cdot 2x - x p_x = 0$$
$$\Rightarrow \text{Equilibrium price of gun: } p_g = {}^1/_2.$$

For firm 2,

$$p_b \cdot 3x - x p_x = 0$$
$$\Rightarrow \text{Equilibrium price of butter: } p_b = {}^1/_3.$$

(b) To find the equilibrium consumption of guns and butter, we olve the consumers' utility maximization problems. Since each firm makes zero profit in equilibrium, a consumer derives his income solely from his/her endowments. Hence, $m_1 = m_2 = p_x \times 10 = 10$.

Consumer 1's utility maximization problem:

$max_{g_1, b_1}\, u_1 = g_1^{.4} b_1^{.6}$ subject to $p_g g_1 + p_b b_1 = m_1$.

The Lagrange function is

$$\mathcal{L}(\lambda, g, b) = {g_1}^{.4} {b_1}^{.6} + \lambda\left(m_1 - p_g g_1 - p_b b_1\right)$$

The first-order conditions are

$$\frac{\partial\mathcal{L}}{\partial g_1} = 0.4{g_1}^{-.6}{b_1}^{.6} - \lambda p_g = 0 \Longrightarrow 0.4{g_1}^{-.6}{b_1}^{.6} = \lambda p_g \qquad (1)$$

$$\frac{\partial\mathcal{L}}{\partial b_1} = 0.6{g_1}^{.4}{b_1}^{-.4} - \lambda p_b = 0 \Longrightarrow 0.6{g_1}^{.4}{b_1}^{-.4} = \lambda p_b \qquad (2)$$

$$\frac{\partial\mathcal{L}(\lambda, g, b)}{\partial\lambda} = m_1 - p_g g_1 - p_b b_1 = 0 \qquad (3)$$

$(1) \div (2)\!: \Longrightarrow b_1 = \frac{p_g}{p_b}\frac{3}{2}g_1.$

Substitute the above condition into (3):

$$p_g g_1 + p_b \frac{p_g}{p_b}\frac{3}{2}g_1 = m_1.$$

Hence, $g_1 = \frac{2m_1}{5p_g} = \frac{20}{5 \cdot 1/2} = 8$, $b_1 = \frac{3m_1}{5p_b} = \frac{30}{5 \cdot 1/3} = 18.$

Similarly, consumer 2's utility maximization problem is:

$$max_{g_2,b_2}\, u_2 = 10 + .5lng_2 + .5lnb_2 \text{ subject to } p_g g_2 + p_b b_2 = m_2.$$

The Lagrange function is:

$$\mathcal{L}(\lambda, g_2, b_2) = 10 + .5lng_2 + .5lnb_2 \;+\; \lambda\big(m_2 - p_g g_2 - p_b b_2\big).$$

The first-order conditions are:

$$\frac{\partial\mathcal{L}}{\partial g_2} = \frac{0.5}{g_2} - \lambda p_g = 0 \Longrightarrow \frac{0.5}{g_2} = \lambda p_g \qquad (4)$$

$$\frac{\partial\mathcal{L}}{\partial b_2} = \frac{0.5}{b_2} - \lambda p_b = 0 \Longrightarrow \frac{0.5}{b_2} = \lambda p_b \qquad (5)$$

$$\frac{\partial\mathcal{L}}{\partial\lambda} = m_2 - p_g g_2 - p_b b_2 = 0 \qquad (6)$$

From (4) and (5), we obtain $b_2 = \frac{p_g}{p_b}g_2$. Substitute this into (6) to obtain consumer 2's consumption of guns:

$$p_g g_2 + p_b \frac{p_g}{p_b} g_2 = m_2 \Rightarrow g_2 = \frac{m_2}{2p_g} = \frac{10}{2 \cdot 1/2} = 10.$$

Consumer 2's consumption of butter is: $b_2 = \frac{m_2}{2p_b} = \frac{10}{2 \cdot 1/3} = 15.$

(c) The total number of guns produced and consumed in equilibrium: $g = g_1 + g_2 = 18$. Using the production function $g = 2x$, we find the amount of oil used by firm 1:

$$x_g = \frac{g}{2} = \frac{18}{2} = 9.$$

The total amount of butter produced and consumed in equilibrium: $b = b_1 + b_2 = 18 + 15 = 33$. Using the production function $b = 3x$, we find the amount of oil used by firm 2:

$$x_b = \frac{b}{3} = \frac{33}{3} = 11.$$

## Answer to the Additional Exercises

1. (a) First, we need to find the money metric indirect utility function $\mu(p;q,m)$. From the consumer's budget constraint, we obtain: $x_0 = m - px_1$. Using this to write the consumer's utility maximization problem as:

$$max_{x_1} u = ax_1 - x_1^2 + m - px_1.$$

The first-order condition is, $a - 2x_1 - p = 0$, which implies

$$x_1^* = \frac{a-p}{2}.$$

Substitute $x_1^*$ into the utility function to obtain the indirect utility function

$$v(p,m) = (a-p)x_1^* - x_1^{*2} + m = \frac{(a-p)^2}{4} + m.$$

Using that we find the expenditure function

$$e(p,u) = u - \frac{(a-p)^2}{4}.$$

Then the money metric indirect utility function is

$$\mu(p;q,m) = v(q,m) - \frac{(a-p)^2}{4} = m + \frac{(a-q)^2}{4} - \frac{(a-p)^2}{4}.$$

The compensating variation:

$$CV = m - \mu(p^1;p^0,m) = m - \left[m + \frac{(a-p^0)^2}{4} - \frac{(a-p^1)^2}{4}\right] = \frac{(a-p^1)^2}{4} - \frac{(a-p^0)^2}{4}.$$

The equivalent variation:

$$EV = \mu(p^0;p^1,m) - m = \left[m + \frac{(a-p^1)^2}{4} - \frac{(a-p^0)^2}{4}\right] - m = \frac{(a-p^1)^2}{4} - \frac{(a-p^0)^2}{4}.$$

Note that the consumer's income does not change (i.e., $m = m'$).

(b) From part (a), we see that $CV = EV$. The result arises because the utility function is quasi-linear.

2. The money metric indirect utility function is:

$$\mu(q_1,q_2;p_1,p_2,u) = 5\left\{\left[\frac{m}{5}\right]^{5/6}\left[\frac{3}{p_1}\right]^{1/2}\left[\frac{2}{p_2}\right]^{1/3}\right\}^{6/5}\left[\frac{q_1}{3}\right]^{3/5}\left[\frac{q_2}{2}\right]^{2/5} = m\left[\frac{q_1}{p_1}\right]^{3/5}\left[\frac{q_2}{p_2}\right]^{2/5}.$$

(a) Here $m = m' = 18$. The prices after the tax cut are $(p_1', p_2') = (0.9, 0.9)$. The compensating variation is

$$CV = 18 - 18\left[\frac{0.9}{1}\right]^{3/5}\left[\frac{0.9}{1}\right]^{2/5} = 1.8.$$

This number tells us that reducing the consumer's income by 1.8 units would compensate him for the tax cut.

(b) The equivalent variation is

$$EV = 18\left[\frac{1}{0.9}\right]^{3/5}\left[\frac{1}{0.9}\right]^{2/5} - 18 = 2.$$

This number says that the tax cut is equivalent to giving the consumer 2 units of additional income.

3. (a) First, we need to find the industry supply function. Note that there is a fixed cost $k$. Consequently, we need to compare MC and AVC. Since $MC = 2q$ and $AVC = q$, we have $MC \geq AVC$ for all $q \geq 0$. Thus, the entire supply function of a firm is determined by $p = MC = 2q$. Solving this equation to obtain the supply function of a firm: $q = p/2$. The industry supply function is

$$Q = Jq = \frac{Jp}{2}.$$

Solving the inverse demand function to obtain:

$$X = \frac{a-p}{b}.$$

Setting $X = Q$ to find the equilibrium price:

$$p^* = \frac{2a}{2+bJ}.$$

(b) With the tax, $p^d = p^s + t$. Then $X = Q$ implies

$$\frac{a-(p^s+t)}{b} = \frac{Jp^s}{2}.$$

Solving the above equation to obtain:

$$p^s = \frac{2a-2t}{2+bJ}.$$

$$p^d = p^s + t = \frac{2a+bJt}{2+bJ}.$$

4. (a) Crusoe Inc.'s (the firm) profit is

$$\pi = pq - wL = 2pL - wL.$$

Note that the production technology exhibits constant returns to scale. The only price that is consistent with the competitive equilibrium is the one that makes $\pi = 0$. Thus, $2p - w = 0$, which yields the equilibrium relative price:

$$\frac{p^*}{w^*} = \frac{1}{2}.$$

(b) Robinson's (the consumer) budget constraint is:

$$px + wR = 24w + \pi$$

Solving Robinson's utility maximization problem to obtain:

$$x = \frac{24w+\pi}{3p}; \quad R = \frac{2(24w+\pi)}{3w}.$$

Use the equilibrium price and note that $\pi = 0$,

$$x^* = \frac{24w^*}{3p^*} = 16; \quad R^* = 16.$$

Hours worked: $L^* = 24 - R^* = 8$.